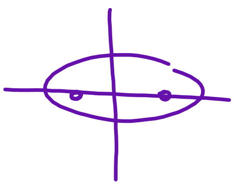
Example: Determine the coordinates of the foci.

a) $9x^2+16y^2-144=0$

convert: $9x^2+16y^2=144$

$$\frac{9x^2}{144}+\frac{16y^2}{144}=1$$



$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$a^2=16$ $b^2=9$

$c^2=16-9$

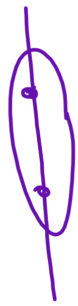
$c^2=7$

$c=\pm\sqrt{7}$

$F_1(-\sqrt{7},0)$ & $F_2(\sqrt{7},0)$

b) $8x^2+2y^2-48=0$

$8x^2+2y^2=48$



$$\frac{8x^2}{48}+\frac{2y^2}{48}=1$$

$$\frac{x^2}{6}+\frac{y^2}{24}=1$$

$c^2=24-6$

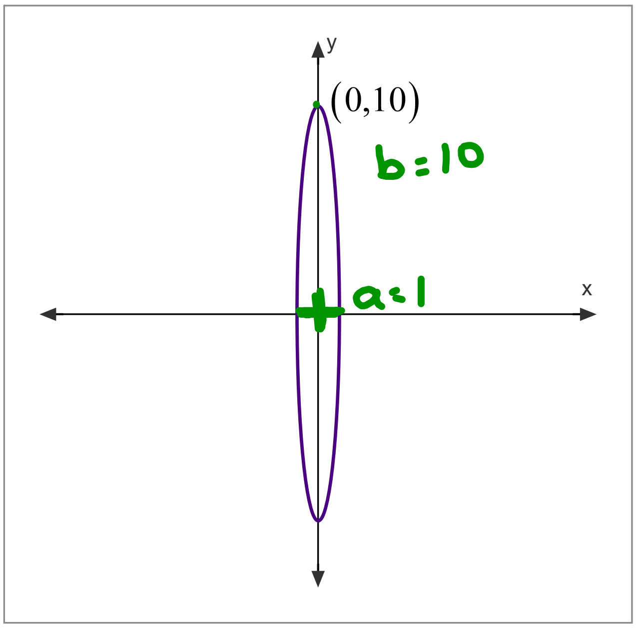
$c^2=18$

$c=\pm\sqrt{18}=\pm3\sqrt{2}$

$F_1(0,3\sqrt{2})$ & $F_2(0,-3\sqrt{2})$

Examples:

1) Find the equation of the ellipse, given that the length of the minor axis is 2 units.



$$x^2 + \frac{y^2}{100} = 1$$

2) Determine the lengths of the major and minor axes of the ellipse whose equation is

$$16x^2 + 25y^2 - 36 = 0$$

$$16x^2 + 25y^2 = 36$$

$$\frac{16x^2}{36} + \frac{25y^2}{36} = 1$$

$$\frac{4x^2}{9} + \frac{25y^2}{36} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{4}{9} \cdot x^2 + \frac{25}{36} \cdot y^2 = 1$$

$$\therefore \frac{x^2}{\frac{9}{4}} + \frac{y^2}{\frac{36}{25}} = 1$$

$a^2 = \frac{9}{4}$ $b^2 = \frac{36}{25}$

$a = \pm \frac{3}{2}$ $b = \pm \frac{6}{5}$

$\therefore$ major axis $= 3$

$\therefore$ minor axis $= \frac{12}{5}$ or 2.4

3) Graph the ellipse given below, including the foci.

$$9x^2 + 25y^2 - 225 = 0$$

$9x^2 + 25y^2 = 225$

$\frac{9x^2}{225} + \frac{25y^2}{225} = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

horizontal $b = \pm 3$

$a = \pm 5$

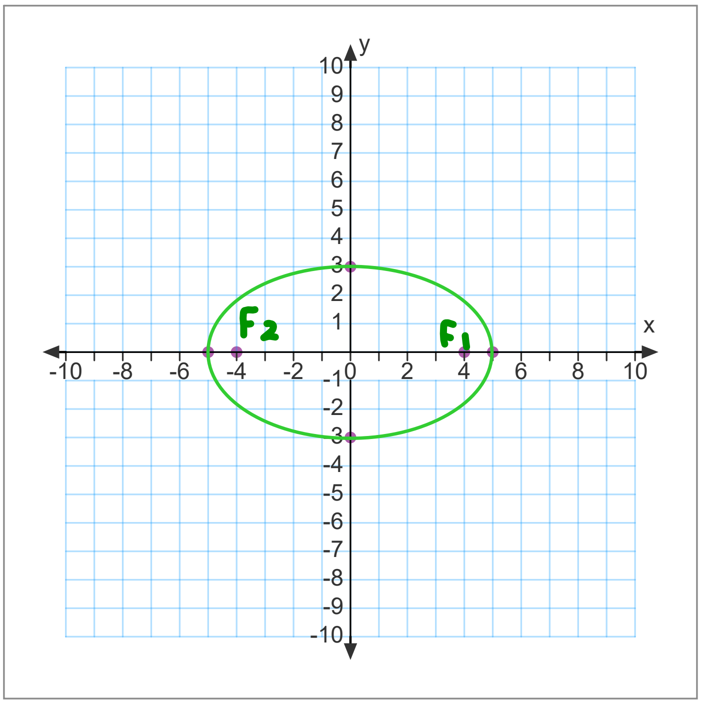
$(-5,0), (5,0)$ $(0,3), (0,-3)$ vertices

$c^2 = 25 - 9$

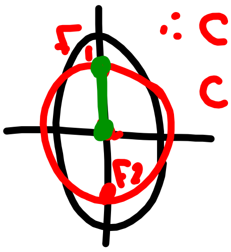
$c^2 = 16$

$c = \pm 4$

$F_1(4,0), F_2(-4,0)$



4) Determine the equation of the circle that has the same centre as the ellipse whose equation is $\frac{x^2}{64}+\frac{y^2}{100}=1$, and passes through the foci of the ellipse.



