# General to Standard Form

Example: Find the centre and radius of the circle.

$$x^2 + y^2 - 4x + 2y - 11 = 0$$

The centre and radius are found in the standard form

$$(x-h)^2 + (y-k)^2 = r^2$$

Rearrange the equation so that the $x$ terms and $y$ terms are together and the constant is on the right hand side.

$$x^2 - 4x + y^2 + 2y = 11$$

Create two perfect square trinomials on the LHS by completing the square.

$$x^2-4x+y^2+2y=11$$

$x^2-4x$

① $-4\div 2=-2$

② $(-2)^2=4$ add

$y^2+2y$

① $2\div 2=1$

② $1^2=1$ add

$$x^2-4x+4+y^2+2y+1=11+\underbrace{4+1}$$

What is added to the LHS must be added to the RHS.

Factor the two trinomials on the LHS and simplify the RHS.

$$(x-2)^2+(y+1)^2=16$$

Centre: $(2,-1)$
Radius: $4$

Convert $x^2+y^2-14x+2y+47=0$ to standard form.

$(x-h)^2+(y-k)^2=r^2$

$x^2-14x \quad +y^2+2y \quad =-47$

① $-14 \div 2 = -7$
$(-7)^2=49$

① $2 \div 2 = 1$
② $1^2 = 1$

$x^2-14x+49+y^2+2y+1=-47+49+1$

$(x-7)^2+(y+1)^2=3$

Find the equation of the circle whose centre is the same as $x^2 + y^2 - 10x - 1 = 0$, but passes through $P(2,-3)$.

$-10 \div 2 = -5$

$(-5)^2 = 25$

$x^2 - 10x + y^2 = 1$

$x^2 - 10x + 25 + y^2 = 1 + 25$

$\boxed{(x-5)^2 + y^2} = 26$

$C(5,0)$

$(x-5)^2 + y^2 = r^2$

$x = 2 \quad y = -3$

$9 + 9 = r^2$

$18 = r$

$\boxed{(x-5)^2 + y^2 = 18}$

$x^2 - 10x + 25 + y^2 = 18$

$\boxed{x^2 + y^2 - 10x + 7 = 0}$

Find the equation of the circle

$$x^2 + y^2 + 12x - 10y - 2 = 0$$

after it has undergone a translation of $t(-6, 1)$.

$\div 2 = 6$, $6^2 = 36$ $\qquad$ $\div 2 = -5$, $(-5)^2 = 25$

$x^2 + 12x + y^2 - 10y = 2$

$x^2 + 12x + 36 + y^2 - 10y + 25 = 2 + 36 + 25$

$(x+6)^2 + (y-5)^2 = 63$

$c(-6, 5) \rightarrow t(-6, 1) \Rightarrow c_2(-12, 6)$

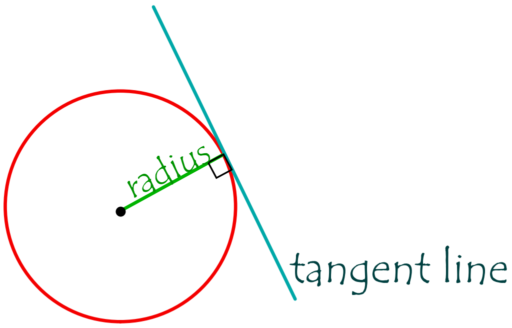
$$\boxed{(x+12)^2 + (y-6)^2 = 63}$$

$x^2 + 24x + 144 + y^2 - 12y + 36 - 63 = 0$

$$\boxed{x^2 + y^2 + 24x - 12y + 117 = 0}$$

# Tangents to Circles

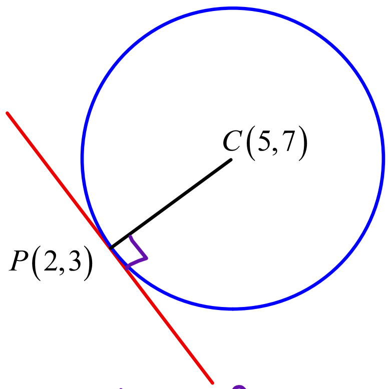
- A tangent line is a line that shares only one point in common with the circle.
- A tangent line is perpendicular to the radius of the circle at the point of contact.



