

## Second-Degree Function: FACTORED FORM

$$f(x) = a(x - x_1)(x - x_2)$$

$a$  is parameter  $a$  (same as the standard and general forms) - it lets us know if the parabola opens up or down and how wide .

$x_1$  and  $x_2$  are the zeros of the function.

What is the vertex of the function  $f(x) = -3(x-4)(x+5)$  ?

The zeros are 4 and  $-5$ .

Since the parabola is symmetrical, the vertex must be exactly halfway between them.

$$h = \frac{x_1 + x_2}{2}$$

The average of the zeros determines the middle.

$$\begin{aligned} \therefore h &= \frac{4 + (-5)}{2} = \frac{-1}{2} & \text{and} & \quad k = f(h) = -3(-0.5 - 4)(-0.5 + 5) \\ & & & \quad k = -3(-4.5)(4.5) \\ & & & \quad k = 60.75 \end{aligned}$$

The vertex of the function is  $V(-0.5, 60.75)$ .

What is the vertex of the function  $g(x) = \frac{1}{2}(x-8)(x-20)$ ?

Zeros:  $\{8, 20\}$

$$h = \frac{8+20}{2} = 14$$

$$k = \frac{1}{2}(14-8)(14-20)$$
$$k = \frac{1}{2}(6)(-6) = -18$$

The vertex is  $V(14, -18)$ .

Convert  $f(x) = -5(x+2)(x-7)$  into...

Need  $h$  &  $k$

$$1) h = \frac{-2+7}{2} = 2.5$$

$$2) k = -5(2.5+2)(2.5-7)$$

$$k = -5(4.5)(-4.5)$$

$$k = 101.25$$

$$\therefore f(x) = -5(x-2.5)^2 + 101.25$$

$$f(x) = -5(x+2)(x-7)$$

FOIL

$$f(x) = -5(x^2 - 5x - 14)$$

$$\therefore f(x) = -5x^2 + 25x + 70$$

Convert  $f(x) = -(x-7)^2 + 9$  into factored form.

$$a = -1$$

Find the zeros (let  $y = 0$ ).

$$0 = -(x-7)^2 + 9$$

$$-9 = -(x-7)^2$$

$$9 = (x-7)^2$$

$$\pm\sqrt{9} = x-7$$

$$\pm 3 = x-7 \longrightarrow 1) 3 = x-7 \quad 2) -3 = x-7$$

$$10 = x_1 \quad 4 = x_2$$

$$f(x) = -(x-10)(x-4)$$

Convert  $y = 3x^2 - 35x - 12$  into factored form.

$a = 3$  Find the zeros (let  $y = 0$ ).

$$0 = 3x^2 - 35x - 12$$

1) By factoring

or

2) Quadratic Equation

$$0 = 3x^2 + x - 36x - 12$$

$$0 = x(3x + 1) - 12(3x + 1)$$

$$0 = (3x + 1)(x - 12)$$

$$x = \frac{-(-35) \pm \sqrt{1225 - 4(3)(-12)}}{6}$$

$$x = \frac{35 \pm \sqrt{1369}}{6} = \frac{35 \pm 37}{6}$$

$$x_1 = -\frac{1}{3} \quad x_2 = 12$$

$$y = 3\left(x + \frac{1}{3}\right)(x - 12)$$

Provide a study of the function  $f(x) = -\frac{3}{4}(x-3)(x+13)$ .

Dom: \_\_\_\_\_

Increasing: \_\_\_\_\_

Ran: \_\_\_\_\_

Decreasing: \_\_\_\_\_

Max: \_\_\_\_\_

y-intercept: \_\_\_\_\_

Min: \_\_\_\_\_

Zero(s): \_\_\_\_\_

Positive: \_\_\_\_\_

Axis of Symmetry: \_\_\_\_\_

Negative: \_\_\_\_\_

Provide a study of the function  $f(x) = 2x(x + 7)$ .

Dom: \_\_\_\_\_

Increasing: \_\_\_\_\_

Ran: \_\_\_\_\_

Decreasing: \_\_\_\_\_

Max: \_\_\_\_\_

y-intercept: \_\_\_\_\_

Min: \_\_\_\_\_

Zero(s): \_\_\_\_\_

Positive: \_\_\_\_\_

Axis of Symmetry: \_\_\_\_\_

Negative: \_\_\_\_\_

## Finding the Rule - Given the Zeros and a Point

What is the equation of the parabola whose zeros are  $-10$  and  $20$  and passes through the point  $(5, 40)$ ?

Given the zeros, use factored form.

$$f(x) = a(x - x_1)(x - x_2)$$

$$f(x) = a(x + 10)(x - 20)$$

$$40 = a(5 + 10)(5 - 20)$$

$$40 = a(15)(-15)$$

$$40 = -225a$$

$$\frac{40}{-225} = -\frac{8}{45} = a$$



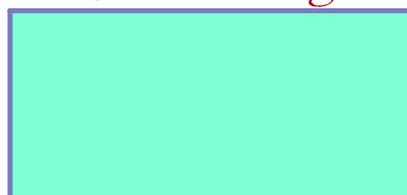
Determine the equation of the parabola whose intercepts are  $(-3,0)$ ,  $(1,0)$  and  $(0,-6)$  ...

- a) in general form
- b) in standard form

We have the zeros and a point, so first we'll find the factored form and then convert.



a) Convert to general



b) We can use the zeros to find  $h$ .

$$h = \frac{x_1 + x_2}{2} = \frac{-3 + 1}{2} = -1$$

Let  $x = -1$  to find  $k$ .

$$f(-1) = 2(-1+3)(-1-1)$$

$$k = 2(2)(-2) = -8$$

Standard form

$$f(x) = 2(x+1)^2 - 8$$