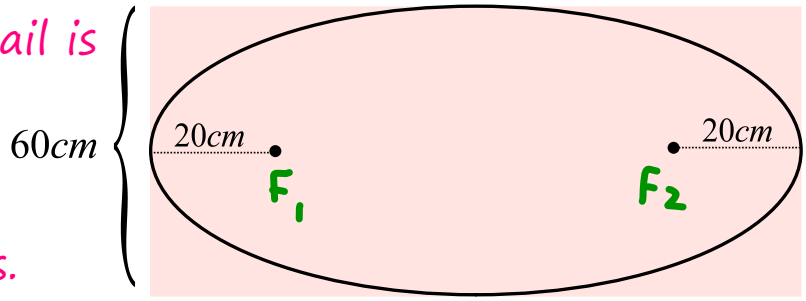
7) A carpenter places two nails on a rectangular piece of wood. Each nail is placed $20\ cm$ from each of the shorter edges.



These shorter edges are $60\ cm$ long.

The carpenter uses these nails as focal points to draw the largest possible ellipse on the rectangular plane.

a) Determine the equation of the ellipse.

$\frac{x^2}{a^2} + \frac{y^2}{900} = 1 \Rightarrow \frac{x^2}{1056.25} + \frac{y^2}{900} = 1$

$c^2 = a^2 - b^2$

or* $c^2 + b^2 = a^2$

$c^2 + b^2 = (c+20)^2$

$c^2 + 900 = (c+20)^2$

$c^2 + 900 = c^2 + 40c + 400$

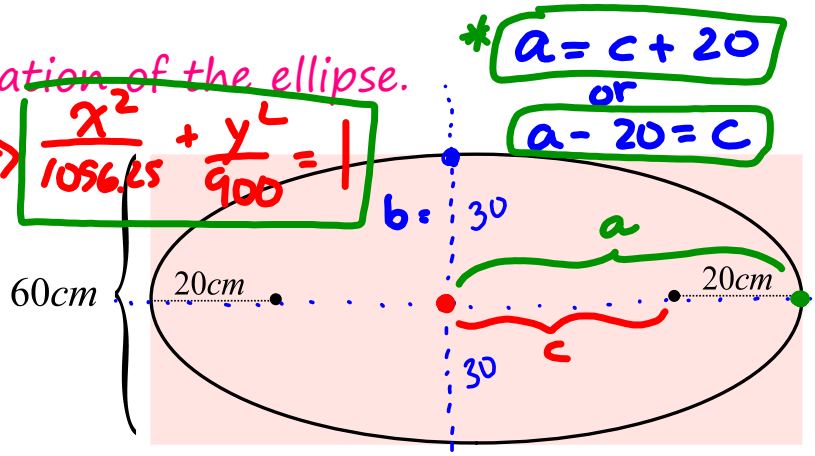
$900 = 40c + 400$

$500 = 40c$

$12.5\text{cm} = c$

* $a = c + 20$ or $a - 20 = c$

$a = 32.5\text{cm}$

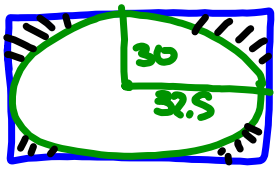


b) Determine the area of the piece of plywood.

$A = 65 \times 60 = 3900\text{cm}^2$

How much wood was left unused?

(Area of ellipse = πab)



Wood: $A = 3900 cm^2$

E: $A = 3063.05 cm^2$

Unused: $3900 - 3063.05$

$= 836.95 cm^2$

Determine the centre of the following ellipse:

$$4x^2 + 49y^2 - 16x + 294y + 261 = 0$$

$4x^2 + 49y^2 - 16x + 294y = -261$

$4x^2 - 16x + 49y^2 + 294y = -261$

$4(x^2 - 4x + 4) + 49(y^2 + 6y + 9) = -261 + 16 + 441$

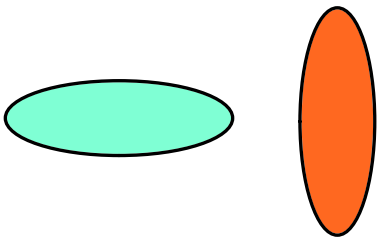
$4(x-2)^2 + 49(y+3)^2 = 196$

$\frac{4(x-2)^2}{196} + \frac{49(y+3)^2}{196} = 1$

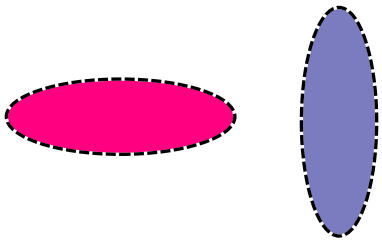
$\frac{(x-2)^2}{49} + \frac{(y+3)^2}{4} = 1$

Ellipses and Inequalities

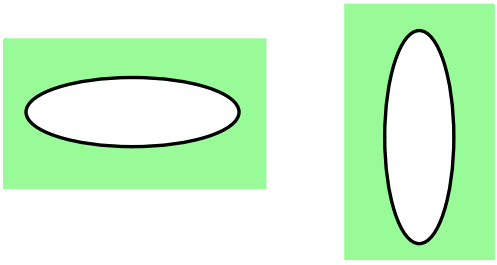
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}< 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}> 1$$