Functions

A relation is a connection between two quantities called variables.

Many relations involve a dependency between the variables. This is when the values of one variable (dependent variable) are determined by the values of the other variable (independent variable).

Recall: there are four ways to describe a relation.

Table of Values

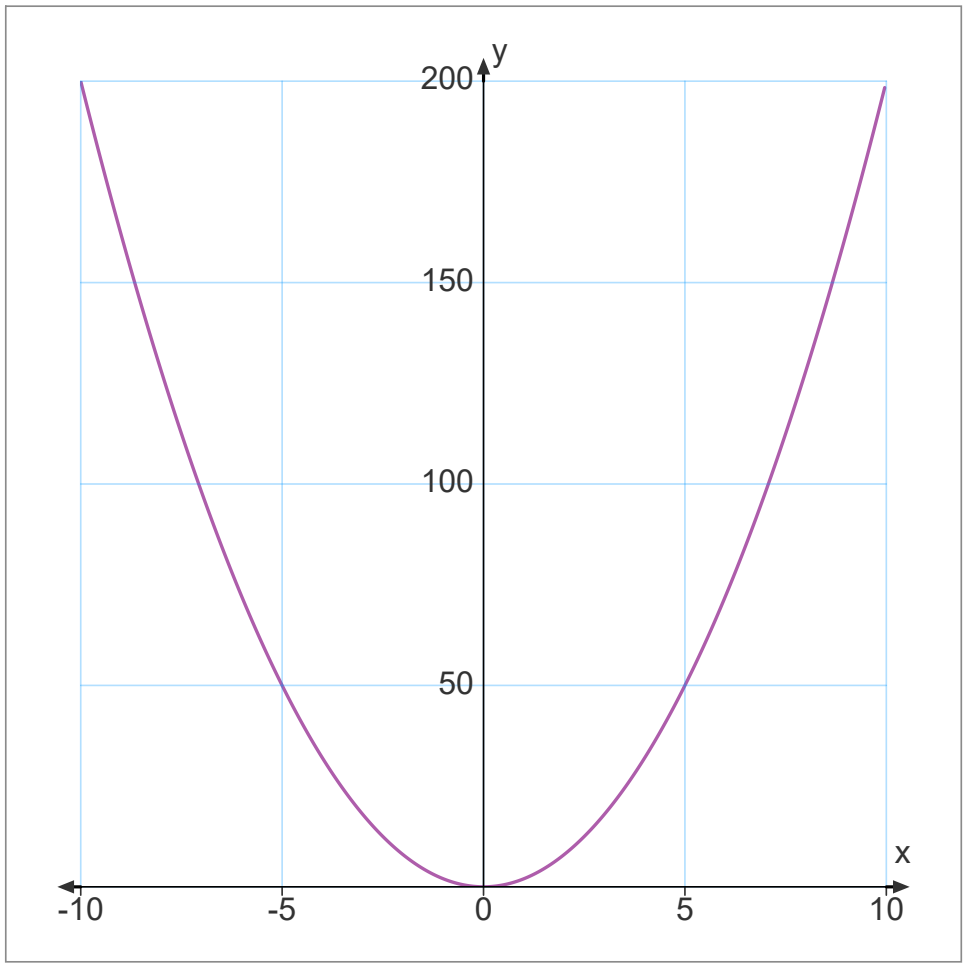
X	Y
4	10
5	7
6	4
7	1
10	-8

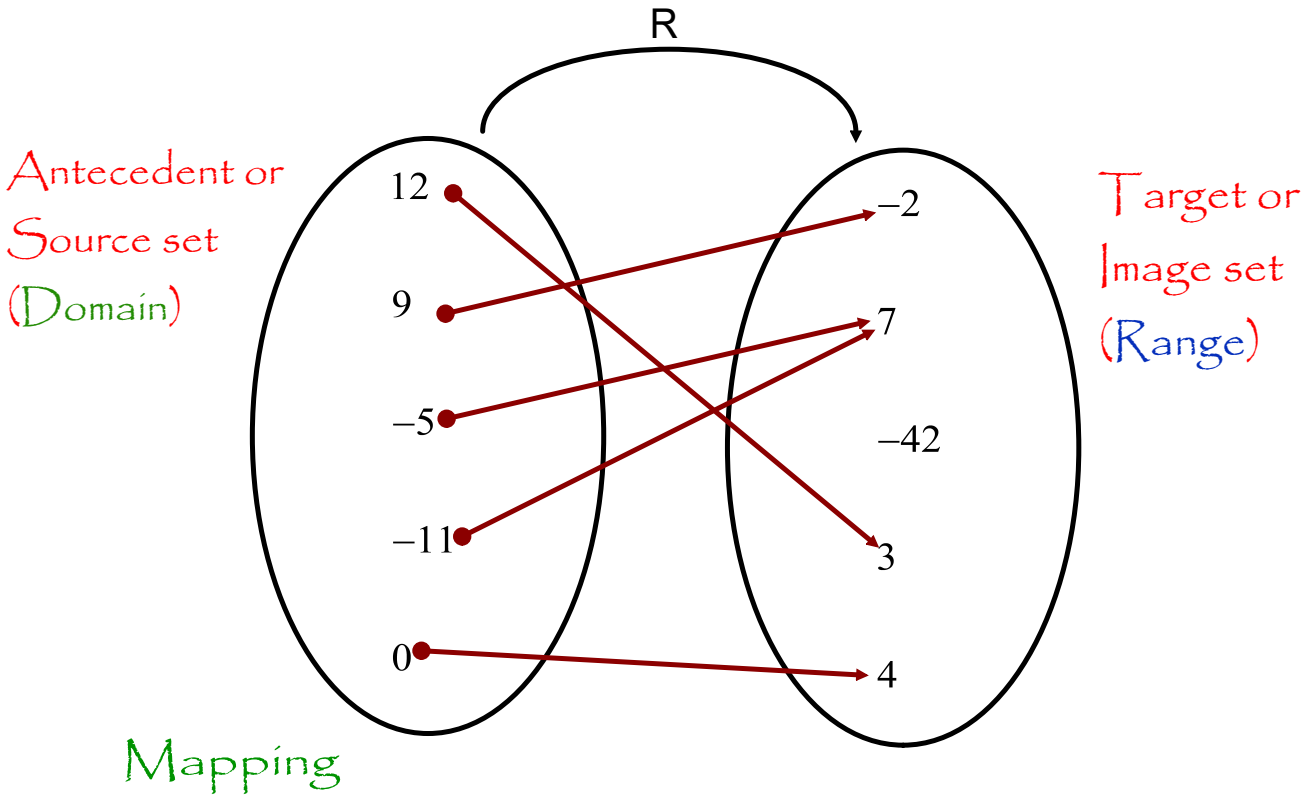
Rule or Equation

