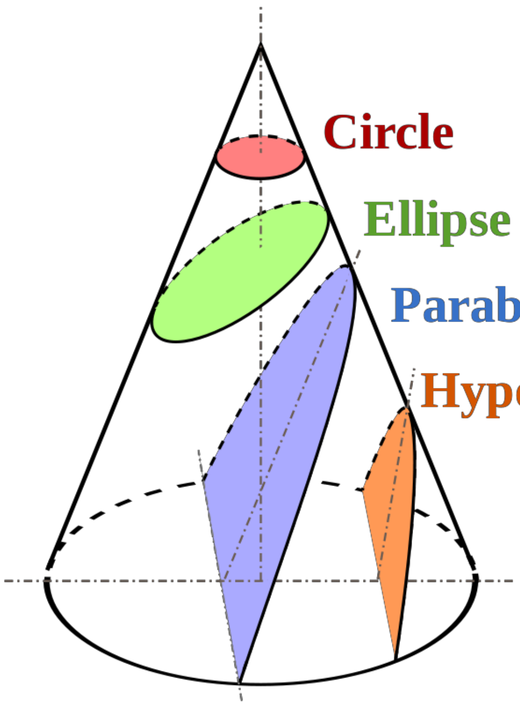
Conic Sections or Geometric Loci



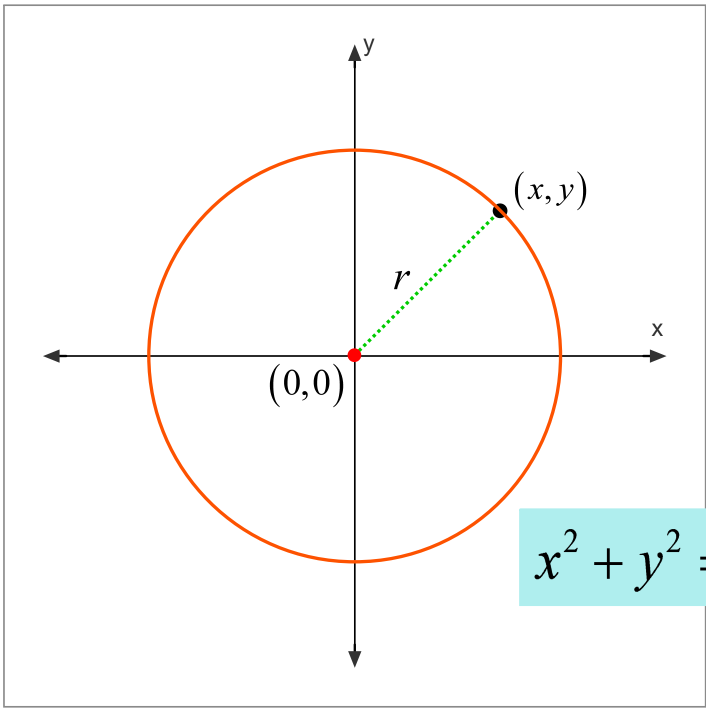
http://www.youtube.com/watch?v=BnWEI5o35G8

http://youtu.be/vREfhjab9Fk

The conic sections are also described as geometric loci.

A geometric locus is a set of points that have a common property.

1) The Circle -- a circle is the set of all points located at an equal distance from a fixed point (the centre)



$$x^2 + y^2 = r^2$$

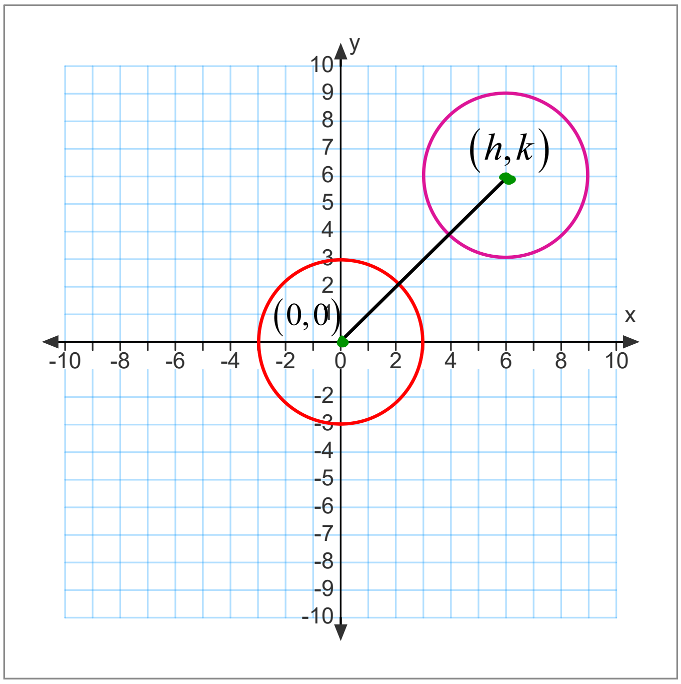
Equation of a circle centred at the origin - standard form

