

Analytic Geometry

CHAPTER

4

Skill Builder

1 The Distance Between Two Points

EVALUATED



- 1 Calculate the distance between the following two points.
A(3,7) B(6,11)

5 units

My Calculations

$$\begin{aligned} d &= \sqrt{(6-3)^2 + (11-7)^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \end{aligned}$$

- 2 Calculate the distance between the following two points:
A(-7,3) B(2,-1)

9.85 units

My Calculations

$$\begin{aligned} d &= \sqrt{(2-(-7))^2 + (-1-3)^2} \\ &= \sqrt{9^2 + (-4)^2} \\ &= \sqrt{81 + 16} \\ &= \sqrt{97} \end{aligned}$$

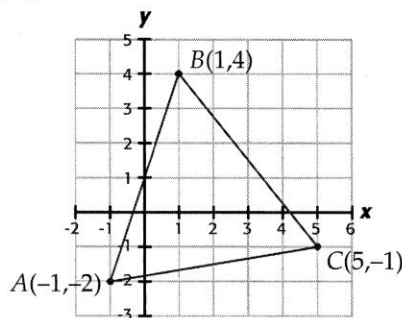
- 3 Calculate the distance between the following two points.
A(-6,2) B(-3,2)

$$x_2 - x_1 = -3 - (-6) = 3 \text{ units}$$

My Calculations

$$\begin{aligned} d &= \sqrt{(-3-(-6))^2 + (2-2)^2} \\ &= \sqrt{3^2 + 0^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

- 4 Find the perimeter of the triangle on the graph below.



18.81 units

My Calculations

$$\begin{aligned} d(A,B) &= \sqrt{(1-(-1))^2 + (4-(-2))^2} \\ &= \sqrt{2^2 + 6^2} \\ &= \sqrt{40} \end{aligned}$$

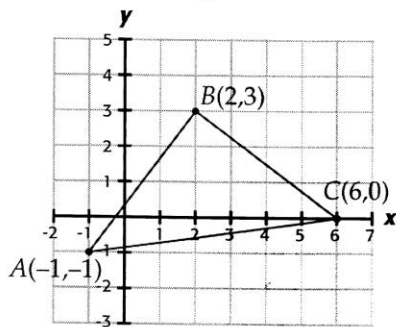
$$\begin{aligned} d(B,C) &= \sqrt{(5-1)^2 + (-1-4)^2} \\ &= \sqrt{4^2 + (-5)^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(5-(-1))^2 + (-1-(-2))^2} \\ &= \sqrt{6^2 + 1^2} \\ &= \sqrt{37} \end{aligned}$$

$$\begin{aligned} \text{perimeter} &= \sqrt{40} + \sqrt{41} + \sqrt{37} \\ &= 18.81 \text{ units} \end{aligned}$$

Skill Builder

- 5 Show that the triangle on the graph below is an isosceles triangle.



My Calculations

$$\begin{aligned} d(A, B) &= \sqrt{(2 - (-1))^2 + (3 - (-1))^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

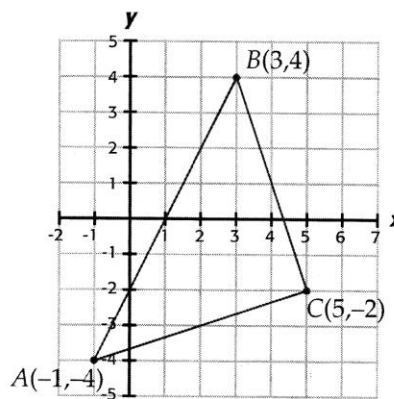
$$\begin{aligned} d(B, C) &= \sqrt{(6 - 2)^2 + (0 - 3)^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} d(A, C) &= \sqrt{(6 - (-1))^2 + (0 - (-1))^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{50} \end{aligned}$$

Isosceles triangles have two congruent sides
 $m\overline{AB} = m\overline{BC}$

$\therefore \triangle ABC$ is isosceles

- 6 Show that the triangle on the graph below is a right angled triangle.



