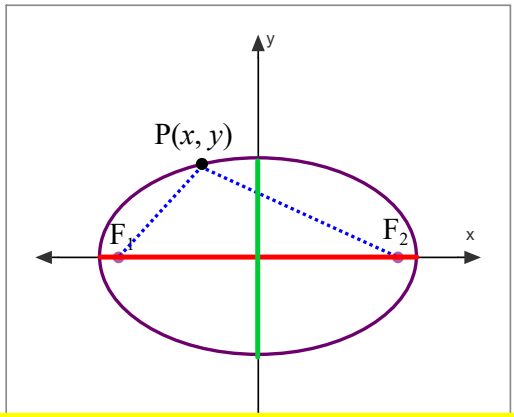
2) The Ellipse

The ellipse is the locus of all points such that the sum of the distances from two fixed points (foci) is constant.

http://youtu.be/7UD8hOs-vaI



$$d(P,F_1)+d(P,F_2)=\text{constant}$$

The ellipse has two axes:

1) major (longer)
2) minor (shorter)

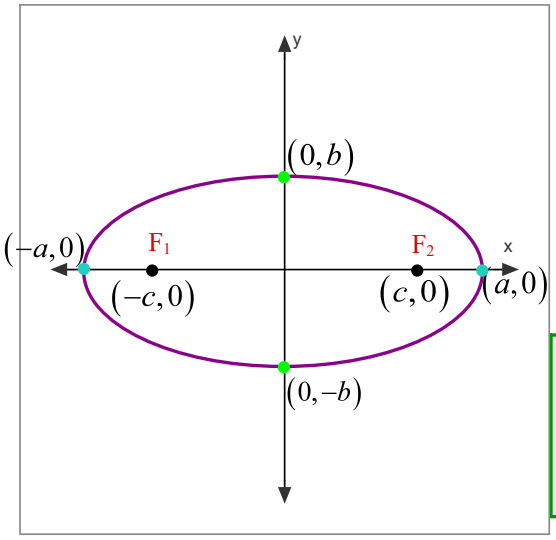
This constant is equal to the length of the major axis.

Equation of the ellipse centred at the origin:

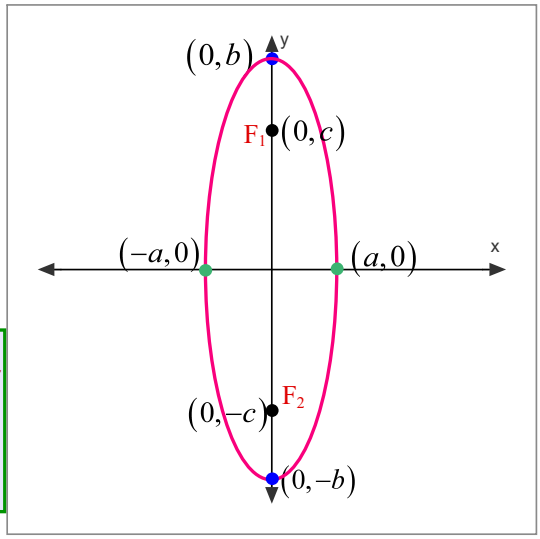
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

a = half the horizontal axis

b = half the vertical axis



The foci lie on the major axis



$\pm a = x$ - intercepts

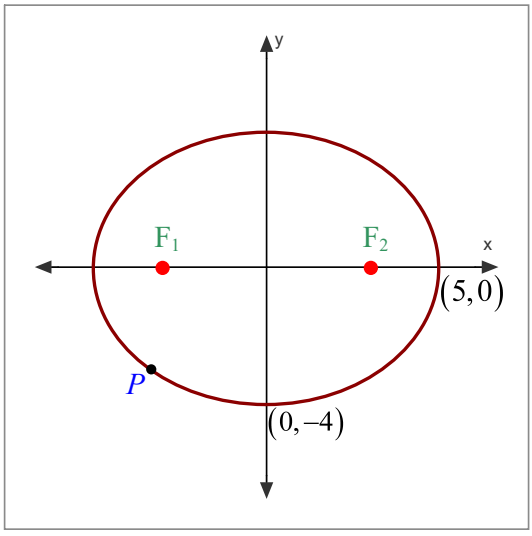
$\pm b = y$ - intercepts

$\pm c =$ coordinate of focus

To determine the coordinates of the foci:

$$\left.\begin{aligned} c^2 &= a^2 - b^2 \\ c^2 &= b^2 - a^2 \end{aligned}\right\}$$ whichever of a or b is bigger (major axis)

Examples:



Find....

