

## Greatest Integer Function (GIF)

Greatest Integer Notation:  $[7]$ ,  $[-3]$ ,  $[4.2]$

(sometimes  $\llbracket -16 \rrbracket$ ).

To evaluate, drop the brackets and replace the real number with the greatest integer less than or equal to the number.



$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$   
no fractions

Examples:

$$[9] = 9$$

$$[-8] = -8$$

$$[6.8] = 6$$

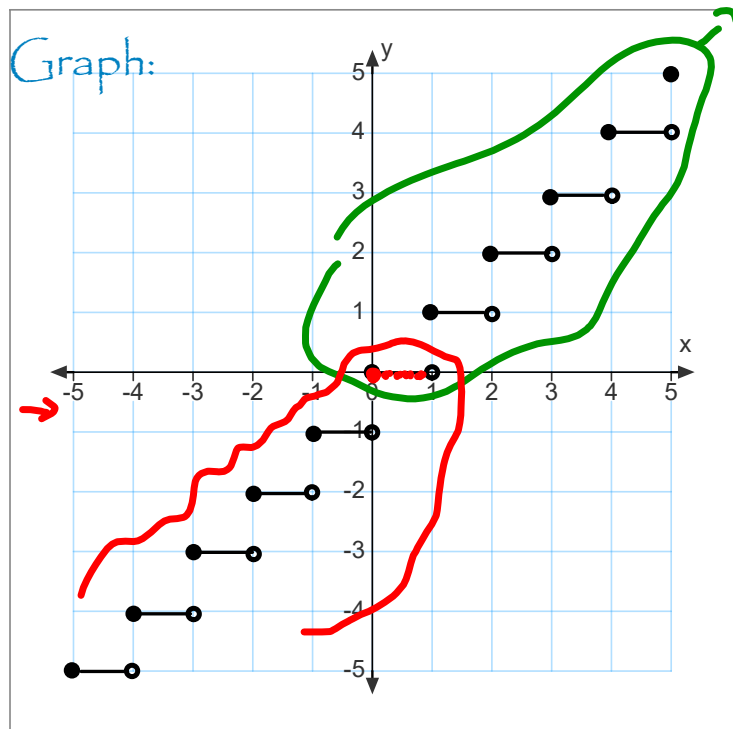
$$[0.953] = 0$$

$$[-3.5] = -4$$

$$[-0.224] = -1$$

# Basic GIF

Rule:  $f(x) = [x]$



**Dom:**  $\mathbb{R}$

**Ran:**  $\mathbb{Z}$  *Integers*

**Inc:**  $\mathbb{R}$

**Dec:** *Never*

**Pos:**  $[0, +\infty[$

**Neg:**  $]-\infty, 1[$

**Max:** *None*

**Min:** *None*

**y-intercept:** *0*

**x-intercept:**  $[0, 1[$



Recall that in a basic function...

Parameters  $a$  and  $b$  are both equal to 1  
and

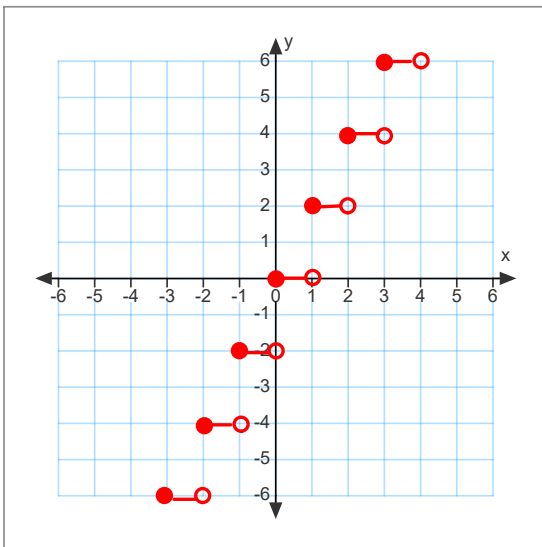
Parameters  $h$  and  $k$  are both equal to 0.