① centre $(0,0)$

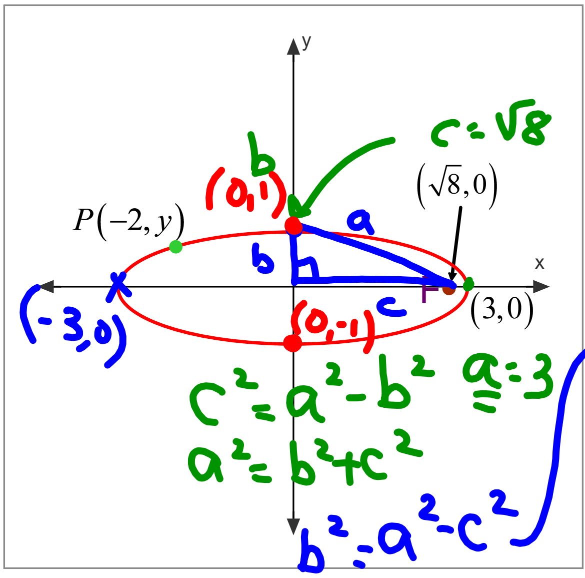
$\therefore\ x^2+y^2=r^2$

$\therefore c=r$

$c^2=r^2$

$c^2=100-64=36$ $\Rightarrow$ Ans: $x^2+y^2=36$

5) Given the graph, determine...



a) the equation of the ellipse

b) the domain and range of the ellipse

c) the value of y in point P

$b^2 = 9 - 8$

$b^2 = 1$

$b = 1$

a) $\frac{x^2}{9} + y^2 = 1$

b) Dom: $[-3, 3]$

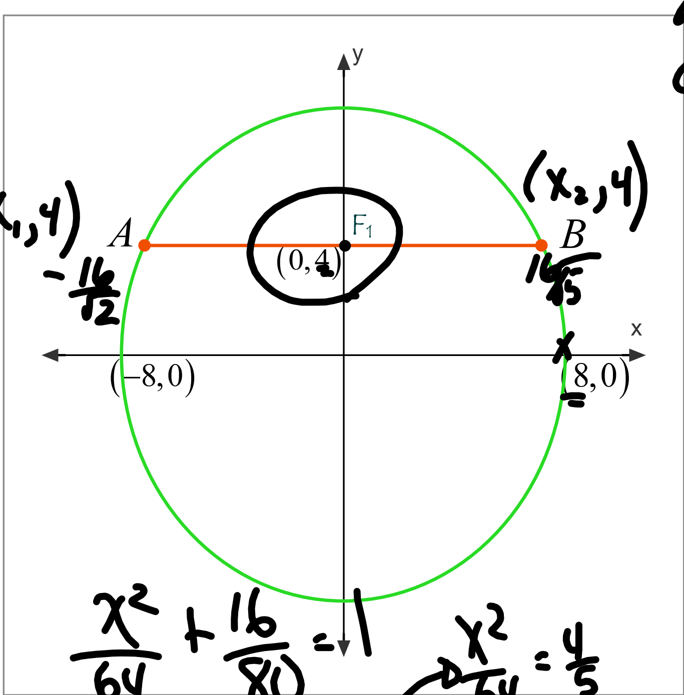
Ran: $[-1, 1]$

c) $\frac{(-2)^2}{9} + y^2 = 1 \Rightarrow \frac{4}{9} + y^2 = 1$

$y^2 = \frac{5}{9} \Rightarrow y = \frac{\pm\sqrt{5}}{3}$

$\therefore y = \sqrt{5}/3$

6) Determine the length of $\overline{AB}$.



$(x_1,4)$ $-\frac{16}{\sqrt{5}}$ $(x_2,4)$ $\frac{16}{\sqrt{5}}$

$a=8$, $c=4$ $\} \Rightarrow b=?$ b^2

$c^2=b^2-a^2$

$a^2+c^2=b^2$

$64+16=b^2$

$80=b^2$

vert.

$\frac{x^2}{64}+\frac{y^2}{80}=1$

let $y=4$

$\frac{x^2}{64}+\frac{y^2}{80}=1$

$\frac{x^2}{64}+\frac{16}{80}=1$

$\frac{x^2}{64}+\frac{1}{5}=1$

$\frac{x^2}{64}=\frac{4}{5}$

$x^2=\frac{256}{5}$

$x=\pm\frac{16}{\sqrt{5}}$

$\therefore$ length $\overline{AB}=\frac{32}{\sqrt{5}}$ or $\frac{32\sqrt{5}}{5}$ or 14.31