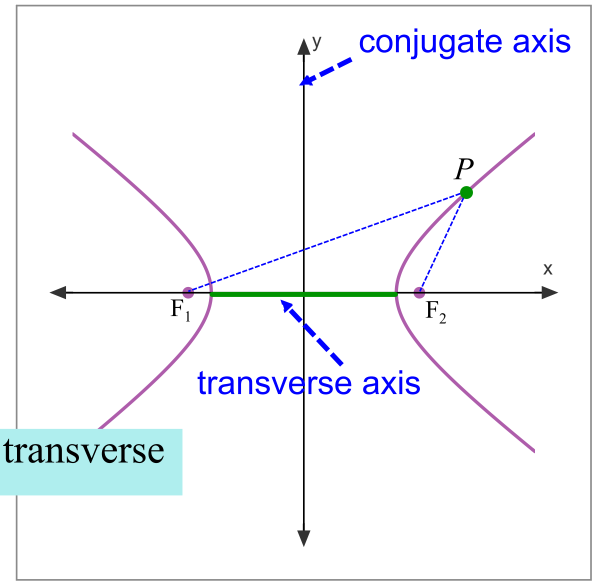
3) The Hyperbola

The locus of all points such that the difference of the distances from two fixed points is constant.

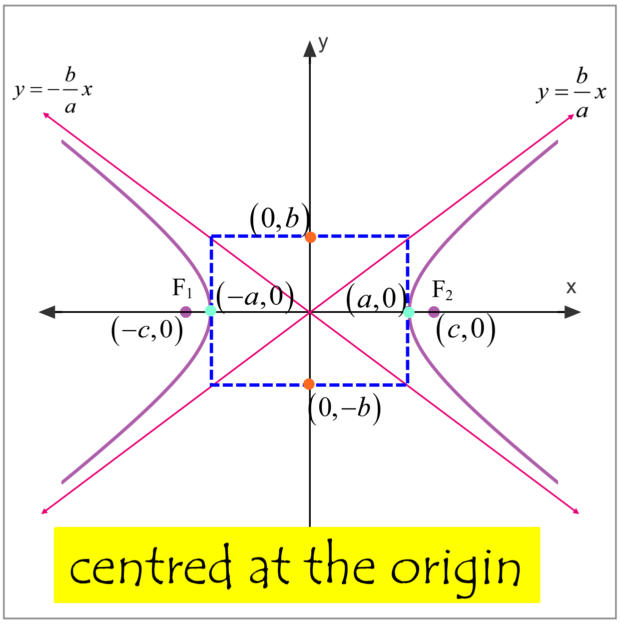
The hyperbola has two axes:

1) transverse axis - contains the vertices & foci
2) conjugate axis



$d(P,F_1) - d(P,F_2) = \text{constant} = \text{length of transverse}$

$2a$ or $2b$



There are ~~two~~ asymptotes

(a line the curve cannot touch) .

Equations: $y=\frac{b}{a}x$ and $y=-\frac{b}{a}x$

A hyperbola can be horizontal...

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Vertices and foci on the x -axis

