Optimisation

or Linear Programming

Example: A furniture manufacturer makes chairs and armchairs. The amount of time spent on making a chair is 3 hours and the amount of time spent on an armchair is 5 hours. In one week, the time spent on finishing these two pieces is equal to 45 hours.

a) Translate this situation into an equation.

x: # of chairs
y: # of armchairs

$$3x + 5y = 45$$

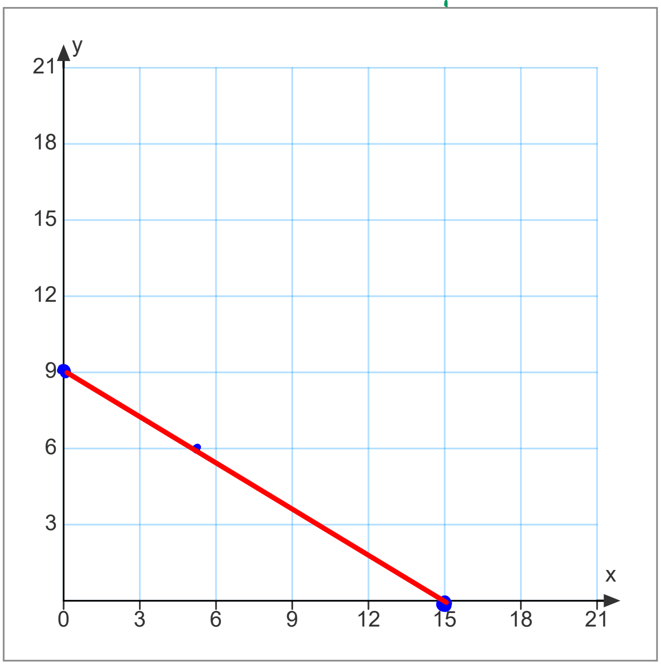
b) Represent this equation on the Cartesian plane.

x	0	15	5	
y	9	0	6	

$3x + 5y = 45$

$15 + 5y = 45$

$5y = 30$



Determine the point of intersection of the following systems of linear equations.

① comparison ② substⁿ ③ eliminⁿ

a) $y = 20 - 7x$

$y = 3x + 5$

$20 - 7x = 3x + 5$

$+7x \quad +7x$

$20 = 10x + 5$

$15 = 10x$

$1.5 = x$

$y = 4.5 + 5$
$= 9.5$

check: $y = 20 - 10.5$
$= 9.5$

$(1.5, 9.5)$

b) $y = 4x + 9$

$3x - 2y = 0$

$3x - 2(4x + 9) = 0$

$3x - 8x - 18 = 0$

$-5x - 18 = 0$

$-5x = 18$

$x = -3.6$

$y = -14.4 + 9 = -5.4$

check: $-10.8 + 10.8 = 0$

$0 = 0$

$(-3.6, -5.4)$

c) $3x + 5y = 29$ $\lfloor \times 2$

$4x + 2y = 6$ $\lfloor \times -5$

L.C.M = 10
{2, 5}

$$
\begin{aligned}
& 6x + 10y = 58 \\
+ \quad & -20x - 10y = -30 \\
\hline
& -14x = 28
\end{aligned}
$$

$x = -2$

$4(-2) + 2y = 6$
$-8 + 2y = 6$
$2y = 14$
$y = 7$

Check: $-6 + 35 = 29$
$29 = 29$

$(-2, 7)$

Example: In basketball, a field goal is worth 2 points, and a penalty shot is only worth 1 point. A team scored 15 times as many field goals as penalty shots for a total of 93 points.

How many of each type of basket did they score?

① x: # of field goals
y: # of penalty shots

② $x = 15y$ $2x + y = 93$

③ Solve the system by sub

$2(15y) + y = 93$

$30y + y = 93$

$31y = 93$

$y = 3$

$x = 45$

④ Answer the question

45 fg & 3 ps.

