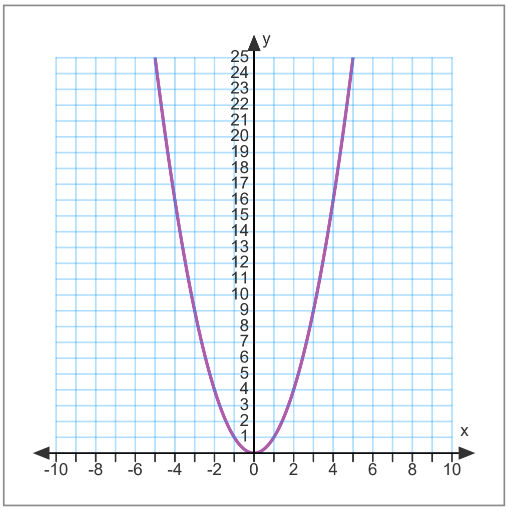
Second-Degree Function

- Also known as: Quadratic Function.

Basic Function Rule: $f(x) = x^2$

Graph:

x	y



Properties
Dom:
Ran:
Increasing:
Decreasing:
Positive:
Negative:
Max:
Min:
y - intercept:
Zero:

Shape is called a parabola .

Transformed Function

Rule: $f(x)=a\left(b(x-h)\right)^2+k$

Because of its symmetry with respect to the y-axis, plus the fact that a and b produce similar effects on the graph, we are able to write the equation without parameter b.

Example:

$$f(x)=2\left(-3(x-4)\right)^2+5$$
$$f(x)=2(-3)^2(x-4)^2+5$$
$$f(x)=2(9)(x-4)^2+5$$
$$f(x)=18(x-4)^2+5$$

Parameters a and b combine to make a "new" a.

Convert each into the form $f(x) = a(x-h)^2 + k$.

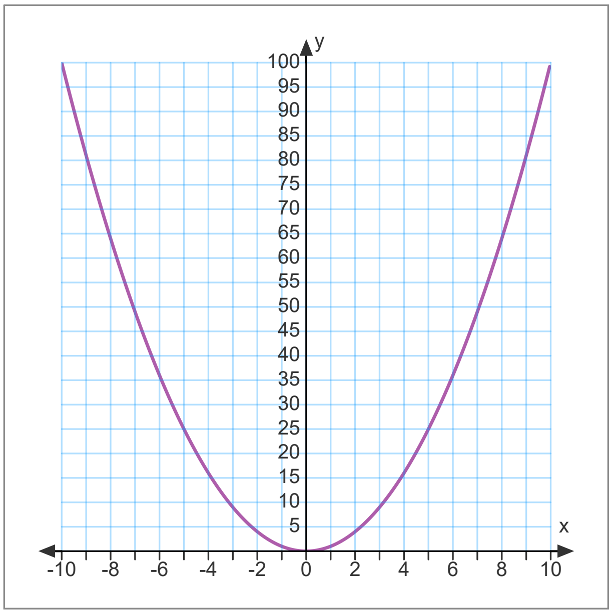
a) $f(x) = -\frac{4}{3}(2(x+5))^2 - 7$

b) $y = 3\left(\frac{1}{2}(x-1)\right)^2 + 10$

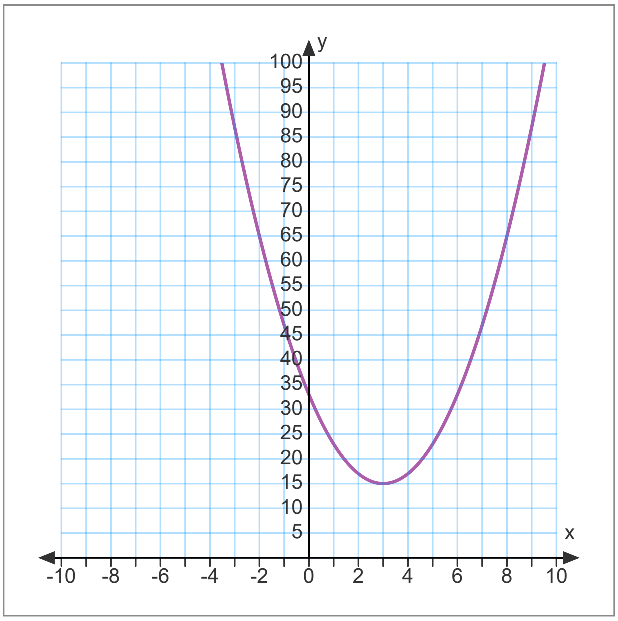
Therefore, the rule for the transformed second-degree function, in STANDARD FORM, is