vertices : $(a, 0)$ and $(-a, 0)$

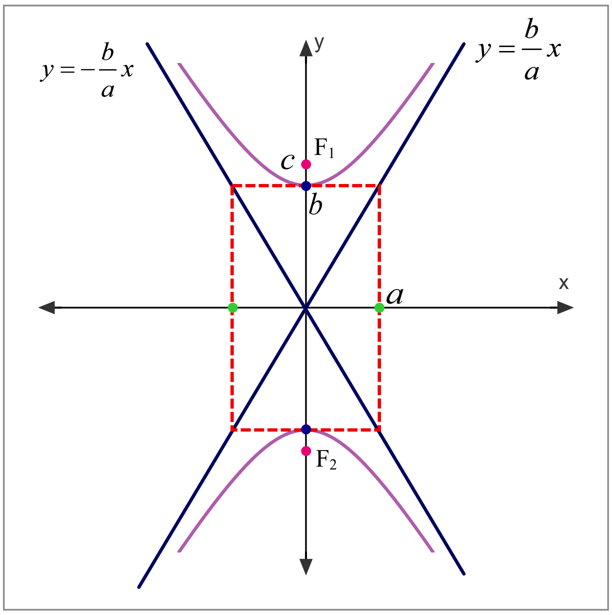
foci : $(c, 0)$ and $(-c, 0)$

or vertical

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

or

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



Vertices and foci on the y-axis

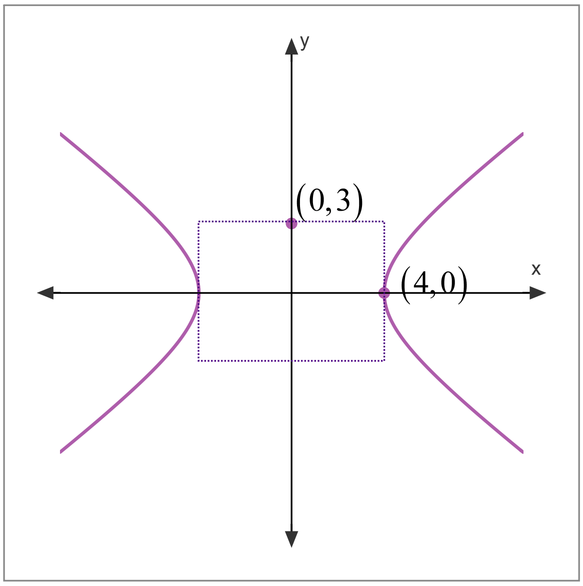
vertices : (0, b) and (0, −b)

foci : (0, c) and (0, −c)

Coordinates of the focus: $c^2 = a^2 + b^2$

(c is the distance from the centre to the focus)

Example:

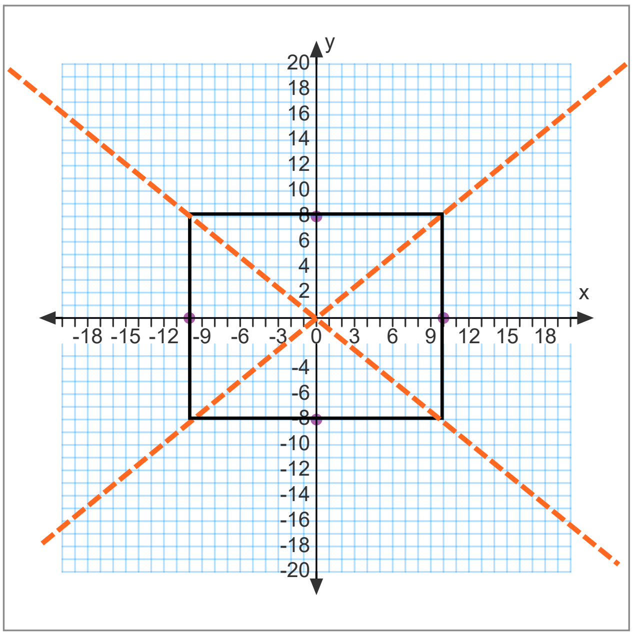


a) Find the coordinates of the foci.
b) Find the equation of the hyperbola.
c) Find the equations of the asymptotes.

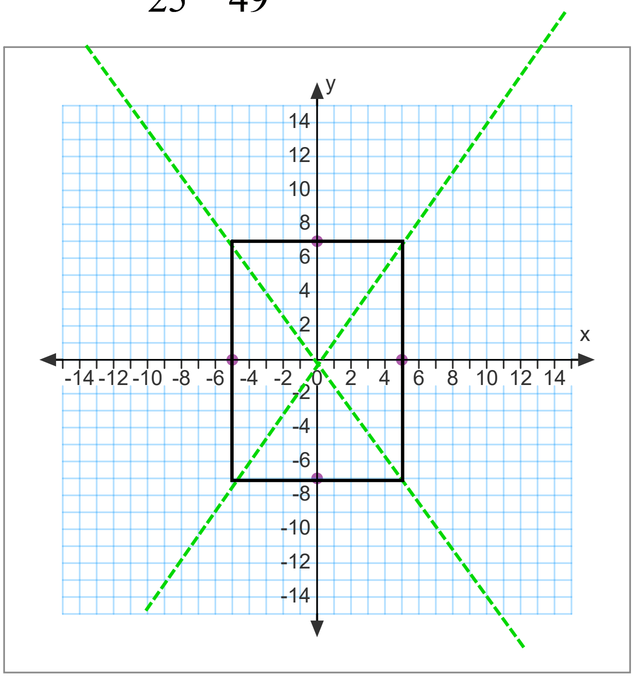
Example: a) Sketch each hyperbola, including the asymptotes.

b) Determine the focal distance for each hyperbola.

