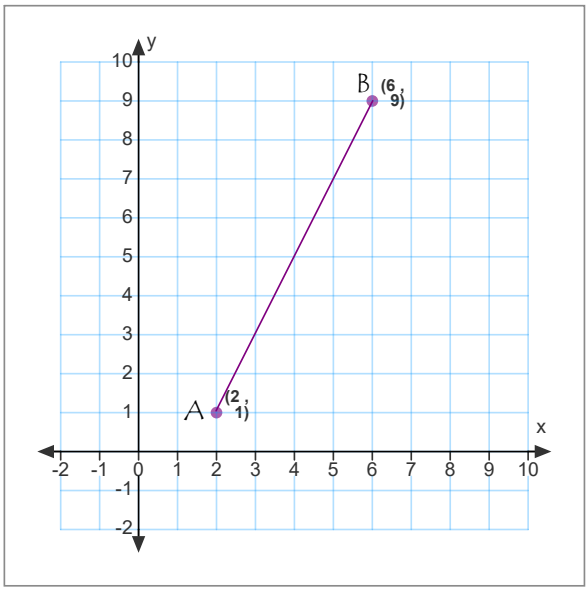
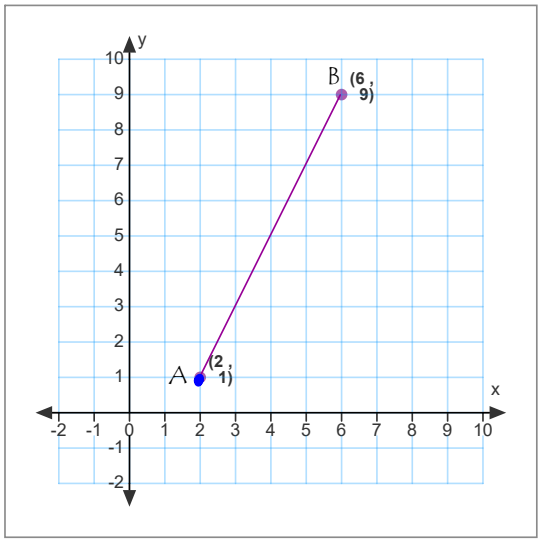
# Division Point of a Segment

a) part to whole - the position of a point of division can be found using a fraction.

Example: Determine the co-ordinates of a point P that is located $\frac{3}{4}$ of the way along segment AB, where A(2, 1) and B(6, 9).



* Because A is the starting point, we have to call point A $(x_1, y_1)$ and point B $(x_2, y_2)$.

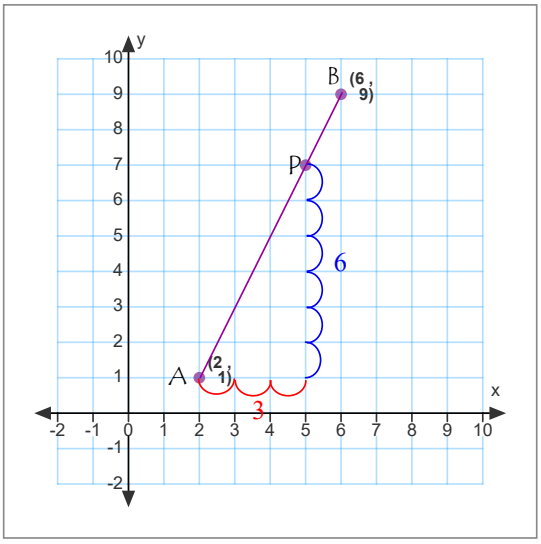



* 1) horizontally

$$\frac{3}{4}(x_2 - x_1) = \frac{3}{4}(6-2)$$
$$= \frac{3}{4}(4)$$
$$= 3$$

* 2) vertically

$$\frac{3}{4}(y_2 - y_1) = \frac{3}{4}(9-1)$$
$$= \frac{3}{4}(8)$$
$$= 6$$




* Therefore, point P can be found at (5, 7)

# Division Point Formula

$x$ co-ordinate of division point: $x_1 + \frac{a}{b}(x_2 - x_1)$

$y$ co-ordinate of division point: $y_1 + \frac{a}{b}(y_2 - y_1)$

where $\frac{a}{b}$ is the fraction of the distance from start to end

Example: Determine the co-ordinates of a point that is found $\frac{2}{3}$ of the way on segment $QR$ if $Q(6, 1)$ and $R(-3, 4)$.