$$f(x) = a(x-h)^2 + k$$

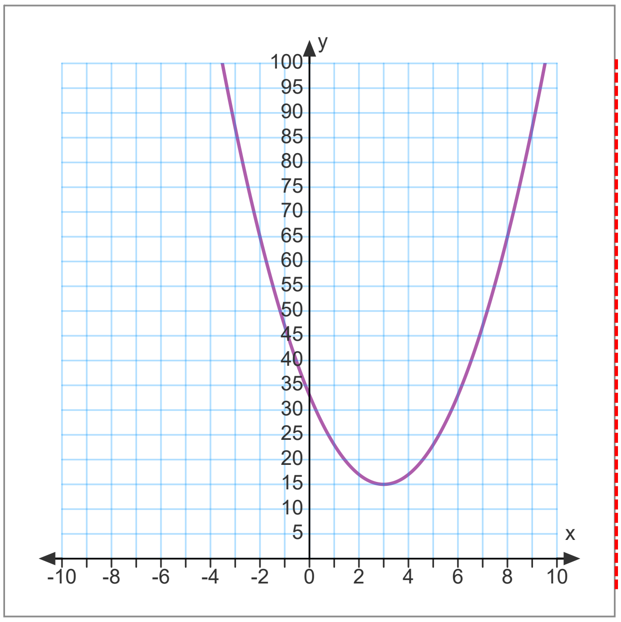
$$f(x) = x^2$$



$$f(x) = 2(x-3)^2 + 15$$



$f(x)=2(x-3)^2+15$



a determines how the parabola opens: up, down, wide, thin.

h & k determine the vertex of the parabola.

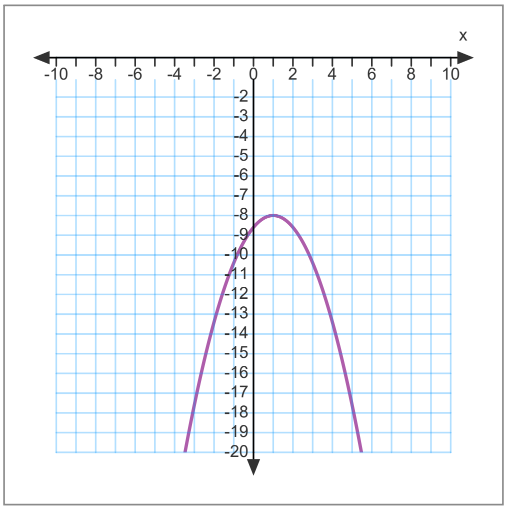
$V(h,k)$

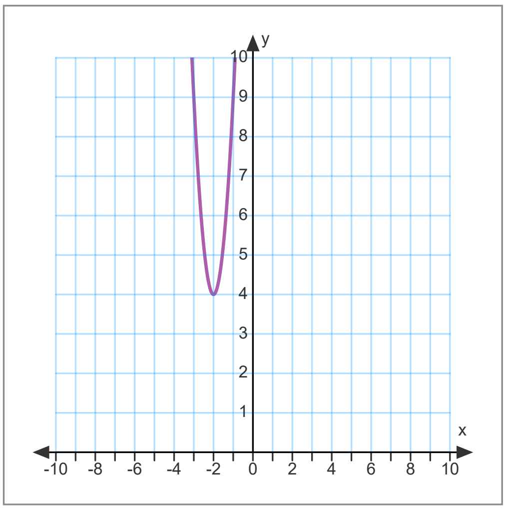
Axis of symmetry: the vertical line passing through the vertex. Its equation is $x=h$.