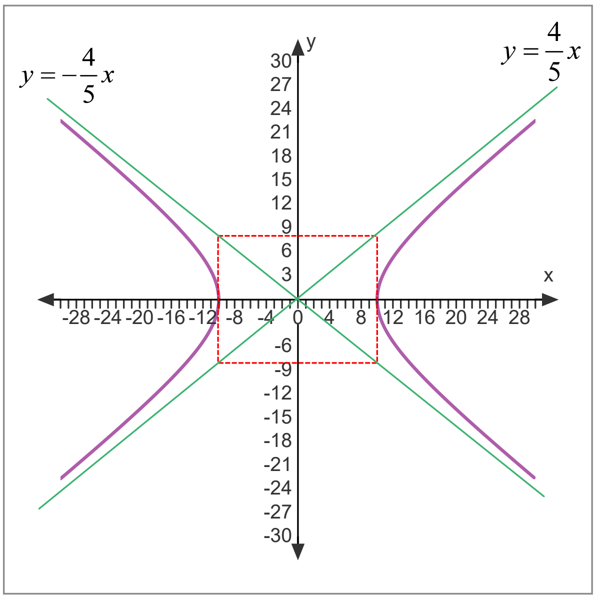
i) $\frac{x^2}{100} - \frac{y^2}{64} = 1$



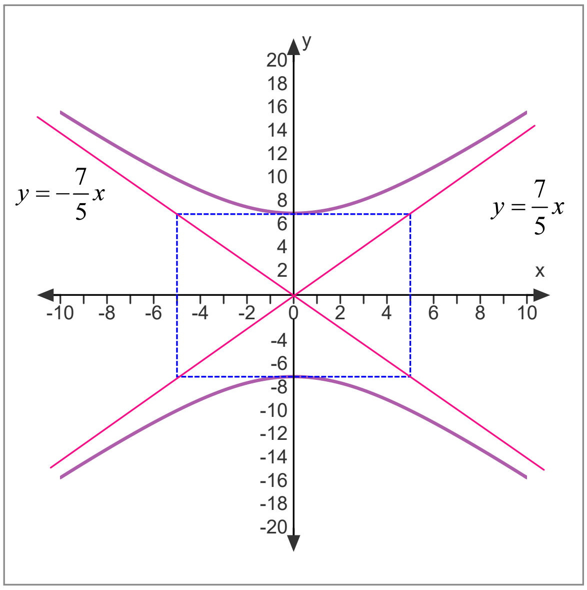
ii) $\frac{x^2}{25} - \frac{y^2}{49} = -1$



i)


$y=-\frac{4}{5}x$
$y=\frac{4}{5}x$
y
x
30
27
24
21
18
15
12
9
6
3
-6
-9
-12
-15
-18
-21
-24
-27
-30
-28 -24 -20 -16 -12 -8 -4 0 4 8 12 16 20 24 28

ii)


$y=-\frac{7}{5}x$
$y=\frac{7}{5}x$
y
x
20
18
16
14
12
10
8
6
4
2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-10 -8 -6 -4 -2 0 2 4 6 8 10

Standard to General Form

$$\frac{x^2}{49}-\frac{y^2}{9}=1$$

Standard form

$$441\left(\frac{x^2}{49}-\frac{y^2}{9}=1\right)$$

Multiply to get rid of the denominators

$$9x^2-49y^2=441$$

$$9x^2-49y^2-441=0$$

General Form

Convert $\frac{x^2}{4} - \frac{y^2}{25} = -1$ to general form.

General Form: $Ax^2 - By^2 \pm C = 0$ + C ⇒ vertical

- C ⇒ horizontal

General to Standard Form

$5x^2 - 40y^2 + 160 = 0$ general form

$5x^2 - 40y^2 = -160$ Make the RHS equal to ± 1

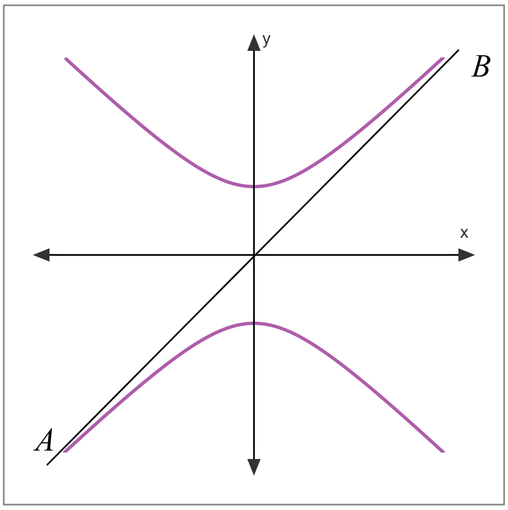
$\frac{5x^2}{160} - \frac{40y^2}{160} = -1$ Reduce the fractions