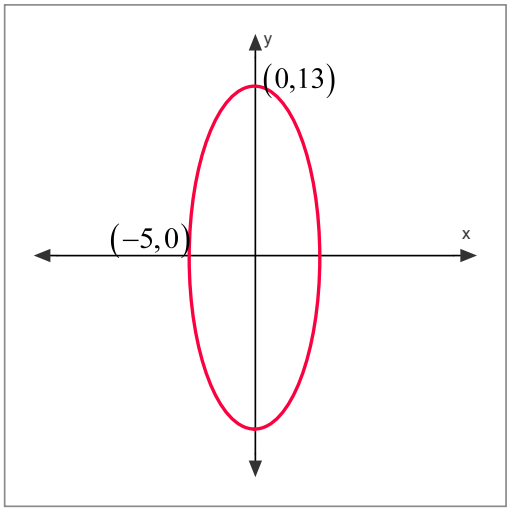
a) length of major axis

b) length of minor axis

c) coordinates of foci

d) total distance from P to both foci

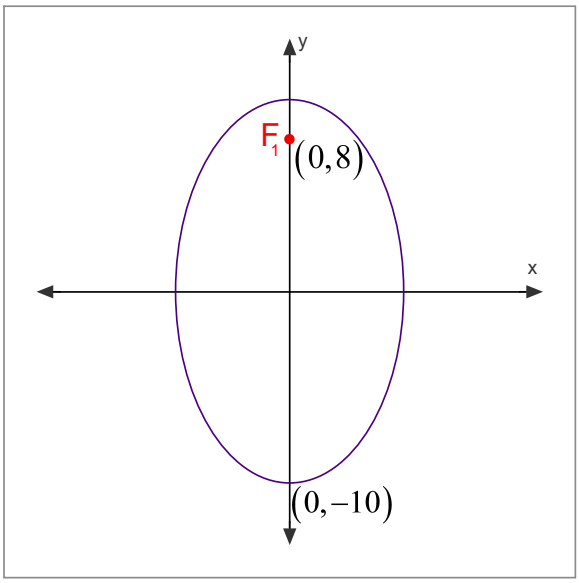
e) the equation of the locus (ellipse)



Determine...

a) the coordinates of the foci

b) the equation of the ellipse



Determine...

a) the length of the minor axis

b) the equation of the ellipse

The Ellipse - General Form

$\frac{x^2}{4}+\frac{y^2}{9}=1$ Standard form

$36\left(\frac{x^2}{4}+\frac{y^2}{9}=1\right)$ Get rid of the denominators

$9x^2+4y^2=36$

$9x^2+4y^2-36=0$ General Form

General Form: $Ax^2 + By^2 - C = 0$

Example: Convert to standard form

$$16x^2 + 25y^2 - 400 = 0$$

Make RHS equal to 1 by dividing

$$\frac{16x^2}{400} + \frac{25y^2}{400} = 1$$

Reduce the fractions

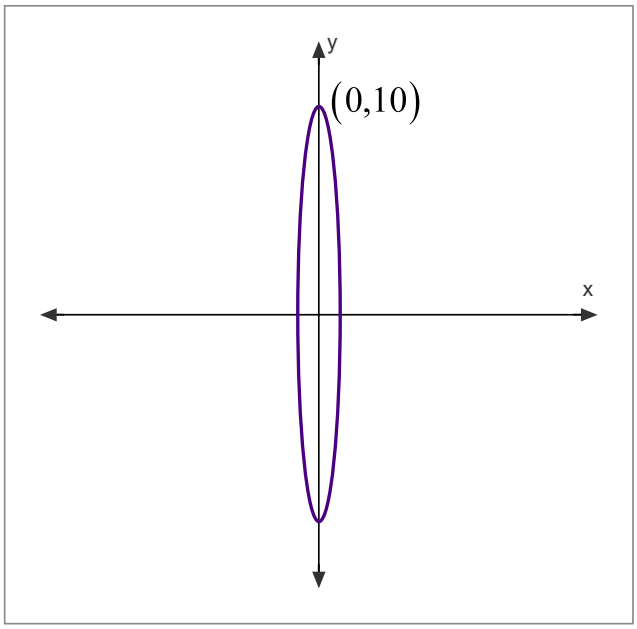
$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Example: Determine the coordinates of the foci.

a) $9x^2 + 16y^2 - 144 = 0$ b) $8x^2 + 2y^2 - 48 = 0$

Examples:

1) Find the equation of the ellipse, given that the length of the minor axis is 2 units.

