General to Standard Form

Example: Find the centre and radius of the circle.

$(x-2)^2+(y-1)^2=7$

$x^2-4x+4+y^2-2y+1=7$

$$x^2+y^2-4x+2y\overset{+11}{-11}=0^{+11}$$

The centre and radius are found in the standard form

$$(x-h)^2+(y-k)^2=r^2$$

Rearrange the equation so that the x terms and y terms are together and the constant is on the right hand side.

$$x^2-4x+y^2+2y=11$$

Create two perfect square trinomials on the LHS by completing the square.

$(x+n)^2 = x^2 + 2nx + n^2$

$$x^2 - 4x + y^2 + 2y = 11$$

$-4 \div 2 = -2$

$(-2)^2 = 4$

add

$2 \div 2 = 1$

$(1)^2 = 1$

add

$x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 4 + 1$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = \overbrace{11 + 4 + 1}^{16}$$

What is added to the LHS must be added to the RHS.

Factor the two trinomials on the LHS and simplify the RHS.

$x^2 - 4x + 4$

$(x-2)^2$

$y^2 + 2y + 1$

$(y+1)^2$

$$(x-2)^2 + (y+1)^2 = 16$$

$h = 2$ $k = -1$ $r = \sqrt{16}$

Centre: $(2, -1)$

Radius: 4

Convert

$$x^2+y^2+12x-8y+47=0$$

$x^2+12x+y^2-8y=-47$

$12\div 2=6$ $\quad -8\div 2=-4$

$6^2=36$ $\quad (-4)^2=16$

$x^2+12x+36+y^2-8y+16=-47+36+16$

$(x+6)^2+(y-4)^2=5$

$$x^2+y^2-5x-16y+60=0$$

$x^2-5x+y^2-16y=-60$

$-5\div 2=-2.5$ $\quad -16\div 2=-8$

$(-2.5)^2=6.25$ $\quad (-8)^2=64$

$x^2-5x+6.25+y^2-16y+64=-60+6.25+64$

$(x-2.5)^2+(y-8)^2=10.25$

Find the equation of the circle whose centre is the same as $x^2+y^2-10x-1=0$, but passes through $P(2,-3)$.

① Find the centre :

$x^2-10x+y^2=1$

$-10\div 2=-5$

$(-5)^2=25$

$(x-5)^2$

$\to h=5$

$(x-5)^2+y^2=$

$C(5,0)$

$x=2 \quad y=-3$

$(2-5)^2+(-3)^2=r^2$

$(-3)^2+(-3)^2=18$

st: $\boxed{(x-5)^2+y^2=18}$

$x^2-10x+25+y^2=18$

gen: $\boxed{x^2+y^2-10x+7=0}$

Find the equation of the circle

$$\boxed{x^2 + y^2 + 12x - 10y - 2 = 0}$$

after it has undergone a translation of $t(-6, 1)$.

left up

centre ∴ same radius

⇓

$h + -6$

$k + 1$

new centre

$x^2 + 12x + y^2 - 10y = 2$

$x^2 + 12x + 36 + y^2 - 10y + 25 = 2 + 36 + 25$

$(x + 6)^2 + (y - 5)^2 = 63$

old $c(-6, 5)$

new $c(-6 + -6, 5 + 1)$

$c(-12, 6)$

$\Rightarrow (x + 12)^2 + (y - 6)^2 = 63$

Tangents to Circles

- A tangent line is a line that shares only one point in common with the circle.
- A tangent line is perpendicular to the radius of the circle at the point of contact.