$\frac{x^2}{32} - \frac{y^2}{4} = -1$ Standard Form

Convert $9x^2 - 16y^2 - 144 = 0$ to standard form.

Ex: Determine the equation of the asymptotes of the hyperbola whose equation is $100x^2 - 441y^2 + 225 = 0$.

Ex: Given a hyperbola whose equation $6x^2 - 4y^2 + 48 = 0$, determine the equation of asymptote $\overline{AB}$.

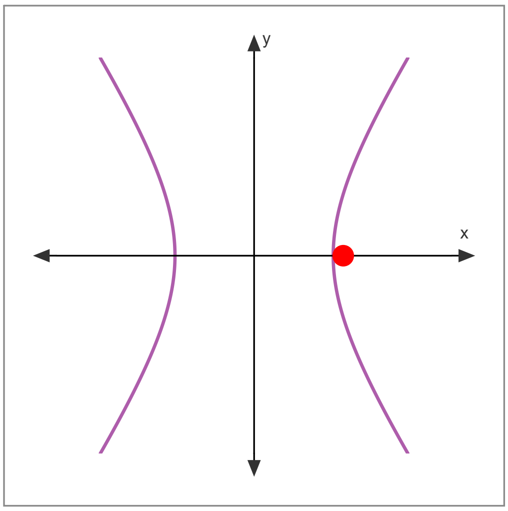


Application Problems

1) The lawn of a park covers a region whose edges follow the equation $\frac{x^2}{64} - \frac{y^2}{36} = 1$.

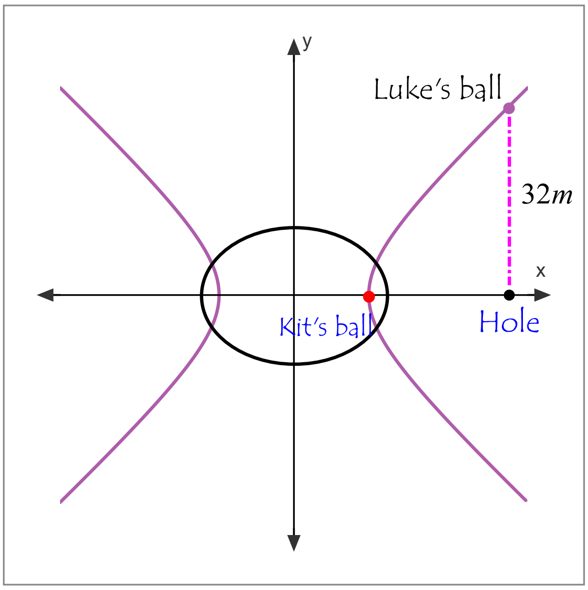
On the right branch, a circular flower bed is to be planted. The endpoints of the diameter of the circle coincide with the focus and vertex of the hyperbola.

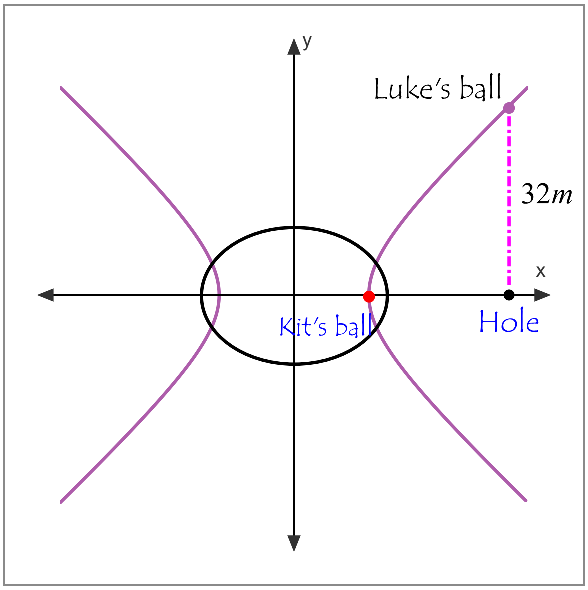
What is the equation of this circle?



