# Determining the Solution Set of a Linear Inequality

A local swimming pool uses a mixture of chlorine and bromine to purify the water. A litre of chlorine costs \$10 and a litre of bromine costs \$16. The pool manager buys a total of at least \$240 worth of these products.

1. Variables:

$x$: # of L of Cl
$y$: # of L of Br

2. Constraints:

$10x + 16y \geq 240$

$\left.\begin{array}{l} x \geq 0 \\ y \geq 0 \end{array}\right\}$ Quadrant 1

1. Graph the line. Recall that if the inequality includes the ~~equal sign~~, the line is drawn solid, but for a strict inequality, the line is broken.

$10x + 16y = 240$

$16y = 240$
$y = 15$

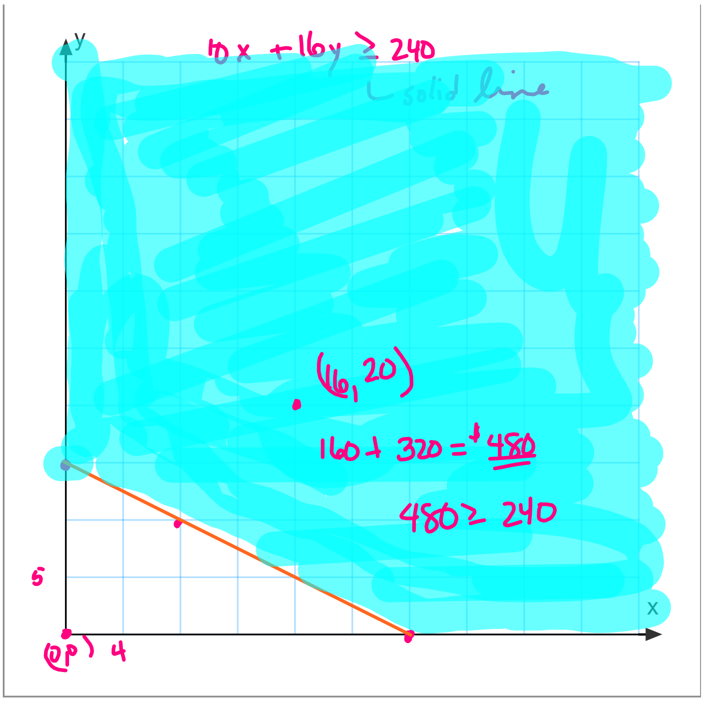
$10x = 240$
$x = 24$

$80 + 16y = 240$
$16y = 160$

| x | y | |
|---|---|---|
| 0 | 15 | y-int |
| 24 | 0 | x-int |
| 8 | 10 | |

2. Choose a point. Test the point in the ~~inequality~~.

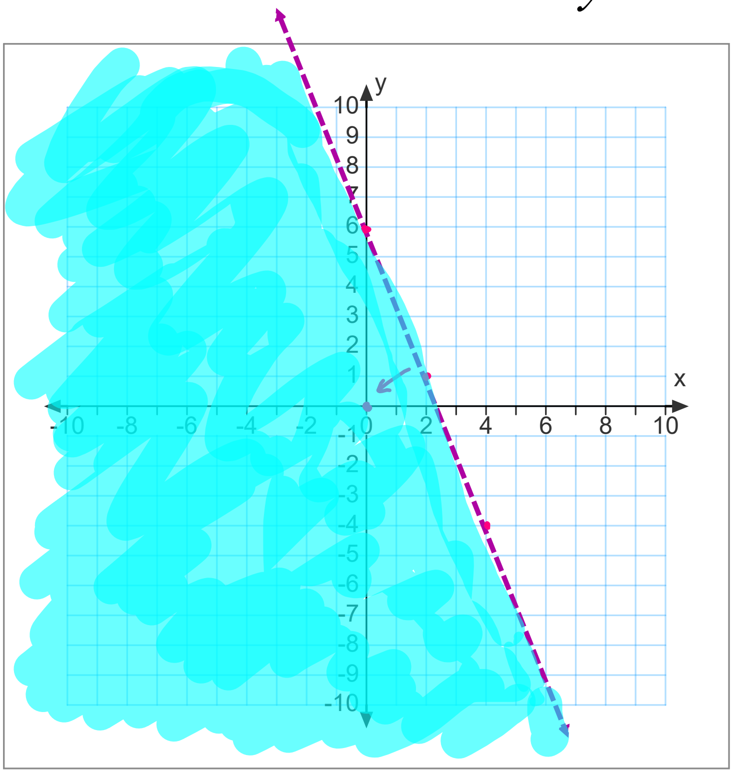
test $(0,0)$ $10(0) + 16(0) \geq 240$
$0 \geq 240$
False