r is the radius of the circle

Example 1: Given $x^2 + y^2 = 169$, find the radius.

Example 2: Find the equation of the circle centred at the origin, given that (2, 3) is on the circle.

Translated Circle - circle not centred at the origin



Equation becomes...

$$(x-h)^2+(y-k)^2=r^2$$

(h,k) is the centre

Example 1: What is the centre and radius of the following circles?

a) $(x-1)^2+(y+5)^2=16$

b) $(x-4)^2+(y+7)^2-36=0$

c) $(x-3)^2+y^2=49$

Example 2: Find the domain and range of the circle

$$(x-2)^2+(y-3)^2=9$$

Expand the equation of the circle

$$(x-5)^2+(y+1)^2=4$$

$$x^2-10x+25+y^2+2y+1=4$$

$$x^2+y^2-10x+2y+26=4$$

$$x^2+y^2-10x+2y+22=0$$

General equation of a circle

$$x^2+y^2+Ax+By+C=0$$

Example 1: Convert to general form

a) $(x-2)^2+(y+2)^2=1$

b) $x^2+(y-3)^2=20$

Example 2: Given a point on the circle, P(4, 2) and the centre (-2, 5), determine the equation of the circle in standard and general form.

Example 3: Find the equation of the circle whose centre is the same as $(x-5)^2 + y^2 = 26$, but passes through the point $P(1,-2)$.

Example 4: Are $x^2 + y^2 - 6x - 8y - 39 = 0$ and $(x-3)^2 + (y-4)^2 = 64$ the equations for the same circle?

Example 5: A circle whose equation is $\left(x-3\right)^2+\left(y-k\right)^2=25$ passes through the point $P(6,-2)$. Find the value(s) of k.

Example 6: Find the equation of the circle

$$(x+2)^2+(y-4)^2-25=0$$

after it has undergone a translation of $t(-8, 5)$.

General to Standard Form

Example: Find the centre and radius of the circle.

$$x^2 + y^2 - 4x + 2y - 11 = 0$$

The centre and radius are found in the standard form

$$(x-h)^2 + (y-k)^2 = r^2$$

Rearrange the equation so that the x terms and y terms are together and the constant is on the right hand side.

$$x^2 - 4x + y^2 + 2y = 11$$

Create two perfect square trinomials on the LHS by <u>completing the square</u>.

$$x^2 - 4x + y^2 + 2y = 11$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 4 + 1$$

What is added to the LHS must be added to the RHS.

Factor the two trinomials on the LHS and simplify the RHS.

$$(x-2)^2+(y+1)^2=16$$

Centre: $(2,-1)$
Radius: 4

Convert $x^2 + y^2 - 14x + 2y + 47 = 0$ to standard form.

Find the equation of the circle whose centre is the same as $x^2 + y^2 - 10x - 1 = 0$, but passes through $P(2, -3)$.

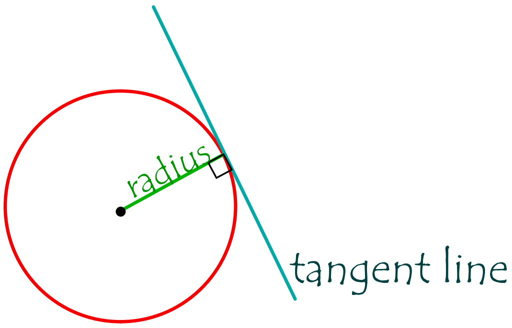
Find the equation of the circle

$$x^2 + y^2 + 12x - 10y - 2 = 0$$

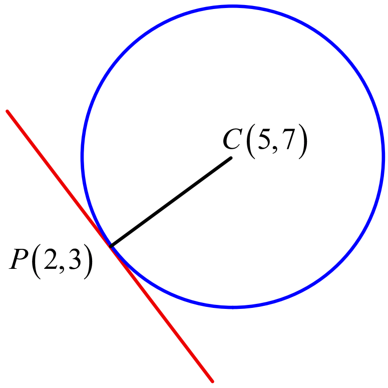
after it has undergone a translation of $t(-6, 1)$.