$$y = -3x + 22$$

Graph

Cartesian Plane




R
Antecedent or
Source set
(Domain)
12
9
−5
−11
0
−2
7
−42
3
4
Target or
Image set
(Range)
Mapping

Verbal Description

The cost of parking is $2 for the first hour, or part thereof, and $1 for each consecutive hour, or part thereof.

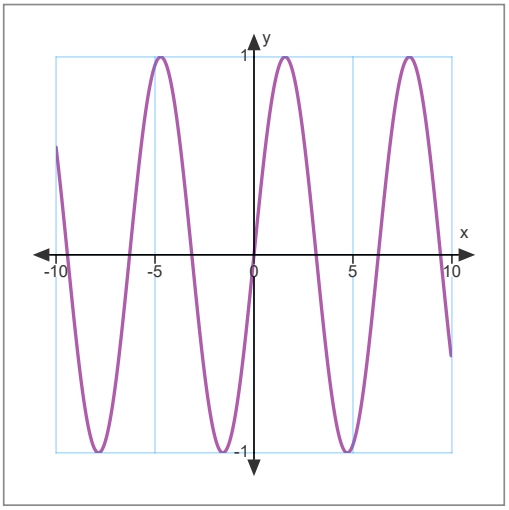
Function A relation in which each value of the independent variable is associated with one and only one value of the dependent variable.