Example: Determine the equation of the tangent line.

equation of a line: $y = mx + b$

$m$:  $b$:




① slope of radius $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{7-3}{5-2} = \frac{4}{3}$

② slopes of perpendicular lines are negative reciprocals

$\therefore$ tangent line $= -\frac{3}{4}$

$y = -\frac{3}{4}x + b$

③ fill in point on the line

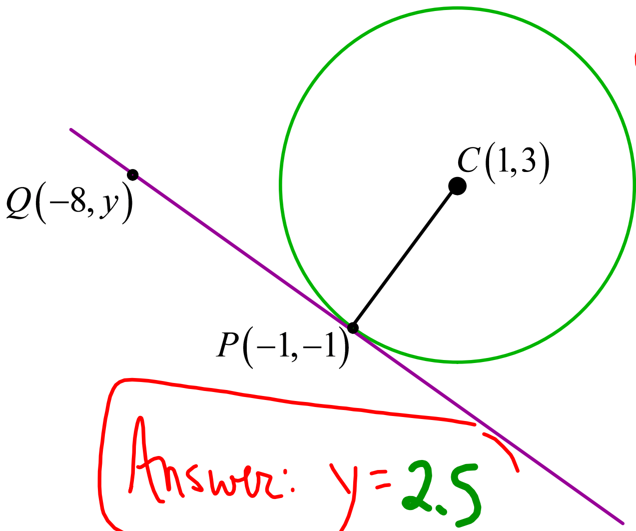
$3 = -\frac{3}{4}(2) + b$

$3 = -\frac{3}{2} + b$

$\frac{9}{2} = b$

$$\boxed{y = -\frac{3}{4}x + \frac{9}{2}}$$

Example: Determine $y$ in the point $Q(-8, y)$.




Answer: $y = 2.5$

① radius : $m = \frac{3+1}{1+1} = \frac{4}{2} = 2$

② tangent : $m = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$-1 = +\frac{1}{2} + b$

$-\frac{3}{2} = b$

$y = -\frac{1}{2}x - \frac{3}{2}$

③ Q : let $x = -8$

$y = -\frac{1}{2}(8) - \frac{3}{2}$

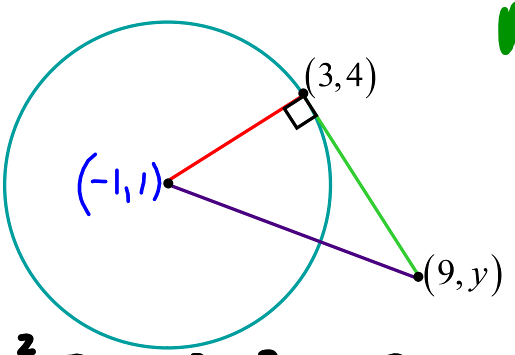
$y = 4 - \frac{3}{2}$

Example: Given the diagram and the equation of the circle: $x^2+y^2+2x-2y-23=0$

Determine: a) the length of the radius.

~~b) the domain and range of the circle.~~

c) the value of $y$.