Tangents to Circles

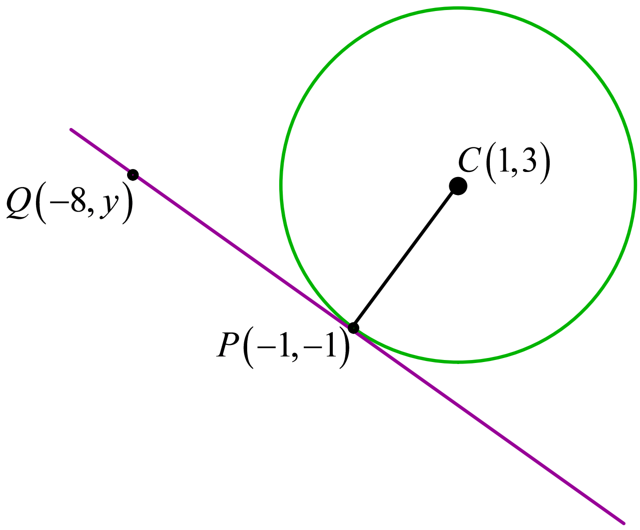
- A tangent line is a line that shares only one point in common with the circle.
- A tangent line is perpendicular to the radius of the circle at the point of contact.



Example: Determine the equation of the tangent line.



Example: Determine y in the point $Q(-8, y)$.



Example: Given the diagram and the equation of the circle:

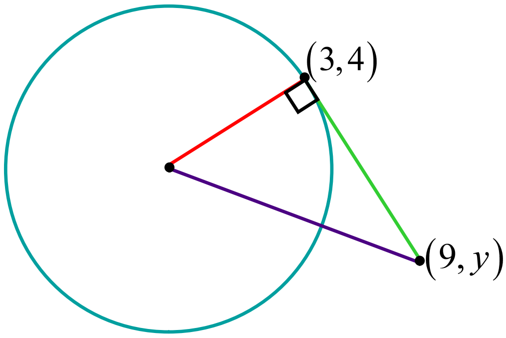
$$x^2 + y^2 + 2x - 2y - 23 = 0$$

Determine:

a) the length of the radius.

b) the domain and range of the circle.

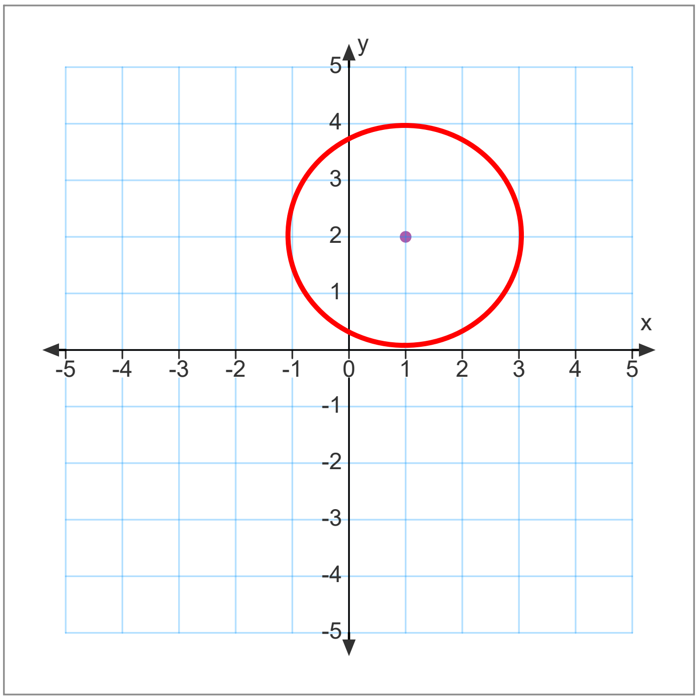
c) the value of y.