2) On a golf course, Luke's ball and Kit's ball lie on a hyperbola. The sand trap is shaped like an ellipse whose equation is $9x^2 + 25y^2 - 3600 = 0$. Luke's ball is $32m$ from the hole. The vertex of the hyperbola coincides with the focus of the ellipse and vice versa.

How far from the hole is Kit's ball?

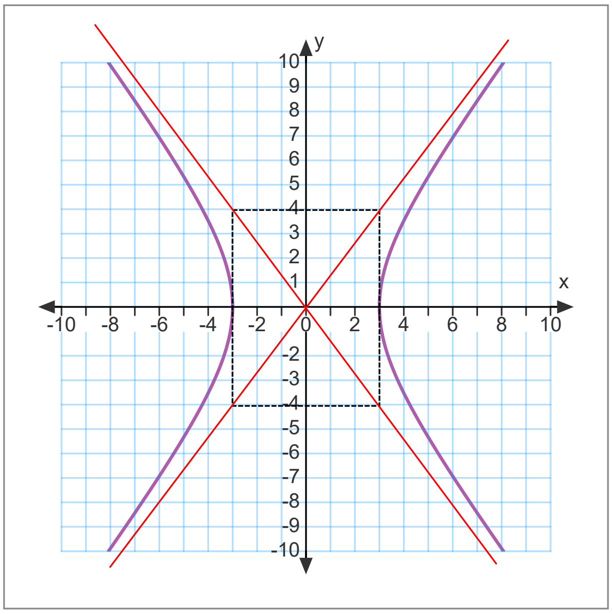



y
Luke's ball
32m
x
Kit's ball
Hole

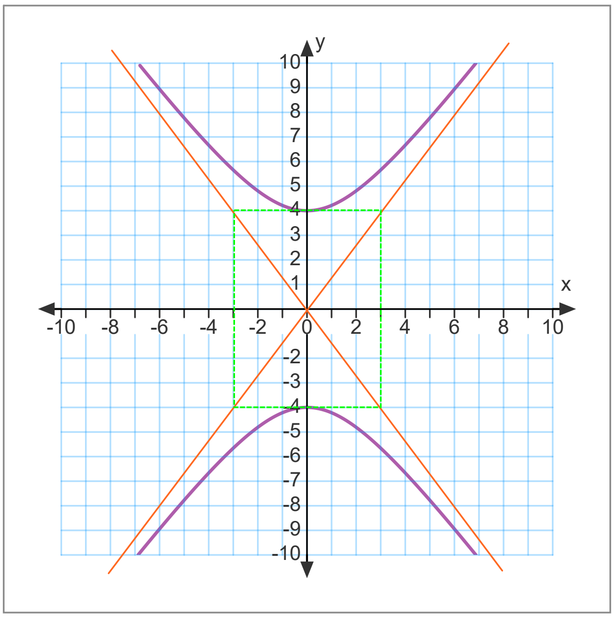
The Hyperbola and Inequalities

Examples:

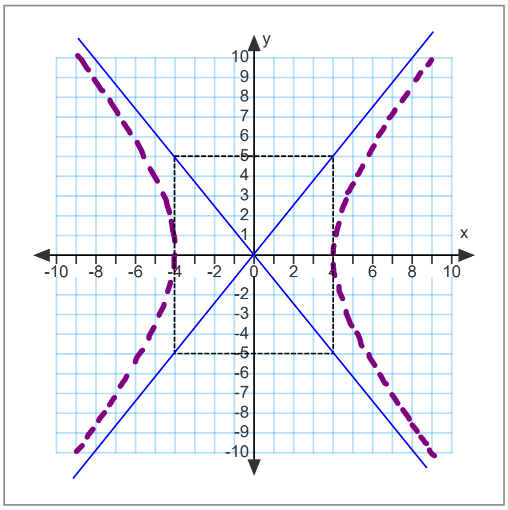
$$\frac{x^2}{9}-\frac{y^2}{16}\geq 1$$



$$\frac{x^2}{9}-\frac{y^2}{16}\geq -1$$



$$\frac{x^2}{16} - \frac{y^2}{25} < 1$$



$$\frac{x^2}{16} - \frac{y^2}{25} < -1$$