3. Shade on the side of the line where the inequality is TRUE.

Example: Graph the solution set of the inequality

$$5x + 2y < 12$$




$5x + 2y = 12$

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |

Graph: $5x+2y=12$

$$\frac{2y}{2} = -\frac{5x}{2} + \frac{12}{2}$$

$$\boxed{y = -\frac{5}{2}x + 6}$$

$-\frac{5}{2}$ or $\frac{5}{-2}$

$\frac{\text{rise}}{\text{run}}$

$5x + 2y < 12 \rightarrow$ broken

Test $(0,0) \Rightarrow 0 + 0 < 12$

$0 < 12$ True

## Systems of Linear Inequalities

Solving a system of linear inequalities means finding all the points that satisfy all the inequalities.

Example: Determine the solution set of the following system.

$$y > -x$$
$$y \leq x$$

$y > -x$

⇓ broken line

$y = -x$

| x | y |
|---|---|
| 0 | 0 |
| 5 | -5 |
| -8 | 8 |

Test (0, 5)

$5 > 0$ True

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$y \le x$

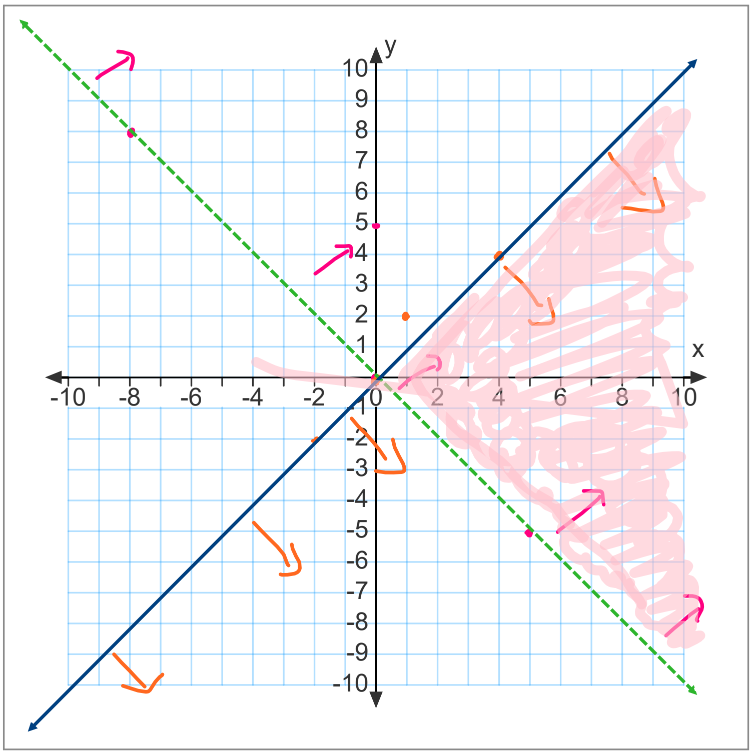
solid line

$y = x$

| x | y |
|---|---|
| 0 | 0 |
| 4 | 4 |
| -2 | -2 |

Test (1, 2)