Not a Function

x	y
3	6
4	9
7	12
4	14
9	15

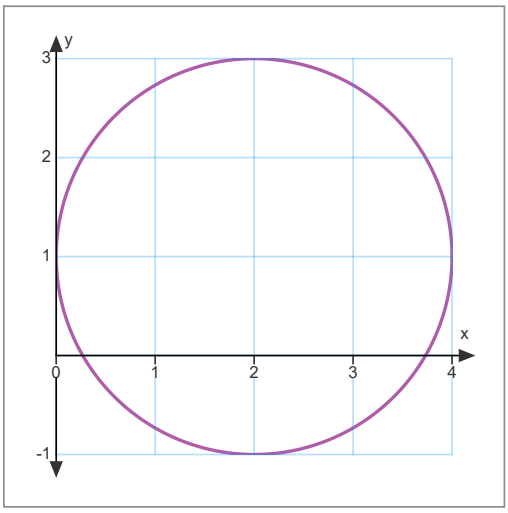
Function

x	y
3	12
4	15
5	9
6	12



Not a Function

Function

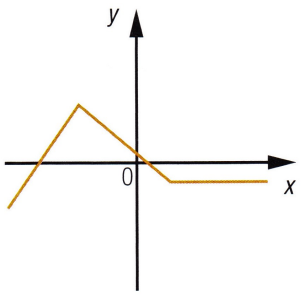


Vertical Line Test for a Graph

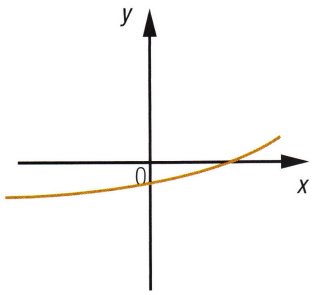
If you draw a vertical line through the graph of a relation, and that line can only pass through the curve once (no matter where the line is drawn), then the relation is a function.

Identify which of the following graphs represents a function.

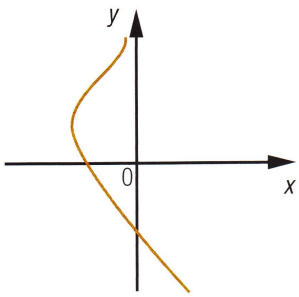
A



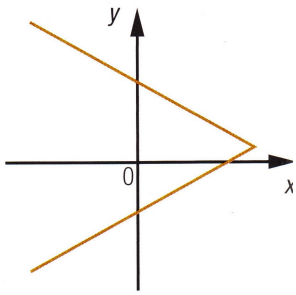
B



C



D



Function Notation - is a way of writing the rule of a function. Instead of using y, we use $f(x)$ (or $g(x)$, or $h(x)$, etc.)

e.g.  becomes 

Why? 1: It lets us know that a relation is a function.

2: It can be used to show a point that is on the curve (graph).

e.g. $f(4)=40$ means that when $x=4,\ y=40$, or $(4,40)$.

Example: If $f(x) = 2x^3 - 6$ determine $f(4)$.

$$f(4) = 2(4^3) - 6$$
$$f(4) = 2 \cdot 64 - 6$$
$$f(4) = 128 - 6$$
$$f(4) = 122$$

If $f(x) = 48$, determine x.

$$48 = 2x^3 - 6$$
$$54 = 2x^3$$
$$27 = x^3$$
$$\sqrt[3]{27} = x$$
$$3 = x$$

Interval Notation : Used for Real Numbers $(\mathbb{R})$; to show where something starts and something ends (an interval).

We use square brackets and/or parentheses.

Examples : $[4,22)$ or $[4,22[$: 4 is included, but 22 is not.

$(-3,10]$ or $]-3,10]$: 10 is included but -3 is not.

$[2,7]$: both values are included.

$]0,5[$: both values are excluded.