Example: Using the parameters, describe the graph of each quadratic function.

a) $f(x) = -\frac{3}{5}(x-1)^2 - 8$

b) [redacted]

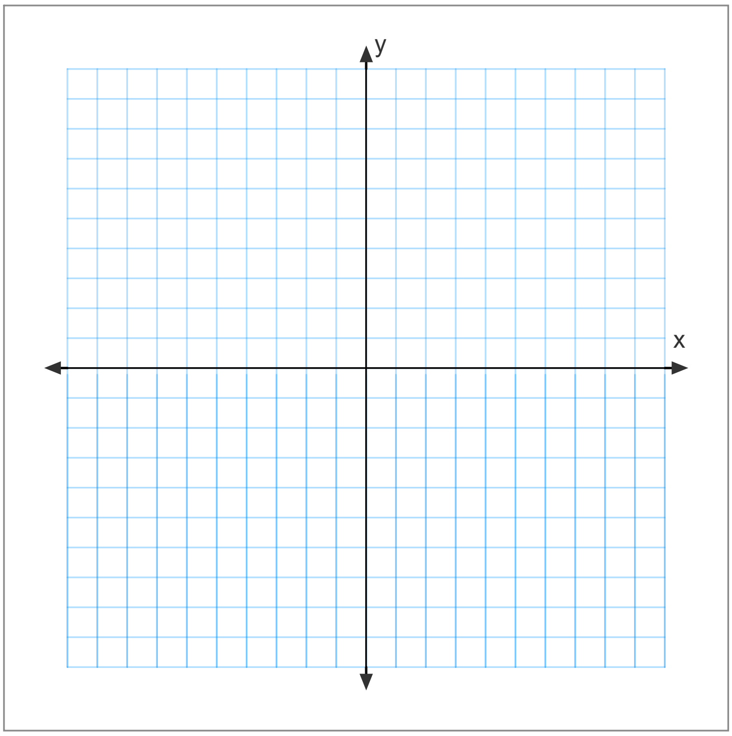




Graphing a Second-Degree Function

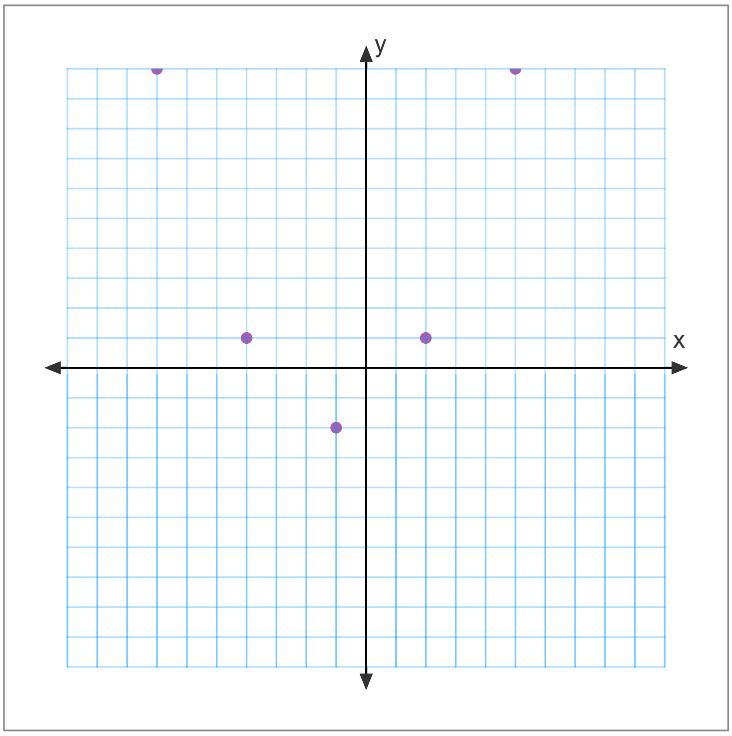
Example: $y=-(x-4)^2+2$

x	y



Example: Graph the function $f(x)=\frac{2}{3}(x+1)^2-4$.

