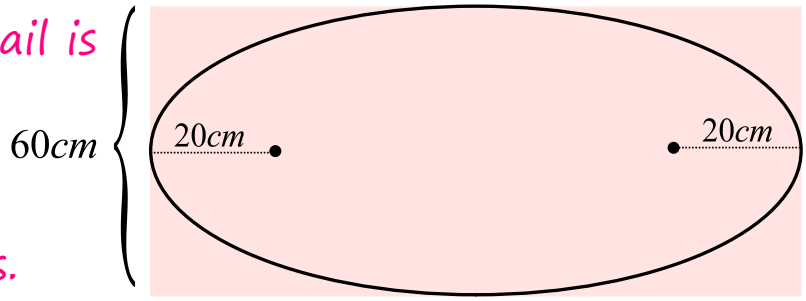
7) A carpenter places two nails on a rectangular piece of wood. Each nail is placed $20\ cm$ from each of the shorter edges.



These shorter edges are $60\ cm$ long.

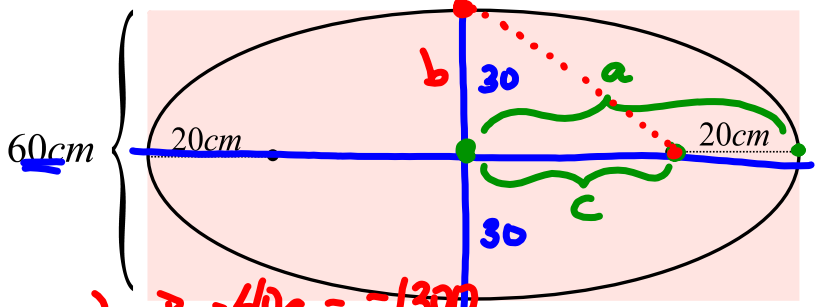
The carpenter uses these nails as focal points to draw the largest possible ellipse on the rectangular plane.

a) Determine the equation of the ellipse.

$b = 30$

$c = a - 20$

$a = c + 20$



or $c^2 = a^2 - b^2$

$b^2 + c^2 = a^2$

$900 + (a-20)^2 = a^2$

$900 + (a^2 - 40a + 400) = a^2$

$a^2 - 40a + 1300 = a^2$

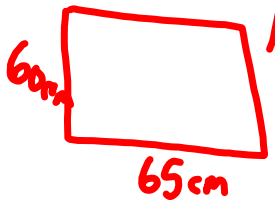
$-40a + 1300 = 0$

$-40a = -1300$

$a = 32.5$

$\frac{x^2}{1056.25} + \frac{y^2}{900} = 1$

b) Determine the area of the piece of plywood.



$A = 60 \times 65$

$= 3900 cm^2$

c) If the area of an ellipse is $A = \pi ab$, then how much wood is unused?

Ellipse: $A = \pi \cdot (32.5)(30) = 3063.05\ cm^2$

Unused = $3900\ cm^2 - 3063.05\ cm^2$

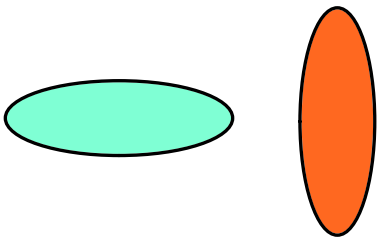
$836.95\ cm^2$

Determine the centre of the following ellipse:

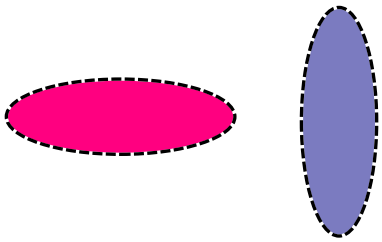
$$4x^2 + 49y^2 - 16x + 294y + 261 = 0$$

Ellipses and Inequalities

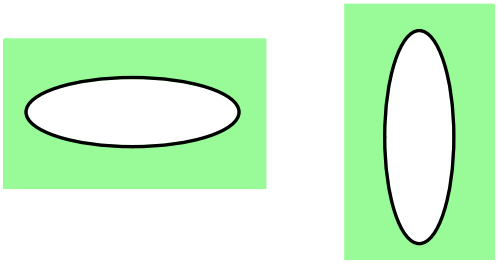
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}< 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}> 1$$