Transformed GIF

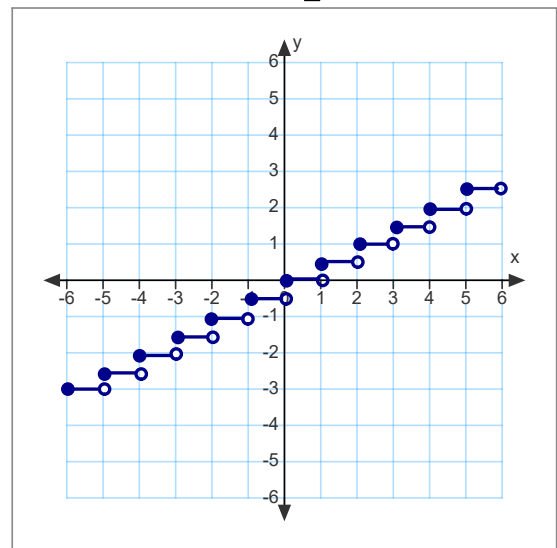
Rule:  $f(x) = a[b(x-h)] + k$

Parameter  $a$

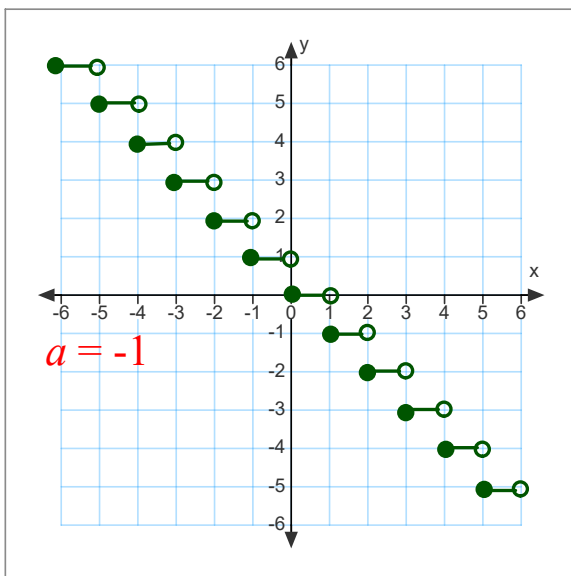
$$f(x) = 2[x]$$



$$f(x) = \frac{1}{2}[x]$$



$$f(x) = -[x]$$



Summary: Parameter  $a$  determines the distance between each step (called the **counterstep**).

The counterstep =  $|a|$

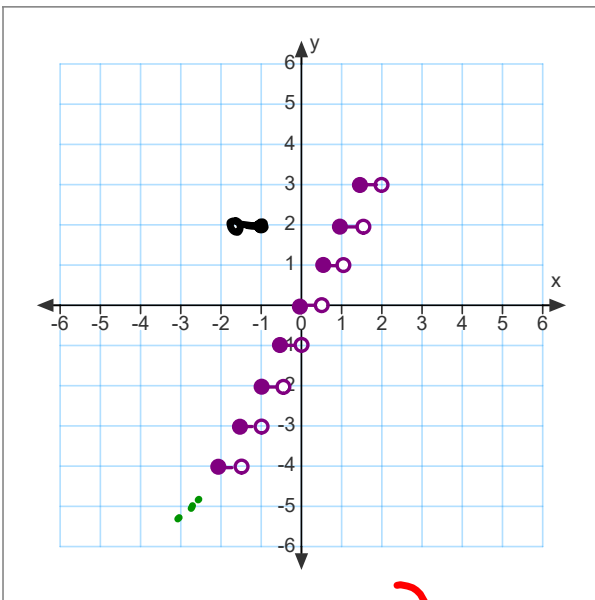
If  $a^+$ , the function is **increasing**.

If  $a^-$ , the function is reflected about the  $x$ -axis and is **decreasing**.