x	y

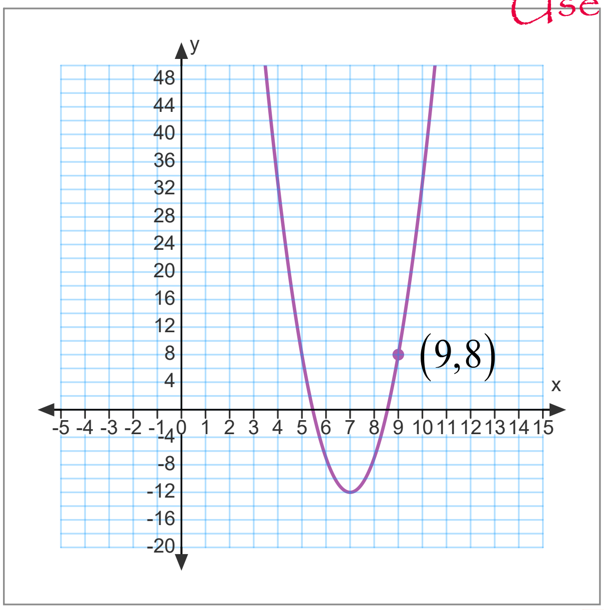


Finding the Rule of a Second-degree Function

- Given the vertex and a point

Use $f(x) = a(x-h)^2 + k$

Example:



$f(x) = 5(x-7)^2 - 12$

Example: What is the equation of the second-degree function whose vertex is $(11, 24)$ and passes through the point $(6, 42)$?

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-11)^2 + 24$$

$$42 = a(6-11)^2 + 24$$

$$42 = a(-5)^2 + 24$$

$$42 = 25a + 24$$

$18 = 25a$ ⟶ $a = \frac{18}{25} = 0.72$

$f(x) = \frac{18}{25}(x-11)^2 + 24$ or $f(x) = 0.72(x-11)^2 + 24$

Example: What is the equation of the second-degree function whose vertex is $V(-10,-2)$ and whose y-intercept is -12?

The given point is $P(0,-12)$.

$$f(x)=a(x-h)^2+k$$
$$f(x)=a(x+10)^2-2$$
$$-12=a(0+10)^2-2$$

$$-12=100a-2$$
$$-10=100a$$
$$\frac{-10}{100}=-\frac{1}{10}=-0.1=a$$

$$f(x)=-\frac{1}{10}(x+10)^2-2$$

Example: A second-degree function has the following properties:

Ran: $]-\infty, 7]$

Zeros: $\{-1, 9\}$

Axis of symmetry: $x = 4$

Determine the equation of this function.

Finding the Zeros

- This is where the function crosses the x-axis.
- To determine the zeros of a function we let $y = 0$, then solve for (or isolate) x.

Example: $f(x) = -\frac{3}{5}(x-1)^2 + 15$

Do a study of the function $f(x) = -\frac{3}{5}(x-1)^2 + 15$.

Dom: ___________

Ran: ___________

Max: ___________

Min: ___________

Positive: ___________

Negative: ___________

Increasing: ___________

Decreasing: ___________

y-intercept: ___________

Zero(s): ___________

There is also a formula for finding zeros for the standar form.

$f(x) = a(x-h)^2 + k$ Let $y = 0$

$$x = h \pm \sqrt{\frac{-k}{a}}$$

Determine the zeros of the following functions.

a) $f(x)=2(x-5)^2-11$

b) $f(x)=-\frac{2}{5}(x+3)^2+4$

c) $f(x)=4(x-10)^2+7$

d) $f(x)=2(x-17)^2$

a) $f(x) = 2(x-5)^2 - 11$

let $y = 0$

$0 = 2(x-5)^2 - 11$

Solve for x

$x = 5 \pm 2.35$

1. $2.35 \approx x - 5$ 2. $-2.35 \approx x - 5$

$7.35 \approx x$ and $2.65 \approx x$ ← Zeros

b) $f(x) = -\frac{2}{5}(x+3)^2 + 4$

let $y = 0$

$0 = -\frac{2}{5}(x+3)^2 + 4$

Solve for x

[redacted]

1. $3.16 \approx x + 3$

$0.16 \approx x$

2. $-3.16 \approx x + 3$

$-6.16 \approx x$

c) $f(x)=4(x-10)^2+7$

let $y=0$

$0=4(x-10)^2+7$

Solve for x

$-7=4(x-10)^2$

$-1.75=(x-10)^2$

$\pm\sqrt{-1.75}=x-10$

But $\sqrt{-1.75}$ is impossible; therefore no zeros.