Properties of Functions

1) Domain and Range

2) Variation

3) Extrema

4) Sign

5) Intercepts

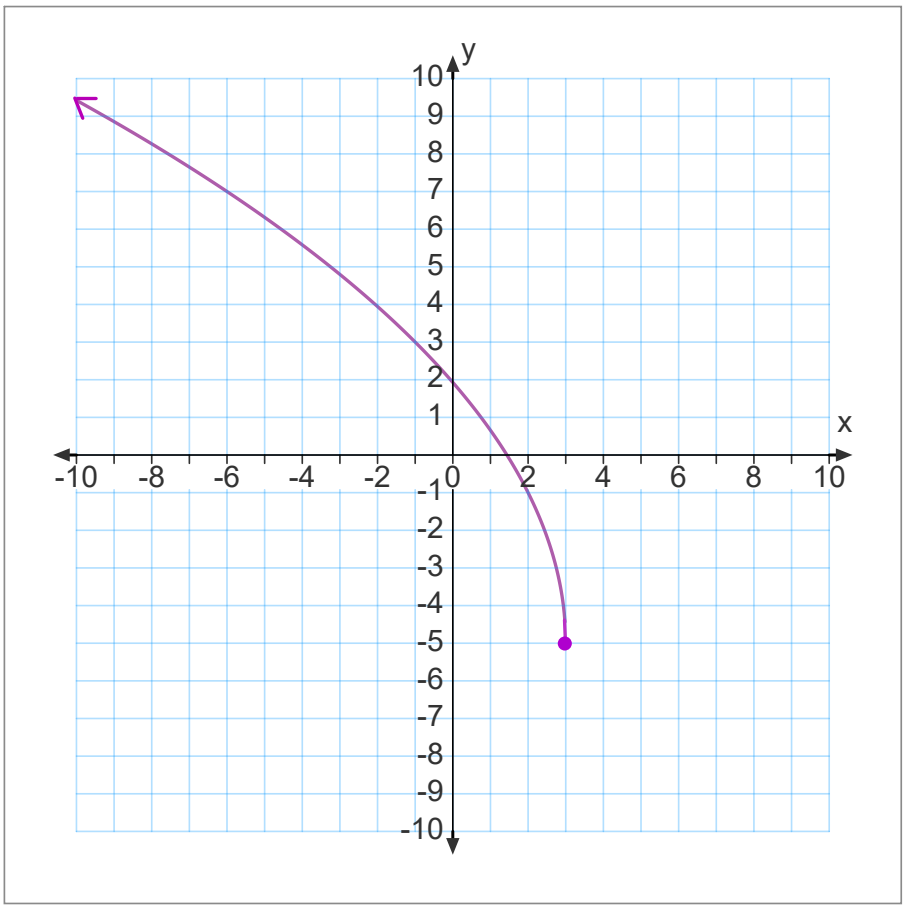
1) Domain and Range

Domain and range refer to the values that the variables are allowed to be.

Domain: these are the values of the independent variable (x-values)

Range: these are the values of the dependent variable (y-values)

Determine the domain and range of the function below.



2) Variation

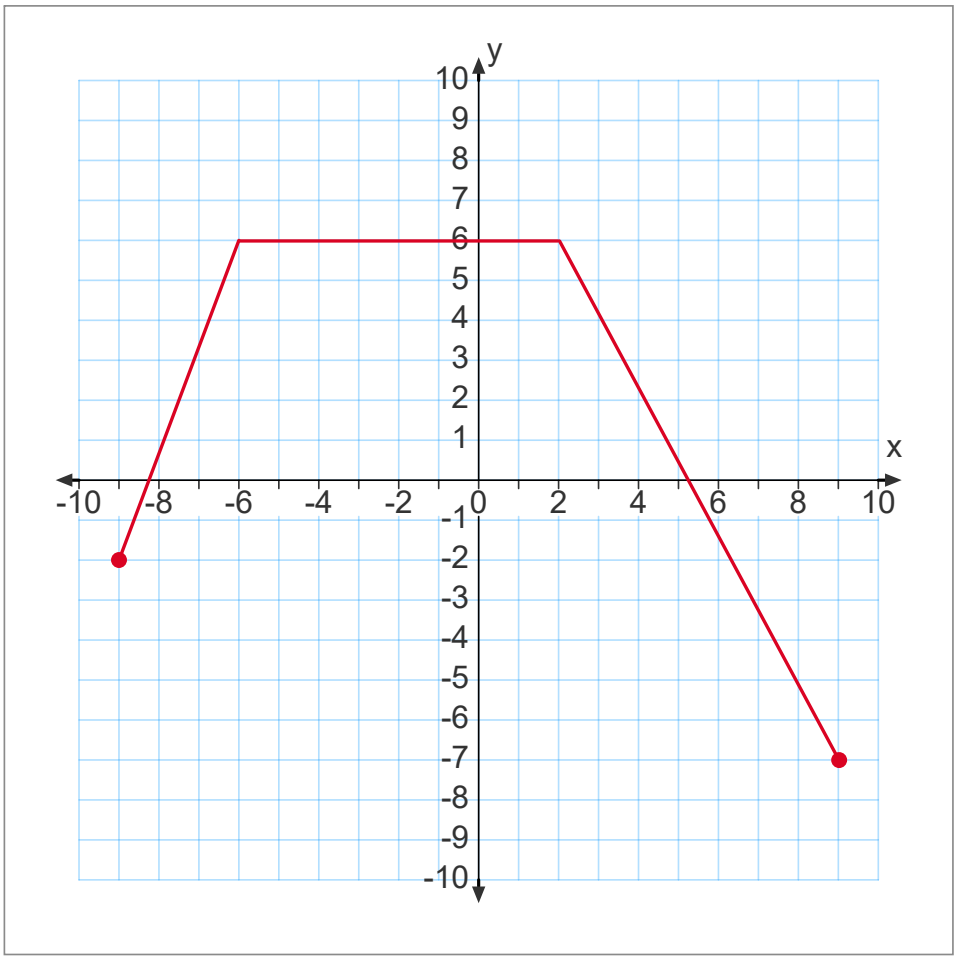
Variation refers to where the function is increasing, decreasing or constant.

