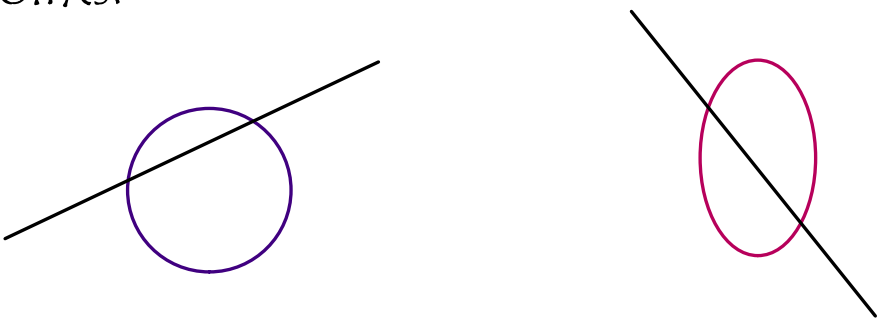
Secant Lines and Circles & Ellipses

A secant line is a line that intersects a curve at two or more points.



A semi-linear system is created. We can use an algebraic method (usually substitution) or a graph to determine the points of intersection.

Example: Determine the points of intersection between the line $y = x + 2$ and the circle $(x-1)^2 + (y-2)^2 = 4$.

By substitution: $(x-1)^2 + (x+2-2)^2 = 4$

$(x-1)^2 + (x)^2 = 4$

$x^2 - 2x + 1 + x^2 = 4$

$2x^2 - 2x + 1 = 4$

Solve $\longrightarrow$ $2x^2 - 2x - 3 = 0$

$$2x^2 - 2x - 3 = 0$$

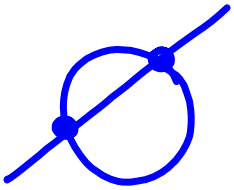
a b c

Factor

$$a \times c = 2 \times (-3) = -6$$

$$b = -2$$

$$\left.\begin{aligned} m \times n &= -6 \\ m + n &= -2 \end{aligned}\right\} \quad \begin{aligned} m &= \\ n &= \end{aligned}$$ ✗



Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$4-(-24)$

$$x = \frac{2 \pm \sqrt{4 - 4(2)(-3)}}{4} = \frac{2 \pm \sqrt{28}}{4}$$

$$x_1 = \frac{2 + \sqrt{28}}{4} \qquad x_2 = \frac{2 - \sqrt{28}}{4}$$

$$x_1 = \frac{2 + 2\sqrt{7}}{4} \qquad x_2 = \frac{2 - 2\sqrt{7}}{4}$$

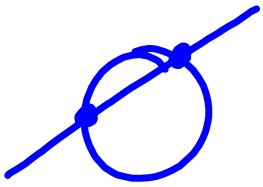
$$x_1 = \frac{1 + \sqrt{7}}{2} \approx 1.82 \qquad x_2 = \frac{1 - \sqrt{7}}{2} \approx -0.82$$

Find y for each x-value.

$$y = x + 2$$

$$y_1 = 1.82 + 2 \qquad y_2 = -0.82 + 2$$

$$y_1 = 3.82 \qquad y_2 = 1.18$$



Therefore, the line intersects the circle at points $(1.82, 3.82)$ & $(-0.82, 1.18)$.

Example: Determine the points of intersection between the line $y = -2x + 1$ and the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

Convert the ellipse to general form.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1 \longrightarrow 225 \times \left(\frac{x^2}{9} + \frac{y^2}{25} = 1 \right)$$

$$25x^2 + 9y^2 = 225$$

Substitute $y = -2x + 1$. $\longrightarrow$ $25x^2 + 9(-2x + 1)^2 = 225$

$$25x^2 + 9(-2x+1)^2 = 225$$

$$25x^2 + 9(4x^2 - 4x + 1) = 225$$

$$25x^2 + 36x^2 - 36x + 9 = 225$$

$$61x^2 - 36x + 9 = 225$$

$$61x^2 - 36x - 216 = 0$$

$$x = \frac{36 \pm \sqrt{(-36)^2 - 4(61)(-216)}}{122}$$

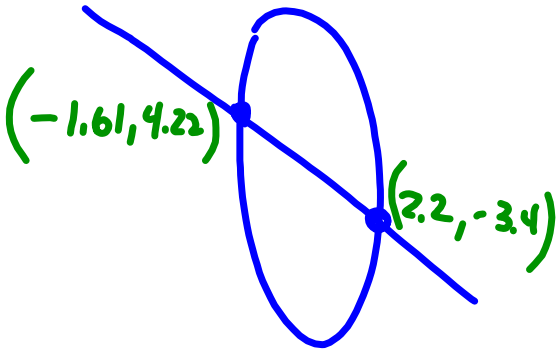
$$x = \frac{36 \pm \sqrt{54000}}{122}$$

$$x = \frac{36 \pm \sqrt{54000}}{122} \quad \begin{cases} x_1 = \dfrac{36 + 232.38}{122} = \dfrac{268.38}{122} = 2.2 \\ x_2 = \dfrac{36 - 232.38}{122} = \dfrac{-196.38}{122} = -1.61 \end{cases}$$

Determine the y-value for each x.

$$y = -2x + 1 \quad \begin{cases} y_1 = -2(2.2) + 1 = -3.4 \\ y_2 = -2(-1.61) + 1 = 4.22 \end{cases}$$

The line and ellipse intersect at points $(2.2,-3.4)$ and $(-1.61,4.22)$.



Determine the points of intersection between ...

a) the line $y = 3x - 12$ and the circle $(x-2)^2 + (y+6)^2 = 90$.

b) the line $y = \frac{1}{2}x + 4$ and the ellipse $\frac{x^2}{20} + \frac{y^2}{35} = 1$.