Optimisation

Symbols:

$<$ less than, fewer than

$>$ greater than, more than, exceeds

$\leq$ less than or equal to, at most, maximum of, no more than

$\geq$ greater than or equal to, at least, minimum of, no less than

Examples with two variables:

1) At a school dance, students paid \$5 and guests paid \$8. The proceeds were <u>more than</u> \$1 200.

x: # of students

y: # of guests

$5x + 8y > 1200$

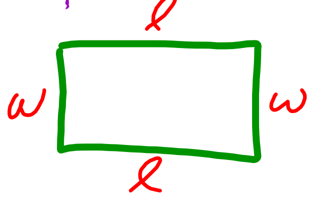
2) At a high school, at least twice as many girls as boys take chemistry.

x : # of girls

y : # of boys

$$x \geq 2y$$

3) The perimeter of a rectangle is less than $60cm$.



$$2\ell + 2w < 60$$

4) Rose and Eric went to New York and Boston. They spent at least twice as much time in New York as in Boston.

x: time in NY
y: time in Boston

$$x \geq 2y$$

The inequalities presented in a situation are known as constraints - conditions that must be met.

Most situations have two extra constraints that are not mentioned. These are called the non-negative constraints and they exist when it is not possible for the variables to take on negative values.

$$x \geq 0$$
$$y \geq 0$$

Example: The maximum number of seats in a plane is 100. There must be at least 4 times as many seats in economy class as in business class.

x : # of economy seats
y : # of business seats

The constraints are:

$x + y \leq 100$ $\qquad$ $x \geq 0$

$x \geq 4y$ $\qquad$ $y \geq 0$

Determining the Solution Set of a Linear Inequality

A local swimming pool uses a mixture of chlorine and bromine to purify the water. A litre of chlorine costs \$10 and a litre of bromine costs \$16. The pool manager buys a total of at least \$240 worth of these products.

1. Variables:

x: Amount of Cl, in L
y: Amount of Br, in L

2. Constraints:

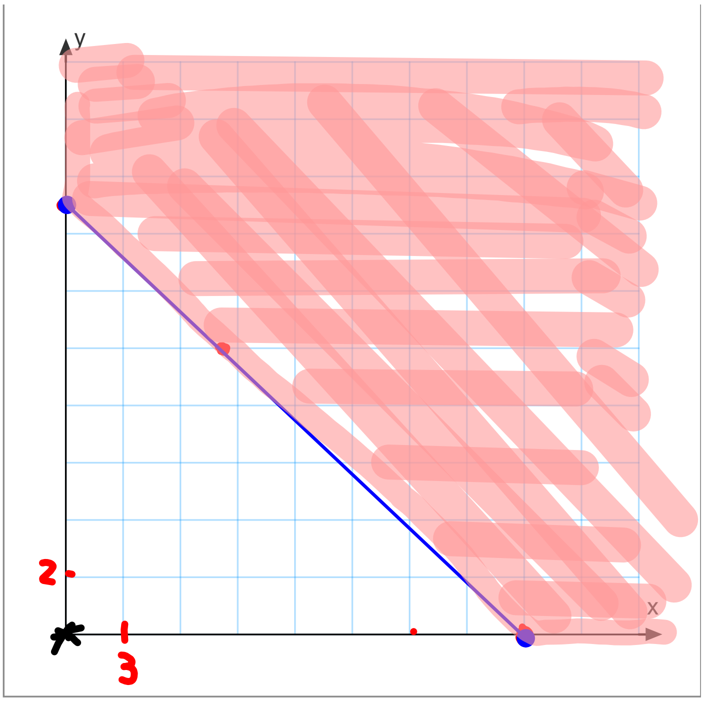
$10x + 16y \geq 240$

$\boxed{\begin{matrix} x \geq 0 \\ y \geq 0 \end{matrix}}$ first quadrant

$10x + 16y = 240$

1. Graph the line. Recall that if the inequality includes the equal sign, the line is drawn solid, but for a strict inequality, the line is broken.