$$(x_1, y_1) = Q(6,1) \qquad \frac{a}{b} = \frac{2}{3}$$

$$x = 6 + \frac{2}{3}(-3-6) \qquad y = 1 + \frac{2}{3}(4-1)$$

$$= 6 + \frac{2}{3}(-9) \qquad = 1 + \frac{2}{3}(3)$$

$$= 6 + (-6) \qquad = 1 + 2$$

$$= 0 \qquad = 3$$

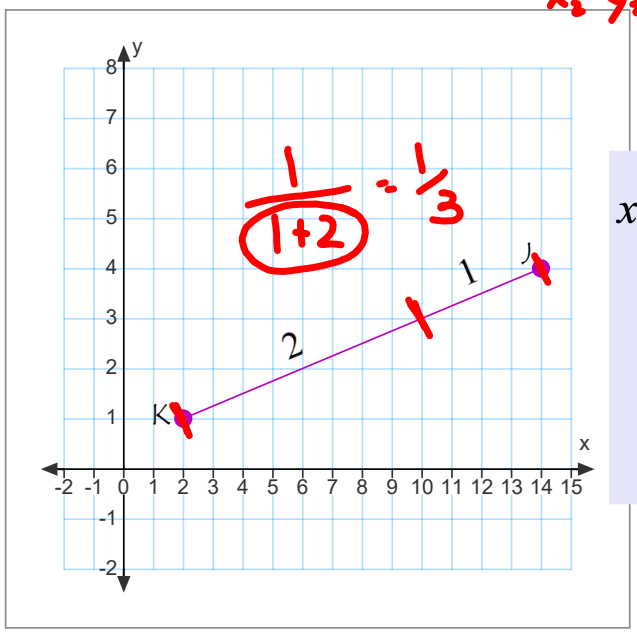
$$\therefore (0,3)$$

b) part to part – the division point can be found using a ratio.

Example: Find the co-ordinates of point P which divides segment JK in a ratio of 1:2. The co-ordinates of J are (14, 4) and those of K are (2, 1).

$x_1, y_1$ $x_2, y_2$

$$\frac{a}{b} = \frac{1}{1+2} = \frac{1}{3}$$




$$x = 14 + \frac{1}{3}(2-14)$$
$$= 14 + \frac{1}{3}(-12)$$
$$= 14 + (-4)$$
$$= 10$$

$$y = 4 + \frac{1}{3}(1-4)$$
$$= 4 + \frac{1}{3}(-3)$$
$$= 4 + (-1)$$
$$= 3$$

$$\therefore P(10,3)$$

When a point divides a line segment in a ratio of m : n, then the fraction used in the division point formula becomes ...

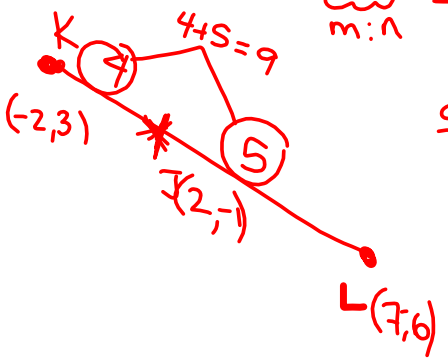
$$\frac{a}{b} = \frac{m}{m+n}$$

Example: Determine the co-ordinates of point J that divides segment KL in a ratio of 4:5. K(-2,3) & L(7, -6).

4:5 → m:n

K(-2,3) → $x_1, y_1$

$x_2, y_2$




$\frac{a}{b} = \frac{m}{m+n} = \frac{4}{9}$

$x = -2 + \frac{4}{9}(7 - -2)$
$= -2 + \frac{4}{9}(9)$ [4×9÷9]
$= -2 + 4$
$= 2$

$y = 3 + \frac{4}{9}(-6 - 3)$
$= 3 + \frac{4}{9}(-9)$
$= 3 + -4$
$= -1$

$J(2, -1)$

Determine the coordinates of point P that is located $\frac{2}{5}$ of the way between points A$(-6,7)$ and B$(9,-3)$.

$x_1, y_1$ $x_2, y_2$

$x = -6 + \frac{2}{5}(9 - -6)$

$x = -6 + \frac{2}{5}(15)$

$x = -6 + \frac{30}{5}$

$x = -6 + 6$

$x = 0$

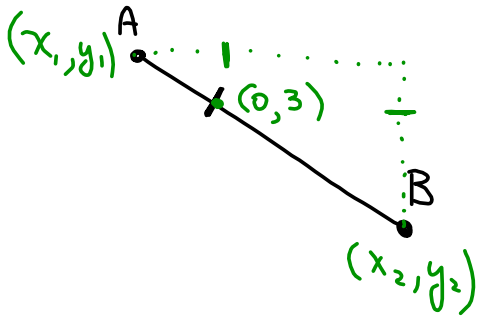
$y = 7 + \frac{2}{5}(-3 - 7)$

$y = 7 + \frac{2}{5}(-10)$

$y = 7 + \frac{-20}{5}$

$y = 7 + -4$

$y = 3$



Determine the coordinates of point P that is located $\frac{3}{8}$ of the way along segment AB if A$(2,4)$ and B$(10,-20)$.

$x_1, y_1$ $x_2$ $y_2$

$x = 2 + \frac{3}{8}(10-2)$

$x = 2 + \frac{3}{8}(8)$

$x = 2 + 3 \times 8 \div 8$

$x = 2 + 3$

$x = 5$

$y = 4 + \frac{3}{8}(-20-4)$

$y = 4 + \frac{3}{8}(-24)$

$y = 4 + 3 \cdot -24 \div 8$

$y = 4 + -9$

$y = -5$

$P(5,-5)$

Determine the coordinates of point P that divides segment BA in a ratio of 6:5, given $A(-1,3)$ and $B(10,25)$.

$x_1, y_1$ $x_2, y_2$

$\frac{a}{b} = \frac{6}{6+5} = \frac{6}{11}$

$x = 10 + \frac{6}{11}(-1-10)$

$= 10 + \frac{6}{11}(-11)$

$= 10 + 6 \times -11 \div 11$

$= 10 + -6$

$= 4$

$y = 25 + \frac{6}{11}(3-25)$

$y = 25 + \frac{6}{11}(-22)$

$y = 25 + 6 \cdot -22 \div 11$

$y = 25 + -12$

$y = 13$

$P(4,13)$