Objective Function

A system of linear inequalities has many solutions. Depending on the situation, some of these solutions (usually one) are better than the others and will be called the optimal solution . What determines this optimal solution is the objective function or objective rule .

Example:

A farmer grows cherries & raspberries on a piece of land that is at most $16ha$ in area. Each hectare of cherries requires 5 days of work and each hectare of raspberries, 3 days of work. The farmer has no more than 60 days available. He decides that the space for raspberries will be at most 3 times the amount of space for cherries. Each hectare of cherries and rasperries produces revenues of $\$3000$ and $\$5000$ respectively. What is the maximum revenue the farmer can earn?

Variables:

x : # of ha of cherries

y : # of ha of raspberries

Constraints:

$x \geq 0$

$y \geq 0$

$x + y \leq 16$

$5x + 3y \leq 60$

$y \leq 3x$

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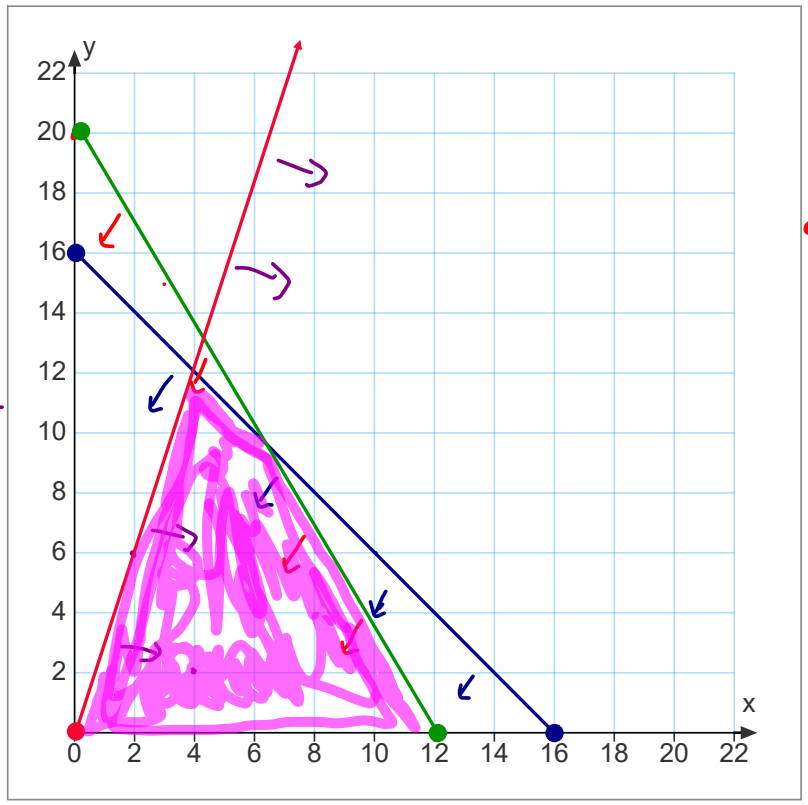
The farmer's objective is to make the maximum revenue possible given the constraints.

The objective rule for this farmer is $R = 3000x + 5000y$



$x \geq 0$
$y \geq 0$
} Q_1

- $x + y \leq 16$
- $5x + 3y \leq 60$
- $y \leq 3x$

x	y
0	16
16	0
10	6

$x + y = 16$ solid

Test (0,0)
$0 \leq 16$ True

x	y
0	20
12	0
3	15

$5x + 3y = 60$ solid

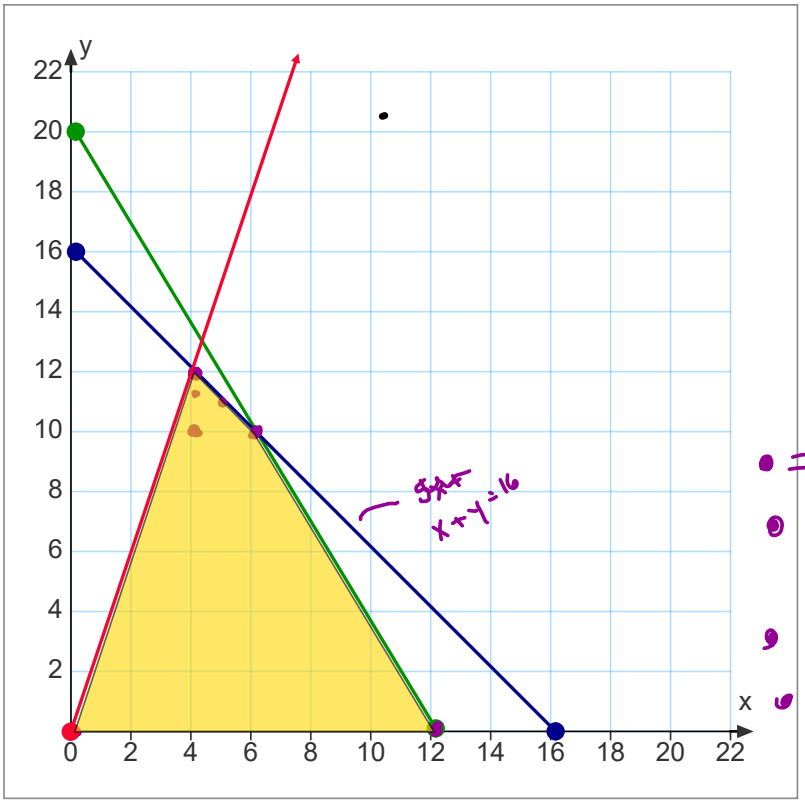
$\rightarrow 15 + 3y = 60$
$3y = 45$

Test (0,0)
$0 \leq 60$ True

x	y
0	0
3	6
5	15

$y = 3x$ solid

Test (4,2)
$2 \leq 12$ True



$3000x + 5000y = 60000$

maximum

$\Rightarrow R = 3000x + 5000y$

Points	Revenue
(9, 12)	27000 + 60000 = $72 000
(6, 10)	18000 + 50 000 = $68 000
(4, 10)	$62 000
(12, 0)	36000
(0, 0)	0
(4, 11.5)	69500
(5, 11)	70 000

The optimal solutions (maximum or minimum) occur on the boundary of the solution set and usually occur only at the vertices.

Ans: The maximum revenue is $72 000.

Solving Optimisation Problems

1. Define the variables.
2. List the constraints.
3. Write the objective function.
4. Graph the polygon of constraints.
5. Determine the coordinates of the vertices of the polygon.
6. Identify the optimal solution - the maximum or minimum that solve the problem.

Example:

Joan wants to give at least 12 chocolates to her children at Easter. She intends to buy at least twice as many dark chocolates as milk chocolates, but no more than 20 dark chocolates.

One milk chocolate costs $2.00 and one dark chocolate costs $4.00.

How many of each type of chocolate should Joan buy in order to minimise her costs ?

x: # of d. choc

y: # of m. choc