$x^2+2x+y^2-2y=23$

$x^2+2x+1+y^2-2y+1=23+1+1$

$(x+1)^2+(y-1)^2=25$

$c(-1,1)$ $r^2=25$

a) $r=5$ units

c) radius: $m=\frac{3}{4}$

tangent: $m=-\frac{4}{3}$

$y=-\frac{4}{3}x+b$

$4=-\frac{4}{3}(3)+b$

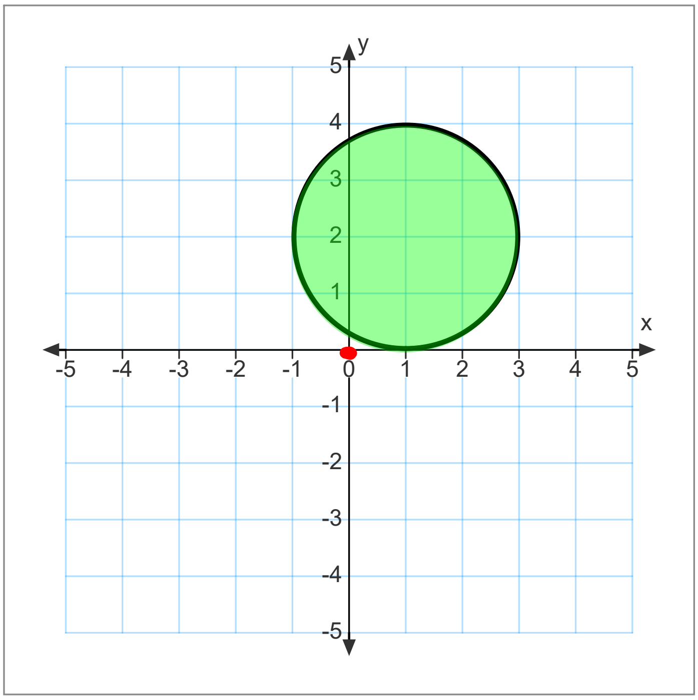
$4=-4+b$

$8=b$

$y=-\frac{4}{3}x+8$

let $x=9$

$y=-12+8=\boxed{-4}$




2) Test a point

test (0,0)

$$(x-1)^2+(y-2)^2 \le 4$$

$$(-1)^2+(-2)^2 \le 4$$

$$1+4 \le 4$$

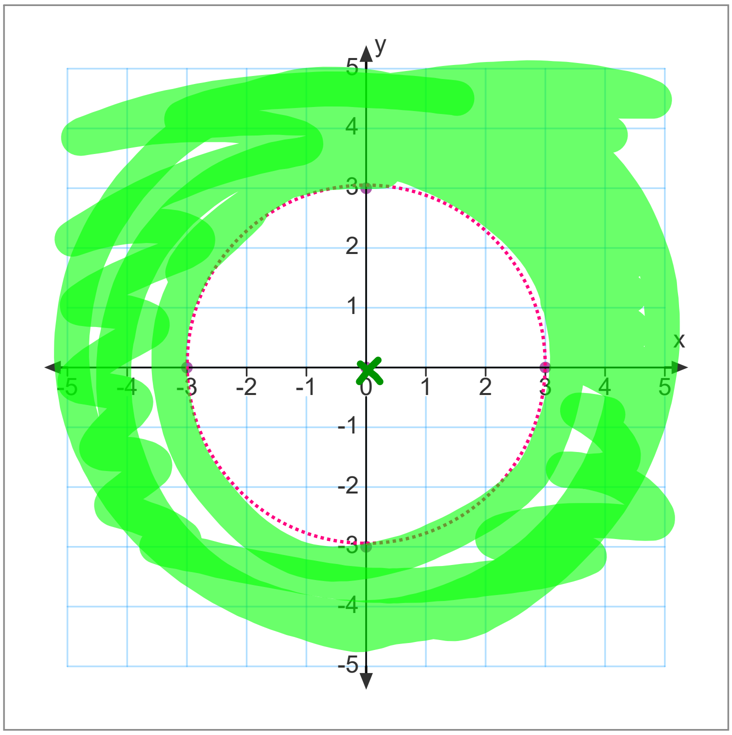
$$5 \le 4 \text{ False}$$

Example: Draw the solution set of $x^2 + y^2 > 9$.

centre $(0,0)$ . $r = 3$

$>$ greater than

dotted

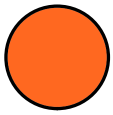



Test $(0,0)$

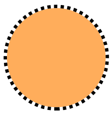
$0 > 9$ false

# Summary

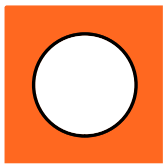
$$(x-h)^2+(y-k)^2 \le r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 < r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 \ge r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 > r^2 \longrightarrow$$