My Calculations

$$\begin{aligned} d(A, B) &= \sqrt{(3 - (-1))^2 + (4 - (-4))^2} \\ &= \sqrt{4^2 + 8^2} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} d(B, C) &= \sqrt{(5 - 3)^2 + (-2 - 4)^2} \\ &= \sqrt{2^2 + (-6)^2} \\ &= \sqrt{40} \end{aligned}$$

$$\begin{aligned} d(A, C) &= \sqrt{(5 - (-1))^2 + (-2 - (-4))^2} \\ &= \sqrt{6^2 + 2^2} \\ &= \sqrt{40} \end{aligned}$$

$$\begin{aligned} (\sqrt{40})^2 + (\sqrt{40})^2 &= (\sqrt{80})^2 \\ a^2 + b^2 &= c^2 \end{aligned}$$

$$\begin{aligned} 40 + 40 &= 80 \\ 80 &= 80 \end{aligned}$$

Pythagoras' Theorem works, so
 $\triangle ABC$ is a right triangle

Skill Builder

2 The Mid-Point of a Line Segment EVALUATED



- 1** Determine the coordinates of the mid-point of the line segment AB where $A = (4, 7)$ and $B = (8, 1)$.

$$(6, 4)$$

My Calculations

$$M\left(\frac{4+8}{2}, \frac{7+1}{2}\right) = M\left(\frac{12}{2}, \frac{8}{2}\right) \\ = M(6, 4)$$

- 2** Determine the coordinates of the mid-point of the line segment AB where $A = (5, 2)$ and $B = (3, 10)$.

$$(4, 6)$$

My Calculations

$$M\left(\frac{5+3}{2}, \frac{2+10}{2}\right) \\ M\left(\frac{8}{2}, \frac{12}{2}\right) = M(4, 6)$$

- 3** Determine the coordinates of the mid-point of the line segment AB where $A = (-3, 2)$ and $B = (5, -4)$.

$$(1, -1)$$

My Calculations

$$M\left(\frac{-3+5}{2}, \frac{2+(-4)}{2}\right) = M\left(\frac{2}{2}, \frac{-2}{2}\right) \\ = M(1, -1)$$

- 4** Determine the coordinates of the mid-point of the line segment AB where $A = (-5, -4)$ and $B = (-1, -10)$.

$$(-3, -7)$$

My Calculations

$$M\left(\frac{-5+(-1)}{2}, \frac{-4+(-10)}{2}\right) \\ M\left(\frac{-6}{2}, \frac{-14}{2}\right) = M(-3, -7)$$

- 5** If point $M(1, 3)$ is the mid-point of segment AB , determine the coordinates of point B if the coordinates of point A are $(5, -1)$.

$$B(-3, 7)$$

My Calculations

$$1 = \frac{x+5}{2} \quad 3 = \frac{-1+y}{2} \\ 2 = x+5 \quad 6 = -1+y \\ -3 = x \quad 7 = y \\ B(-3, 7)$$

- 6** If point $M(-2, -3)$ is the mid-point of segment AB , determine the coordinates of point A if the coordinates of point B are $(-8, -5)$.

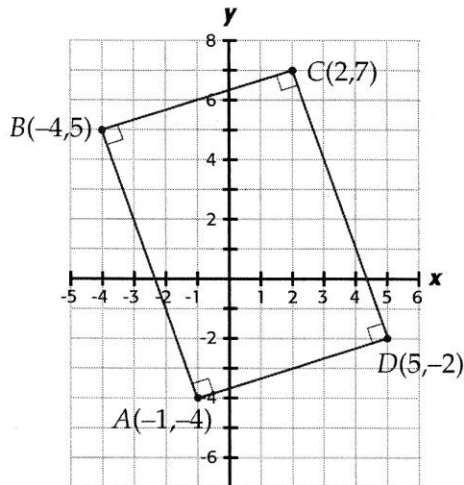
$$A(4, -1)$$

My Calculations

$$-2 = \frac{x+(-8)}{2} \quad -3 = \frac{-5+y}{2} \\ -4 = x+(-8) \quad -6 = -5+y \\ 4 = x \quad y = -1 \\ (4, -1)$$

Skill Builder

- 7** Find the area of the rectangle on the graph below.



