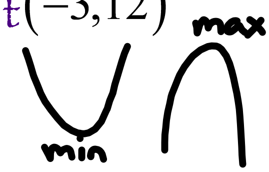
Example: Determine the equation of the second-degree function with a ~~maximum~~ vertex at $(-3,12)$ and another point at $(4,-5)$?



vertex $(-3,12) = (h,k)$

point $(4,-5) \Rightarrow (x,y)$

$$f(x)=a(x-h)^2+k$$

$$f(x)=a(x+3)^2+12$$

$$-5=a(4+3)^2+12$$

$$-5=a(7)^2+12$$

$$-5=49a+12$$

$$-12 \qquad -12$$

$$-17=49a$$

$$\div 49 \qquad \div 49$$

$$\frac{-17}{49}=a$$

$$\boxed{f(x)=-\frac{17}{49}(x+3)^2+12}$$

Finding the Zeros (x- intercepts)

- This is where the function crosses the x-axis.
- To determine the zeros of a function we let $y = 0$, then solve for (or isolate) x.

Example: $f(x) = -\frac{3}{5}(x-1)^2 + 15$ **Find the zeros.**

let y (or f(x)) = 0

$0 = -\frac{3}{5}(x-1)^2 + 15$

$-15 \qquad\qquad -15$

$-15 = -\frac{3}{5}(x-1)^2$

$\div -\frac{3}{5} \qquad \div -\frac{3}{5}$

$x -\frac{5}{3} \qquad x -\frac{5}{3}$

$\div -0.6 \qquad \div -0.6$

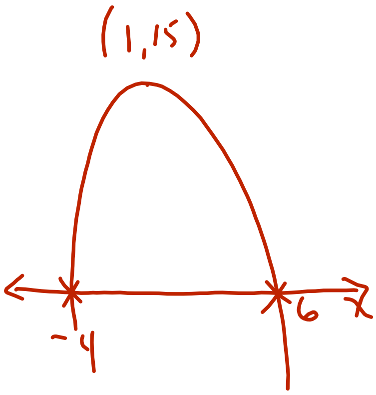
$25 = (x-1)^2$

$\pm\sqrt{25} = \sqrt{(x-1)^2}$

$-5, 5 = x-1$

$-5 = x-1 \qquad 5 = x-1$

$-4 = x \qquad 6 = x$



zeros: $\{-4, 6\}$

There is a formula...

$$f(x) = a(x-h)^2 + k$$

let $y = 0$

$$0 = a(x-h)^2 + k$$

$$-k = a(x-h)^2$$

$$\frac{-k}{a} = (x-h)^2$$

$$\pm\sqrt{-k/a} = x-h$$

$$\boxed{x = h \pm \sqrt{\frac{-k}{a}}}$$

ex: $f(x) = 5(x-3)^2 - 20$

$$0 = 5(x-3)^2 - 20$$

$a = 5 \quad h = 3 \quad -20 = k$

$$x = h \pm \sqrt{\frac{-k}{a}}$$

$$x = 3 \pm \sqrt{\frac{20}{5}}$$

$$x = 3 \pm \sqrt{4}$$

$$x = 3 \pm 2$$

$x_1 = 3+2 \quad x_2 = 3-2$

$\{5, 1\}$

Determine the zeros of the following functions.