$2 \le 1$ False



# Graph the following system of inequalities

- $y \geq -x - 2$ solid

| x | y |
|---|---|
| 0 | -2 |
| 5 | -7 |
| -2 | 0 |

Test: $0 \geq -2$ True

- $y \leq x + 4$
- $3x + y \leq 8$

| x | y |
|---|---|
| 0 | 4 |
| 1 | 5 |
| 2 | 6 |

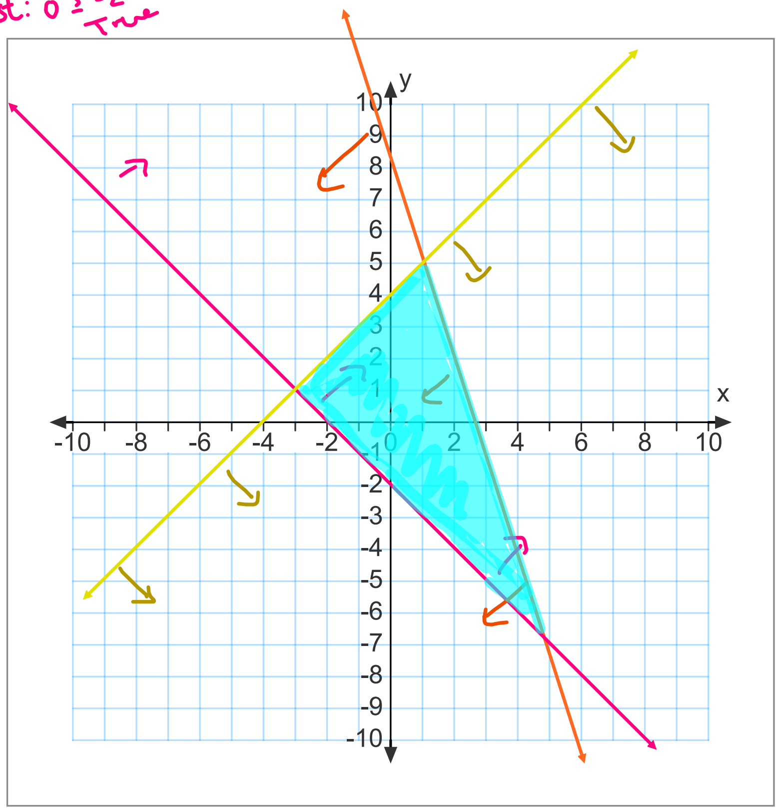
solid

Test: $0 \leq 0 + 4$
$0 \leq 4$ True

| x | y |
|---|---|
| 0 | 8 |
| 1 | 5 |
| 2 | 2 |

solid

Test: $0 + 0 \leq 8$
$0 \leq 8$
True



The figure created by the solution set of all the inequalities is called the polygon of constraints.

Example: A municipal garden grows red roses and white roses. There are ~~at most~~ 400 roses in total. The number of red roses increased by 80 is greater than twice the number of white roses.

Graph the solution set.

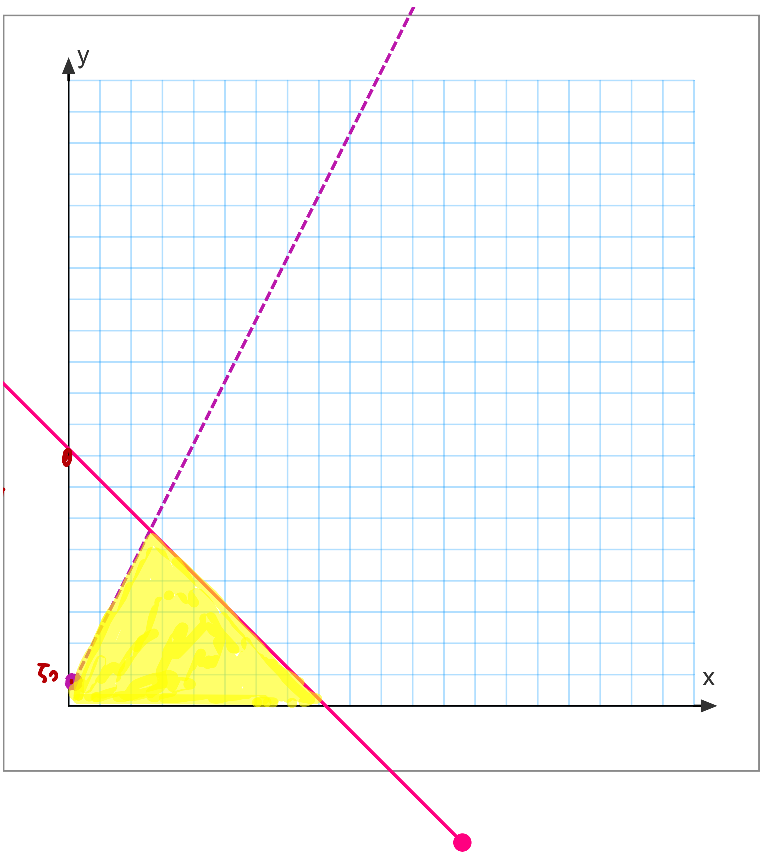
① Variables

$x$: # of red roses

$y$: # of white roses

② Constraints

$$\begin{cases} x + y \le 400 \\ x + 80 > 2y \\ \boxed{\begin{matrix} x \ge 0 \\ y \ge 0 \end{matrix}} \Rightarrow Q_1 \end{cases}$$




| $x + y \le 400$ | $x + 80 > 2y$ |
|---|---|

| x | y |
|---|---|
| 0 | 400 |
| 400 | 0 |
| 200 | 200 |

$0 \le 400$

$\frac{1}{2}x + 40 = y$

$80 > 0$

Not a polygon of constraints (Unbound).