60 square units

My Calculations

Area = length \times width

Length = $d(A, B)$

Width = $d(B, C)$

$$\begin{aligned} d(A, B) &= \sqrt{(-4 - (-1))^2 + (5 - (-4))^2} \\ &= \sqrt{(-3)^2 + 9^2} \\ &= \sqrt{90} \end{aligned}$$

$$\begin{aligned} d(B, C) &= \sqrt{(2 - (-4))^2 + (7 - 5)^2} \\ &= \sqrt{6^2 + 2^2} \\ &= \sqrt{40} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \sqrt{90} \times \sqrt{40} \\ &= \sqrt{3600} \\ &= 60 \text{ square units} \end{aligned}$$

- 8** The endpoints of the diameter of a circle are at the points $A(2, 5)$ and $B(8, 13)$. What is the length of the radius?

5

My Calculations

radius = $\frac{1}{2}$ diameter

$$\begin{aligned} d(A, B) &= \sqrt{(8-2)^2 + (13-5)^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

diameter = 10
radius = 5

- 9** The endpoints of the diameter of a circle are at the points $A(-3, 2)$ and $B(1, 5)$. What is the circumference of the circle?

15.71 units

My Calculations

$C = d \cdot \pi$

$$\begin{aligned} d(A, B) &= \sqrt{(1 - (-3))^2 + (5 - 2)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$C = 5 \cdot \pi$
 $C = 15.71 \text{ units}$

- 10** The centre of a circle is at the point $A(7, -3)$. Point B is on the edge of this circle at $B(2, 9)$. What is the area of this circle?

530.93 square units

My Calculations

radius = $d(A, B)$

$$\begin{aligned} d(A, B) &= \sqrt{(2-7)^2 + (9-(-3))^2} \\ &= \sqrt{(-5)^2 + 12^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Area = πr^2
 $= \pi (13)^2$
 $= 169 \cdot \pi$
 $= 530.93$

EVALUATED

Skill Builder

- 7** If point $M\left(\frac{5}{2}, -\frac{3}{2}\right)$ is the mid-point of segment AB , determine the coordinates of point A if the coordinates of point B are $(0, 0)$.

$$A(5, -3)$$

My Calculations

$$\begin{aligned}\frac{5}{2} &= \frac{0+x}{2} & -\frac{3}{2} &= \frac{y+0}{2} \\ 5 &= 0+x & -3 &= y+0 \\ 5 &= x & -3 &= y \\ & & & (5, -3)\end{aligned}$$

- 8** If point $M\left(-\frac{3}{2}, \frac{1}{2}\right)$ is the mid-point of segment AB , determine the coordinates of point B if the coordinates of point A are $(-4, 5)$.

$$B(1, -4)$$

My Calculations

$$\begin{aligned}-\frac{3}{2} &= \frac{-4+x}{2} & \frac{1}{2} &= \frac{y+5}{2} \\ -3 &= -4+x & 1 &= y+5 \\ 1 &= x & -4 &= y \\ & & & (1, -4)\end{aligned}$$

- 9** A circle is drawn on a Cartesian Plane. The end-points of a diameter of this circle are located at point $A = (-2, -1)$ and point $B = (8, 5)$. What are the coordinates of the centre of this circle?

My Calculations

$$\begin{aligned}\text{centre} &: \left(\frac{-2+8}{2}, \frac{-1+5}{2}\right) \\ &= \left(\frac{6}{2}, \frac{4}{2}\right) \\ &= (3, 2)\end{aligned}$$

- 10** The locations of Dan's, Eric's, and Peter's houses have been plotted on a Cartesian Plane, and a straight line can be drawn through the three locations. What are the coordinates of Eric's house if Dan's house $D(-1, 1)$ is the halfway point between Eric's and Peter's house $P(3, 5)$?

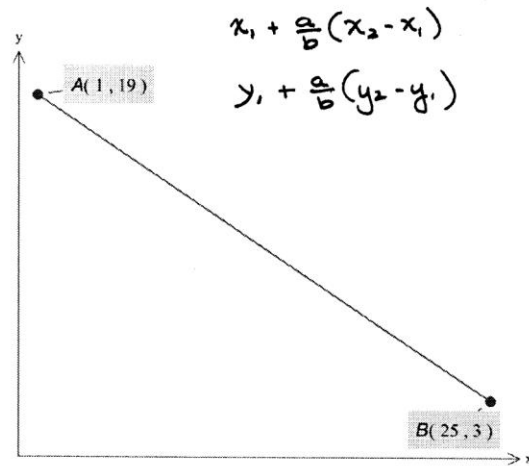
$$E(-5, -3)$$

My Calculations

$$\begin{aligned}\text{Dan: } D \text{ is a midpoint} \\ -1 &= \frac{3+x}{2} & 1 &= \frac{5+y}{2} \\ -2 &= 3+x & 2 &= 5+y \\ -5 &= x & -3 &= y \\ \text{Eric's house} &: (-5, -3)\end{aligned}$$