Increasing: the x-values (interval of the domain) that cause the function to 'go up and to the right'.

Decreasing: the interval of the domain that causes the function to 'go down and to the right'.

Constant: the interval of the domain that causes the function to be horizontal. The function is said to be both increasing and decreasing at the same time.

Determine the variation for the function below.



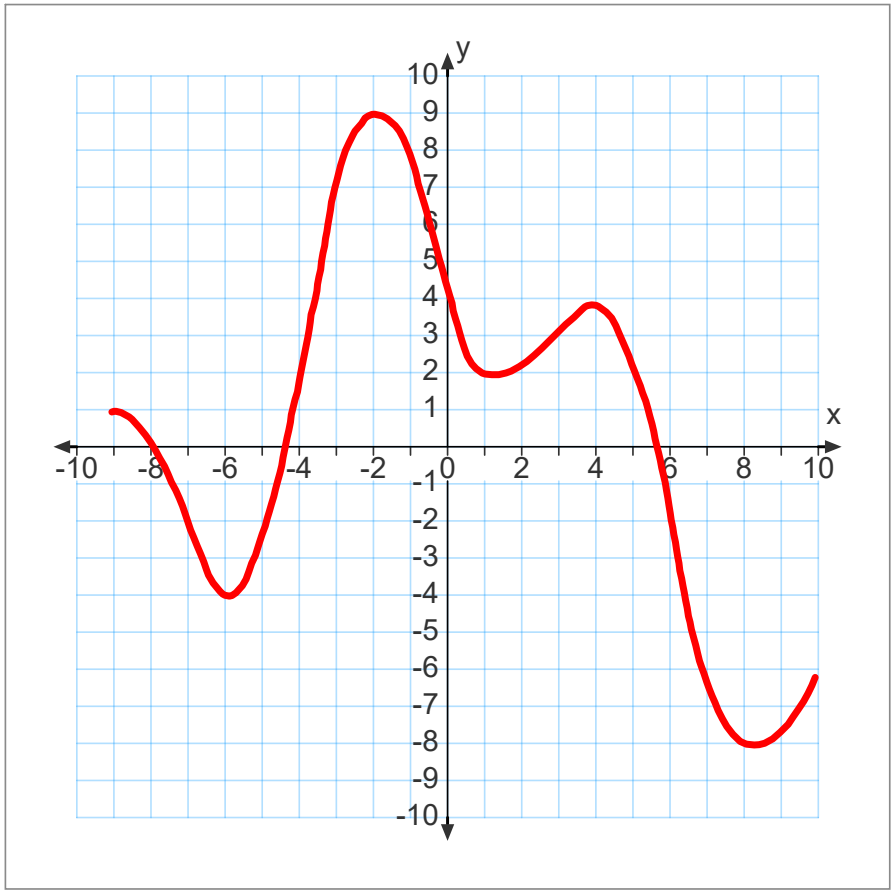
3) Extrema

Extrema refer to the highest (maximum) and lowest (minimum) points on a graph (y-values).

a) Absolute: the very highest or the very lowest points possible.

b) Relative: the highest and lowest points when compared to other points nearby.

Determine the extrema of the function below.