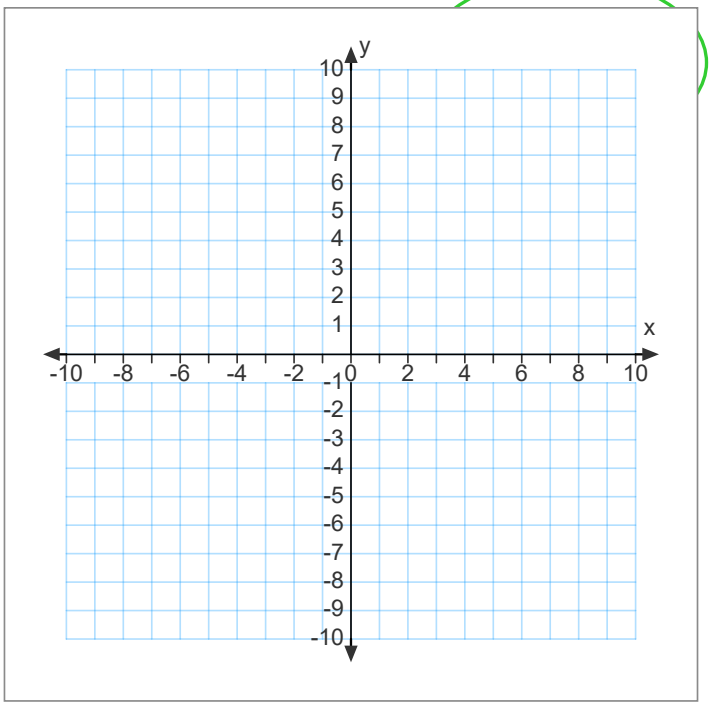


2) Determine the lengths of the major and minor axes of the ellipse whose equation is

$$16x^2 + 25y^2 - 36 = 0$$

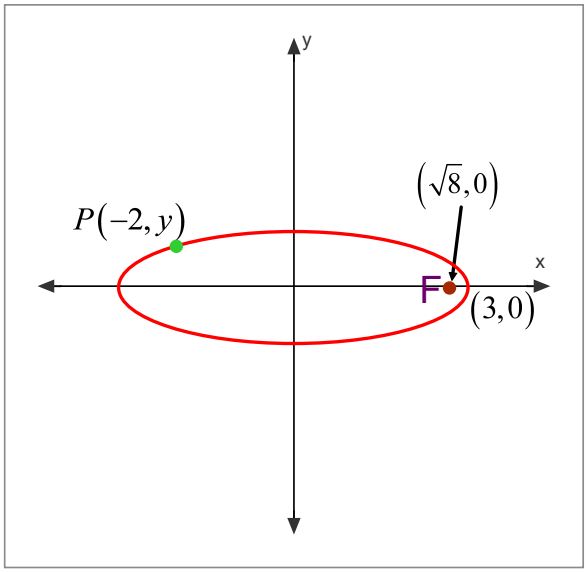
3) Graph the ellipse given below, including the foci.

$$9x^2 + 25y^2 - 225 = 0$$



4) Determine the equation of the circle that has the same centre as the ellipse whose equation is $\frac{x^2}{64}+\frac{y^2}{100}=1$, and passes through the foci of the ellipse.

5) Given the graph, determine...

