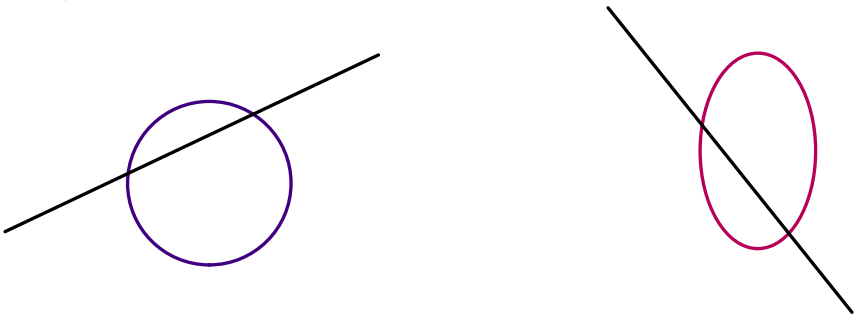
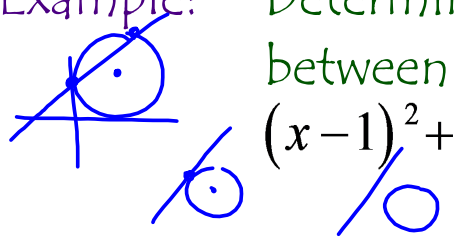
Secant Lines and Circles & Ellipses

A secant line is a line that intersects a curve at two or more points.



A semi-linear system is created. We can use an algebraic method (usually substitution) or a graph to determine the points of intersection.

Example: Determine the points of intersection between the line $y = x+2$ and the circle $(x-1)^2+(y-2)^2=4$.



by substitution

By substitution: $(x-1)^2+(x+2-2)^2=4$

Expand $(x-1)^2 + (x)^2 = 4$

$x^2-2x+1+x^2=4$

① $m \times n = -6$, $m+n = -2$ no

$2x^2-2x+1=4$

① factor
② Quad. eq^n

$2x^2-2x-3=0$

a b c

$$2x^2 - 2x - 3 = 0$$

Factor

$a \times c = 2 \times (-3) = -6$

$b = -2$

$m \times n = -6$
$m + n = -2$
$m =$
$n =$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4(2)(-3)}}{4} = \frac{2 \pm \sqrt{28}}{4}$$

$$x_1 = \frac{2 + \sqrt{28}}{4} \qquad x_2 = \frac{2 - \sqrt{28}}{4}$$

$$x_1 = \frac{2 + 2\sqrt{7}}{4} \qquad x_2 = \frac{2 - 2\sqrt{7}}{4}$$

$$x_1 = \frac{1 + \sqrt{7}}{2} \approx 1.82 \qquad x_2 = \frac{1 - \sqrt{7}}{2} \approx -0.82$$

Find y for each x-value.

$$y = x + 2$$

$y_1 = 1.82 + 2$ $\qquad$ $y_2 = -0.82 + 2$

$y_1 = 3.82$ $\qquad$ $y_2 = 1.18$

Therefore, the line intersects the circle at points $(1.82, 3.82)$ & $(-0.82, 1.18)$.

Example: Determine the points of intersection between the line $y = -2x + 1$ and the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

$\times 225$

Convert the ellipse to general form.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$225 \times \left(\frac{x^2}{9} + \frac{y^2}{25} = 1 \right)$$

$$25x^2 + 9y^2 = 225$$

Substitute $y = -2x + 1$. $\longrightarrow$ $25x^2 + 9(-2x+1)^2 = 225$

Expand

$$25x^2 + 9(-2x+1)^2 = 225$$

$$25x^2 + 9(4x^2 - 4x + 1) = 225$$

$$25x^2 + 36x^2 - 36x + 9 = 225$$

$$61x^2 - 36x + 9 = 225$$

$$61x^2 - 36x - 216 = 0$$

$$x = \frac{36 \pm \sqrt{(-36)^2 - 4(61)(-216)}}{122}$$

$$x = \frac{36 \pm \sqrt{54000}}{122}$$

$$x = \frac{36 \pm \sqrt{54000}}{122} \quad \begin{cases} x_1 = \dfrac{36 + 232.38}{122} = \dfrac{268.38}{122} = 2.2 \\ x_2 = \dfrac{36 - 232.38}{122} = \dfrac{-196.38}{122} = -1.61 \end{cases}$$

Determine the y-value for each x.

$$y = -2x + 1 \quad \begin{cases} y_1 = -2(2.2) + 1 = -3.4 \\ y_2 = -2(-1.61) + 1 = 4.22 \end{cases}$$