$10x + 16y \geq 240$ solid



2. Choose a point. Test the point in the inequality.

Test $(0,0)$

$10(0) + 16(0) \geq 240$

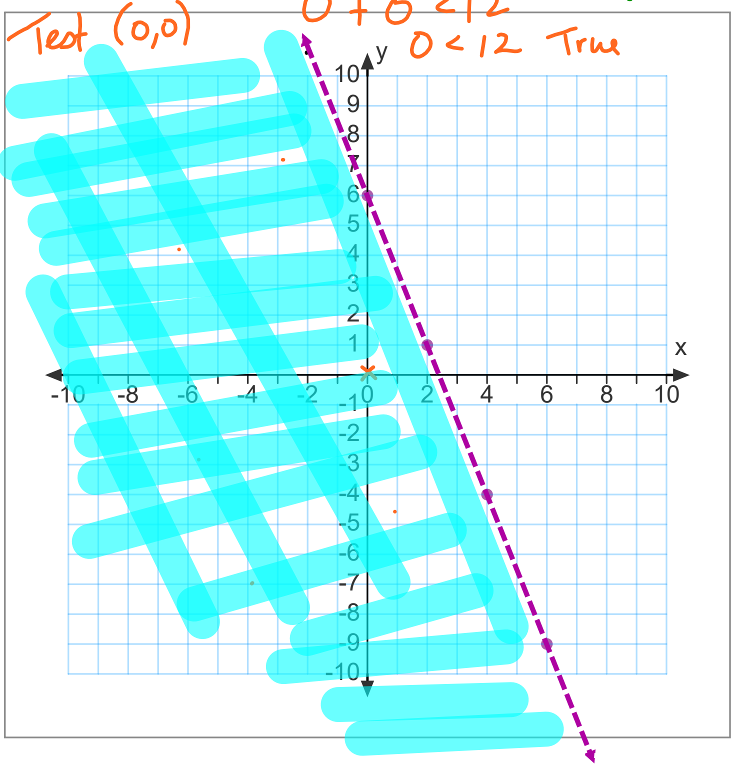
$0 \geq 240$ False

3. Shade on the side of the line where the inequality is TRUE.

Example: Graph the solution set of the inequality

strict inequality $5x + 2y < 12$

Test (0,0) $0 + 0 < 12$ $0 < 12$ True



-10 $2y = 22$

$5x + 2y = 12$

①

x	y
0	6
2	1
-2	11

② $2y = -5x + 12$

$y = -\frac{5}{2}x + 6$

y-int = 6 slope: $-\frac{5}{2}$ $\frac{\text{rise}}{\text{run}}$

Systems of Linear Inequalities

Solving a system of linear inequalities means finding all the points that satisfy all the inequalities.

Example: Determine the solution set of the following system.

$$y > -x$$
$$y \le x$$

Graph the following system of inequalities

① • $y \geq -x - 2$

$y = \frac{-1}{1}x - 2$

$\geq$ solid

shade Test (0,0)

$0 \geq -2$ True

• $y \leq x + 4$

$y = \frac{1}{1}x + 4$

$\leq$ solid

Test (0,0)