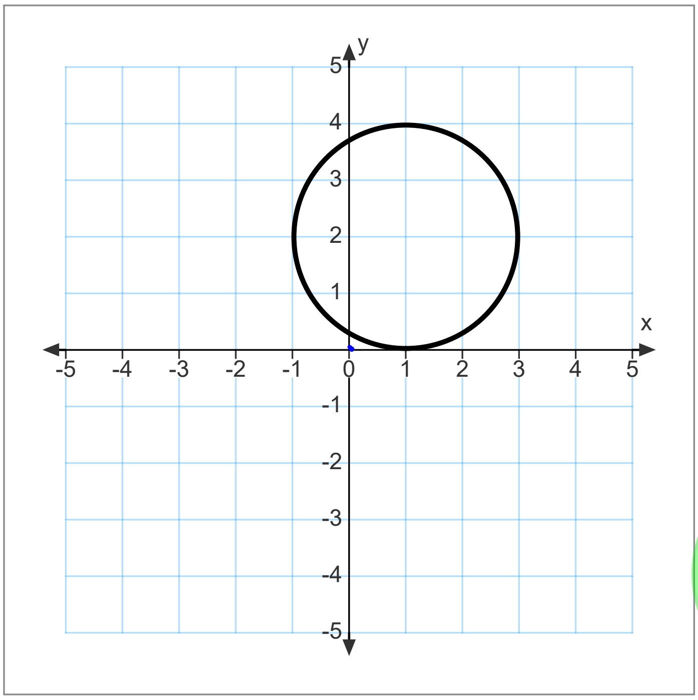


Inequalities and Circles

Example: Draw $(x-1)^2+(y-2)^2 \leq 4$

1. Draw the circle
$$(x-1)^2+(y-2)^2=4$$

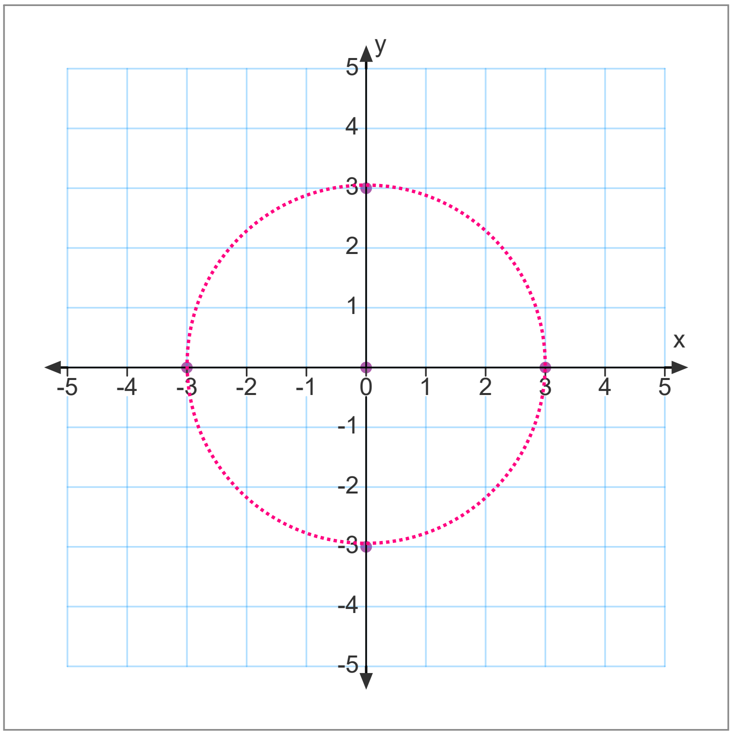




2) Test a point

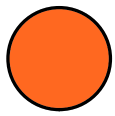
$$(x-1)^2+(y-2)^2 \leq 4$$

Example: Draw the solution set of $x^2 + y^2 > 9$.

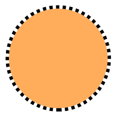


Summary

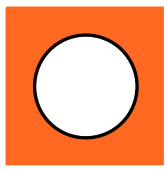
$$(x-h)^2+(y-k)^2 \le r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 < r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 \ge r^2 \longrightarrow$$



$$(x-h)^2+(y-k)^2 > r^2 \longrightarrow$$