Steps: ● — ○

Parameter  $b$

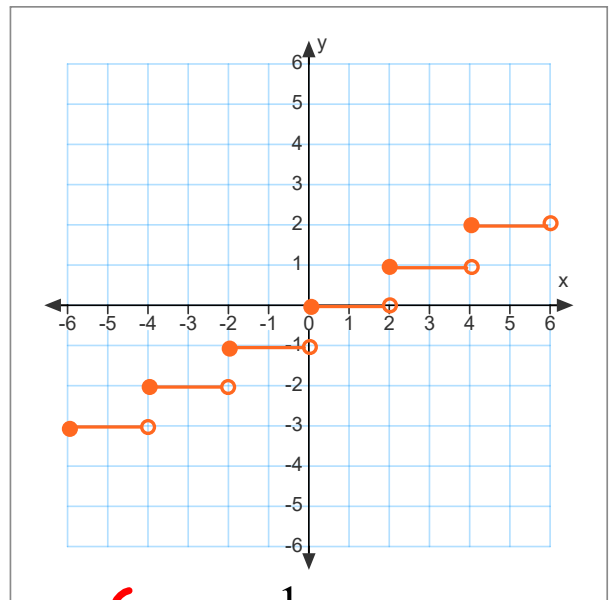
$$f(x) = [2x]$$



$$b = 2$$

$$\text{length} = \frac{1}{2}$$

$$f(x) = \left[ \frac{1}{2}x \right]$$

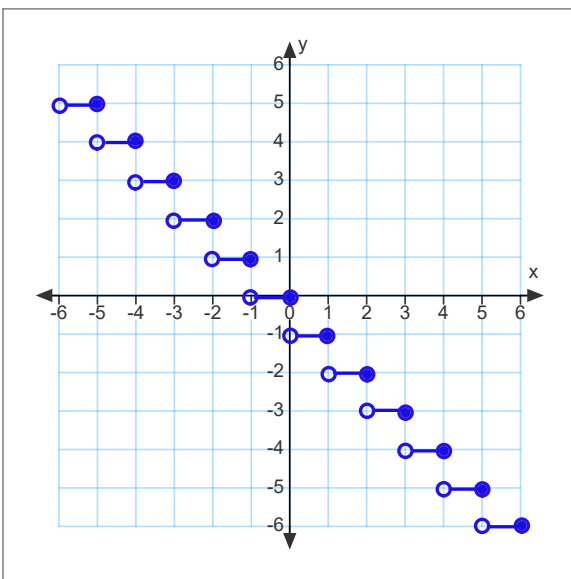


$$b = \frac{1}{2}$$

$$\text{length} = 2$$

*-Reciprocals-*

$$f(x) = [-x]$$



Summary: Parameter  $b$  determines the **length** of each step.

The length =  $\frac{1}{|b|}$  i.e., the reciprocal of

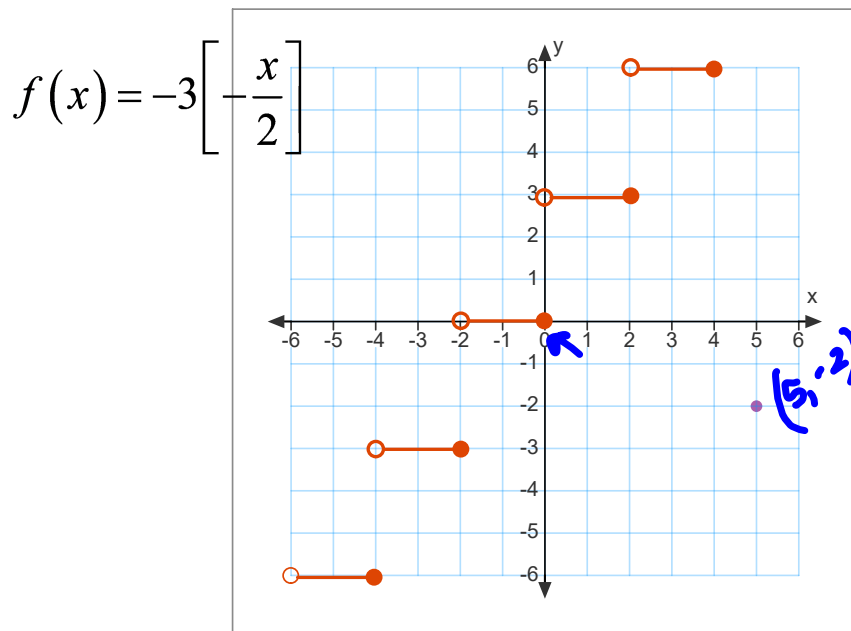
If  $b^+$ , the function  $\frac{b}{}$  is **increasing**.

Steps: ●—○



If  $b^-$ , the function is **decreasing** and is reflected about the  $y$ -axis.

Steps: ○—●

If  $a$  and  $b$  are **both negative**, the function is **increasing** again, **but** each step starts with an **empty dot** and ends with a **full dot**.





Remember: If  $a$  and  $b$  have the same sign, or if  $ab = (+)$ ,  
then the function is increasing:   
If  $a$  and  $b$  have opposite signs, or if  $ab = (-)$ ,  
then the function is decreasing: 

Notice that the origin,  $(0,0)$ , has been a solid (coloured) dot.

Parameter  $h$  translates the function  $h$  units left or right;

Parameter  $k$  translates the function  $k$  units up or down.

This means that  $(h,k)$  will always be a solid dot .

Example:  $f(x) = [x+3] - 8$

$$h = -3$$

$$k = -8$$