$0 \leq 4$ True

• $3x + y \leq 8$

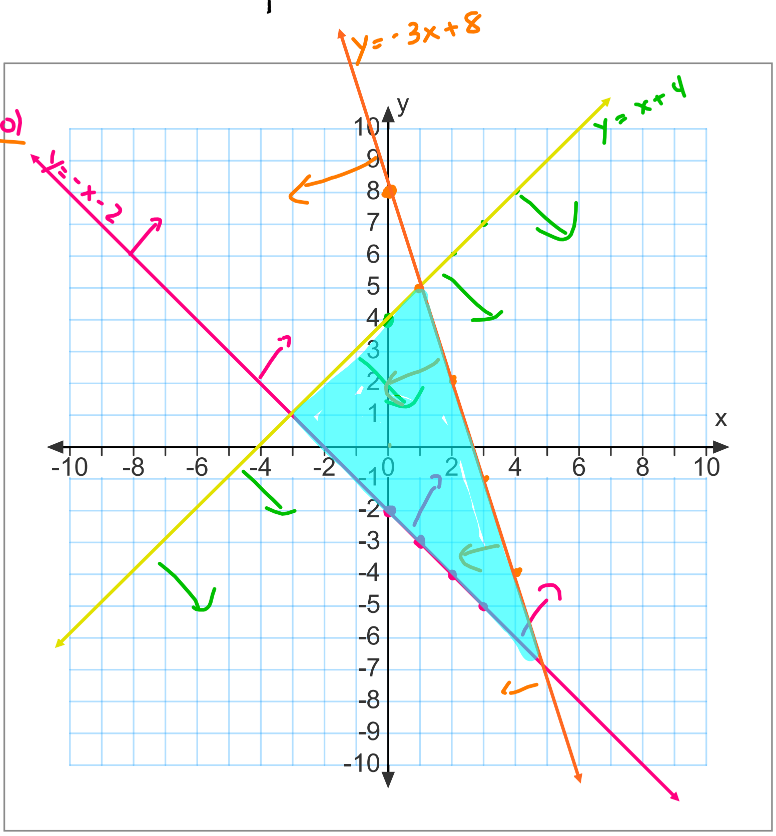
$y = -\frac{3}{1}x + 8$

$\leq$ solid

Test (0,0)

$0 + 0 \leq 8$

$0 \leq 8$ True



The figure created by the solution set of all the inequalities is called the <u>polygon of constraints</u>.

Example: A municipal garden grows red roses and white roses. There are at most 400 roses in total. The number of red roses increased by 80 is greater than twice the number of white roses.

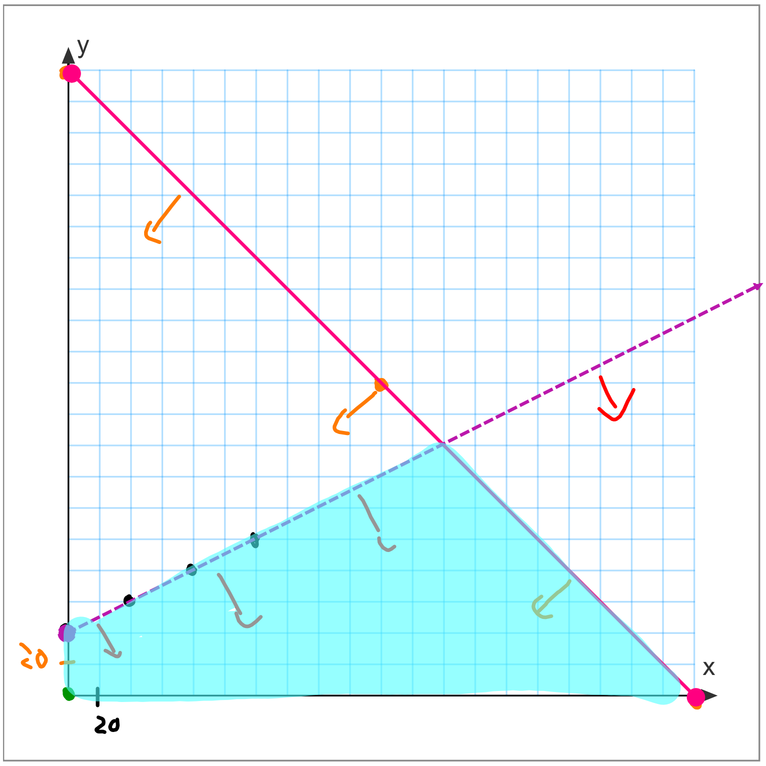
Graph the solution set.

x: # of red roses
y: # of white roses

$x + y \le 400$

$x + 80 > 2y$

$Q_1 \begin{cases} x \ge 0 \\ y \ge 0 \end{cases}$



$x + y \le 400$

solid

x	y
0	400
400	0
200	200

Test (0,0)

$0 \le 400$ True

$x + 80 > 2y$

$x + 80 = 2y$ > dotted

$\frac{1}{2}x + 40 = y$

Test (0,0)

$0 + 80 > 0$

$80 > 0$ True

Not a polygon of constraints (Unbound).