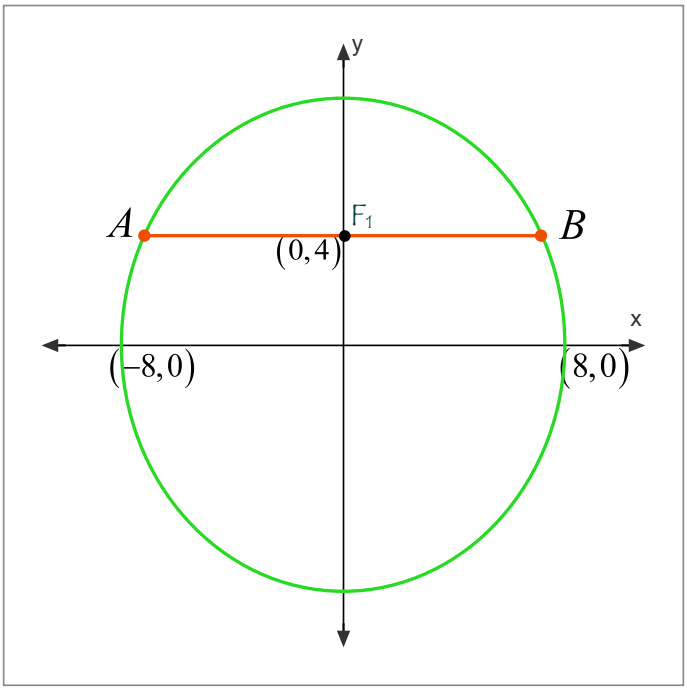


a) the equation of the ellipse

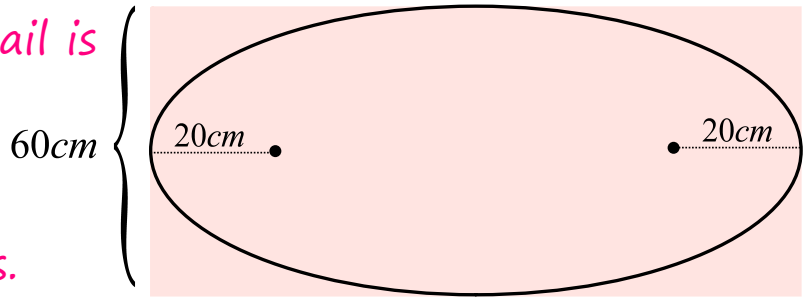
b) the domain and range of the ellipse

c) the value of y in point P

6) Determine the length of $\overline{AB}$.



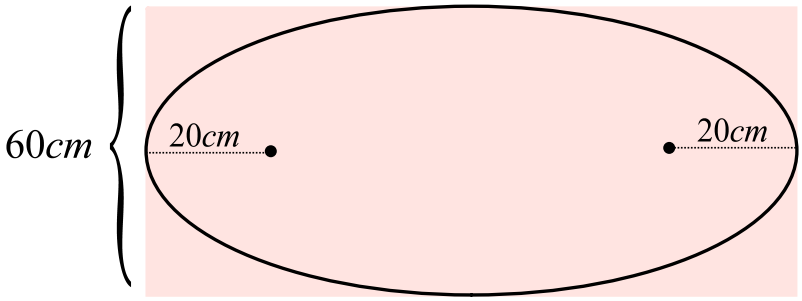
7) A carpenter places two nails on a rectangular piece of wood. Each nail is placed $20\ cm$ from each of the shorter edges.



These shorter edges are $60\ cm$ long.

The carpenter uses these nails as focal points to draw the largest possible ellipse on the rectangular plane.

a) Determine the equation of the ellipse.



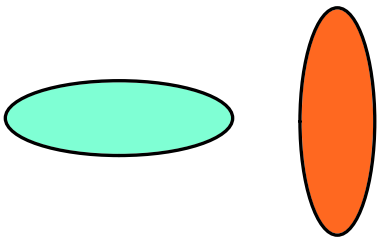
b) Determine the area of the piece of plywood.

Determine the centre of the following ellipse:

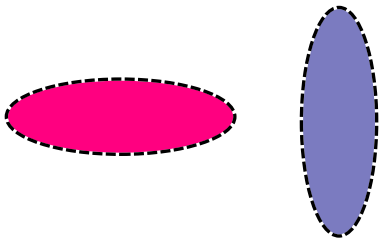
$$4x^2 + 49y^2 - 16x + 294y + 261 = 0$$

Ellipses and Inequalities

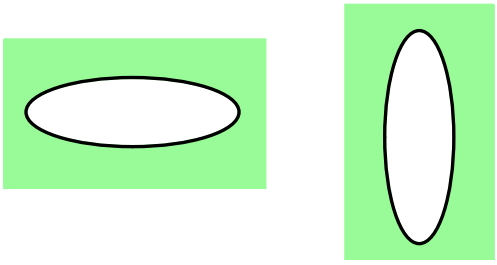
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}< 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}> 1$$