The Division Point of a Line Segment

1. Alex and Benjamin are sitting at home and talking on the phone with each other. The location of each of their houses is indicated on the graph provided (scaled in kilometres). During their phone call they decide to meet up. Over the next thirty minutes, Alex runs one quarter of the way to Benjamin's house and Benjamin rides his bike five eighths of the way to Alex's house.



What are their locations now and how far apart are they?

Show all work:

① Alex: $(x_1, y_1) = (1, 19)$

	x	y
$1 + \frac{1}{4}(25 - 1)$	$1 + \frac{1}{4}(24)$	$19 + \frac{1}{4}(3 - 19)$
	$= 1 + 6$	$= 19 + \frac{1}{4}(-16)$
	$= 7$	$= 19 - 4$
		$= 15$

Alex is at $(7, 15)$

② Benjamin $(x_1, y_1) = (25, 3)$

	x	y
$25 + \frac{5}{8}(1 - 25)$	$3 + \frac{5}{8}(19 - 3)$	
$= 25 + \frac{5}{8}(-24)$	$= 3 + \frac{5}{8}(16)$	
$= 25 - 15$	$= 3 + 10$	
$= 10$	$= 13$	

Benjamin is at $(10, 13)$

③ distance = $\sqrt{(10 - 7)^2 + (13 - 15)^2}$

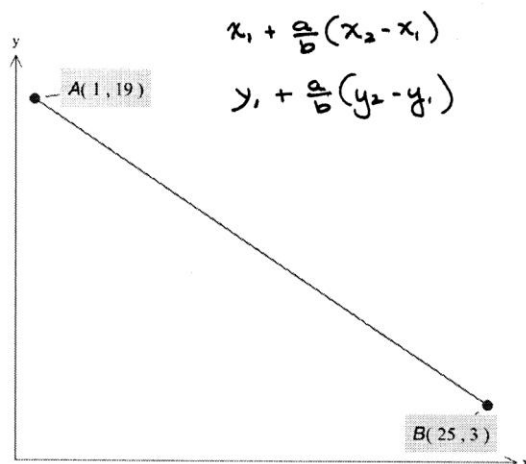
$$\begin{aligned}
 &= \sqrt{3^2 + (-2)^2} \\
 &= \sqrt{9 + 4} \\
 &= \sqrt{13}
 \end{aligned}$$

They are 3.61 km apart.

The Division Point of a Line Segment

1. Alex and Benjamin are sitting at home and talking on the phone with each other. The location of each of their houses is indicated on the graph provided (scaled in kilometres). During their phone call they decide to meet up. Over the next thirty minutes, Alex runs one quarter of the way to Benjamin's house and Benjamin rides his bike five eighths of the way to Alex's house.

What are their locations now and how far apart are they?



Show all work:

① Alex: $(x_1, y_1) = (1, 19)$

x	y
$1 + \frac{1}{4}(25 - 1) = 1 + \frac{1}{4}(24)$ $= 1 + 6$ $= 7$	$19 + \frac{1}{4}(3 - 19)$ $= 19 + \frac{1}{4}(-16)$ $= 19 - 4$ $= 15$

Alex is at $(7, 15)$

② Benjamin $(x_1, y_1) = (25, 3)$

x	y
$25 + \frac{5}{8}(1 - 25)$ $= 25 + \frac{5}{8}(-24)$ $= 25 - 15$ $= 10$	$3 + \frac{5}{8}(19 - 3)$ $= 3 + \frac{5}{8}(16)$ $= 3 + 10$ $= 13$

Benjamin is at $(10, 13)$

③ distance = $\sqrt{(10 - 7)^2 + (13 - 15)^2}$

$$\begin{aligned}
 &= \sqrt{3^2 + (-2)^2} \\
 &= \sqrt{9 + 4} \\
 &= \sqrt{13}
 \end{aligned}$$

They are 3.61 km apart.