a) $f(x)=2(x-5)^2-11$

b) $f(x)=-\frac{2}{5}(x+3)^2+4$

c) $f(x)=4(x-10)^2+7$

d) $f(x)=2(x-17)^2$

a) $f(x) = 2(x-5)^2 - 11$

let $y = 0$

$$0 = 2(x-5)^2 - 11$$

Solve for x

$$11 = 2(x-5)^2$$

$$5.5 = (x-5)^2$$

$$\pm\sqrt{5.5} = x - 5$$

$$\pm 2.35 \approx x - 5$$

1. $2.35 \approx x - 5$

2. $-2.35 \approx x - 5$

$7.35 \approx x$ and $2.65 \approx x$ Zeros

$x-h \Rightarrow x-(-3)$

b) $f(x) = -\frac{2}{5}(x+3)^2 + 4$

$a = -\frac{2}{5} = -0.4$

$h = -3$

$k = 4$

let $y = 0$

$0 = -\frac{2}{5}(x+3)^2 + 4$

Solve for x

$x = h \pm \sqrt{\frac{-k}{a}}$

$x = -3 \pm \sqrt{\frac{-4}{-0.4}}$

$x = -3 \pm \sqrt{10}$

$x = -3 \pm 3.16$

1\. $3.16 \approx x + 3$

$0.16 \approx x$

2\. $-3.16 \approx x + 3$

$-6.16 \approx x$

① $-3+3.16$

0.16

② $-3-3.16$

-6.16

c) $f(x) = 4(x-10)^2 + 7$

let $y = 0$

$0 = 4(x-10)^2 + 7$

$-7 \qquad\qquad -7$

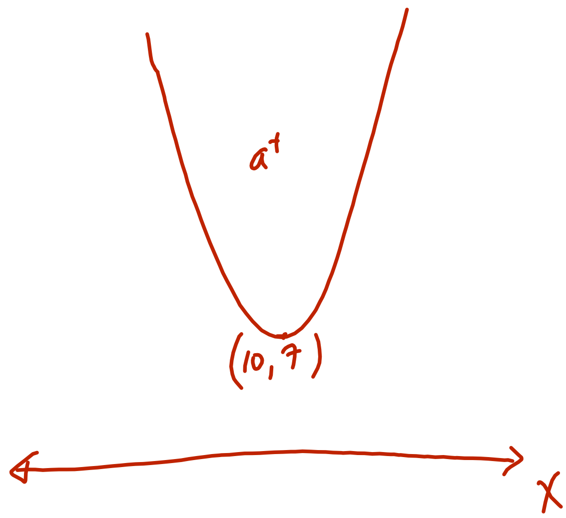
Solve for x

$-7 = 4(x-10)^2$

$\div 4 \quad \div 4$

$-1.75 = (x-10)^2$

$\pm\sqrt{-1.75} = x - 10$



But $\sqrt{-1.75}$ is impossible; therefore no zeros.

Notice that parameters a & k are both positive.
When a & k have the same sign, there are no zeros.

d) $f(x)=2(x-17)^2$

$a=2$

$h=17$

$\boxed{k=0}$

$x=h\pm\sqrt{\frac{-k}{a}}$

$x=17\pm\sqrt{\frac{0}{2}}$

$=17\pm\sqrt{0}$

$=17\pm0$

$=17$

when $k=0$ the vertex is the only zero.

When finding zeros, there are 3 possibilities:

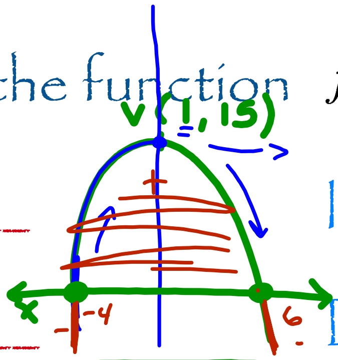
1) there are 2 solutions (a & k have opposite signs)

2) there is one solution (the vertex is the zero; $k = 0$)

3) there are no solutions (a & k have the same sign)

Do a study of the function $f(x) = -\frac{3}{5}(x-1)^2 + 15$.

a, $h=1$, $k=15$



$x = 1 \pm \sqrt{\frac{-15}{-0.6}}$

$x = 1 \pm \sqrt{25}$

$x = 1 \pm 5$

$x = \{-4, 6\}$

Dom: $\mathbb{R}$

Ran: $]-\infty, 15]$

Max: 15

Min: None

Positive: $[-4, 6]$

Negative: $]-\infty, -4] \cup [6, +\infty[$

Increasing: $]-\infty, 1]$

Decreasing: $[1, +\infty[$

$x=0$

y-intercept: 14.4

$y=0$

Zero(s): $\{-4, 6\}$

Axis of symmetry: $x = 1$