Notice that parameters a & k are both positive.
When a & k have the same sign, there are no zeros.

d) $f\left(x\right)=2\left(x-17\right)^2$

When finding zeros, there are 3 possibilities:

1) there are 2 solutions (a & k have opposite signs)

2) there is one solution (the vertex is the zero; $k = 0$)

3) there are no solutions (a & k have the same sign)

Example: Provide a study of the following function.

$$f(x) = -(x+4)^2 + 9$$

Dom: ___________

Ran: ____________

Max: ____________

Min: ____________

Positive: ____________

Negative: ____________

Increasing: ____________

Decreasing: ___________

y-intercept: ___________

Zero(s): ____________

Example: Provide a study of the following function.

$$f(x)=\frac{2}{5}(x+6)^2+3$$

Dom: ___________

Ran: ____________

Max: ____________

Min: ____________

Positive: ____________

Negative: ____________

Increasing: ____________

Decreasing: ___________

y-intercept: ____________

Zero(s): ____________

Axis of Symmetry: ______

$$g(x) = -6(x-27)^2 + 54$$

Dom: ___________

Ran: ___________

Max: ___________

Min: ___________

Positive: ___________

Negative: ___________

Increasing: ___________

Decreasing: ___________

y-intercept: ___________

Zero(s): ___________

Axis of Symmetry: ______

$$h(x) = 2.75(x-10)^2 - 11$$

Dom: ___________

Ran: ___________

Max: ___________

Min: ___________

Positive: ___________

Negative: ___________

Increasing: ___________

Decreasing: ___________

y-intercept: ___________

Zero(s): ___________

Axis of Symmetry: ______

Example: A baseball is struck at a height of $1.1m$ above the ground. Three seconds later, it reaches its maximum height of $45.2m$. The height of the ball's trajectory is a second degree function of time.

We know: 1. Vertex

2. Point

Can find: Equation

$$f(t) = a(t-3)^2 + 45.2$$

$$1.1 = a(0-3)^2 + 45.2$$

$$1.1 = a(-3)^2 + 45.2$$

$$1.1 = 9a + 45.2$$

$$-44.1 = 9a$$

$$-4.9 = a$$

$$\therefore f(t) = -4.9(t-3)^2 + 45.2$$

a) What is the ball's height after **2** seconds?

$$f(t) = -4.9(t-3)^2 + 45.2$$

let $t = 2$

$$f(2) = -4.9(2-3)^2 + 45.2$$

$$f(2) = -4.9(-1)^2 + 45.2$$

$$f(2) = -4.9 + 45.2$$

$$f(2) = 40.3$$

The ball's height is $40.3m$.

b) At what time would a player catch the ball **2.2*m*** above the ground ?

$$f(t) = -4.9(t-3)^2 + 45.2$$

let $y = 2.2$

$$2.2 = -4.9(t-3)^2 + 45.2$$

$$-43 = -4.9(t-3)^2$$

$$8.78 \approx (t-3)^2$$

$$\pm 2.96 \approx t-3$$

1) $2.96 = t-3$

$5.96 = t$

2) $-2.96 = t-3$

$0.04 = t$

The player catches the ball after 5.96 seconds.

c) If no one catches the ball, when will it hit the ground?

$$f(t) = -4.9(t-3)^2 + 45.2$$

let $y = 0$

$$0 = -4.9(t-3)^2 + 45.2$$

$$-45.2 = -4.9(t-3)^2$$

$$9.22 \approx (t-3)^2$$

$$\pm 3.04 \approx t - 3 \longrightarrow$$

1)

2)

The ball hits the ground after approximately $6.04\ s$.

d) What is the domain and range of this situation?

Domain: ___________

Range: ___________

Determine the rule of the second-degree function whose table of values is...

x	y
0	-6
1	6
2	10
3	6
4	-6

Example: The elevation of a firework was measured from the time of launch until it exploded. Its trajectory can be described by the function

$$h(t) = -5(t-4.3)^2 + 93$$

a) What was the maximum height reached by this firework? ________

b) When was the maximum height achieved? ________

c) At what height was the firework launched? ________

d) How many seconds after launch did the the firework explode if its height was 90.55m?

$$h(t) = -5(t-4.3)^2 + 93$$

let $y = 90.55$

$$90.55 = -5(t-4.3)^2 + 93$$