4) Sign

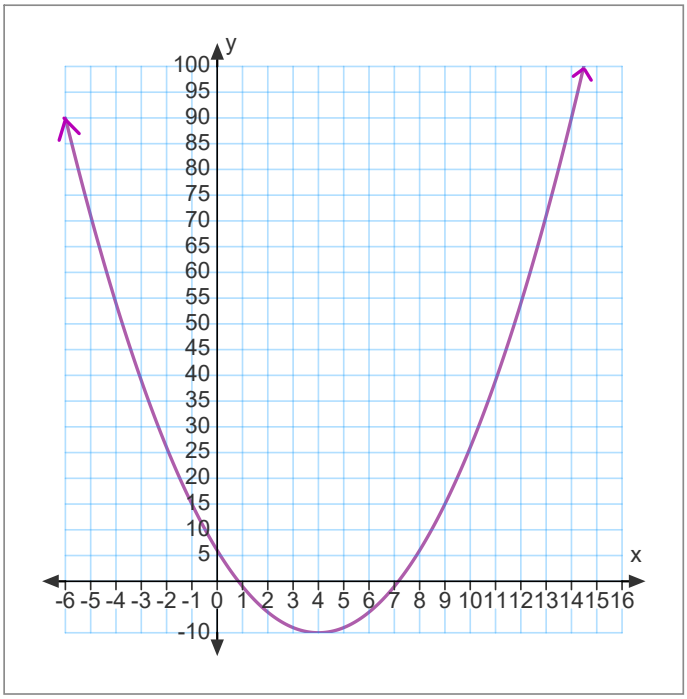
The sign of a function refers to where the function is positive and negative.

The reference is the x-axis.

Any part of the function above the x-axis is positive and any part below the x-axis is negative.

We use interval notation and x-values to describe this property.

Determine the sign of the function below.



5) Intercepts

The intercepts of a function refer to where the function crosses the axes .

a) y-intercept

AKA: initial value, value at 0

This is where the function crosses the y-axis $(x = 0)$.

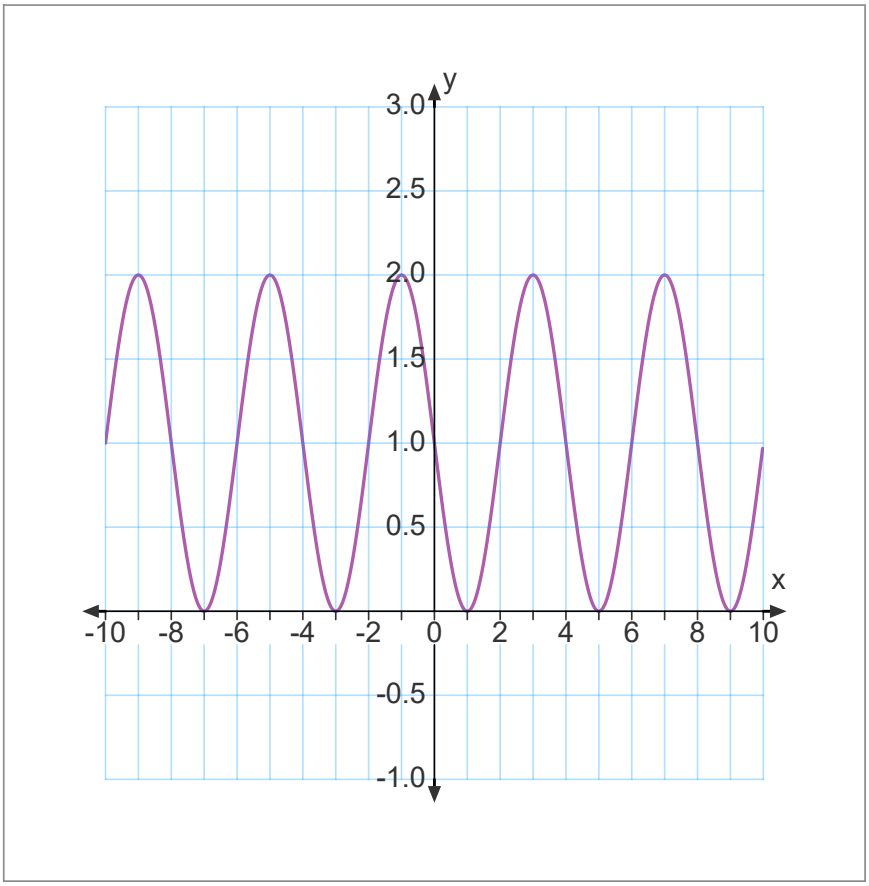
There can only be one y-intercept.

b) x-intercept AKA: zero, root

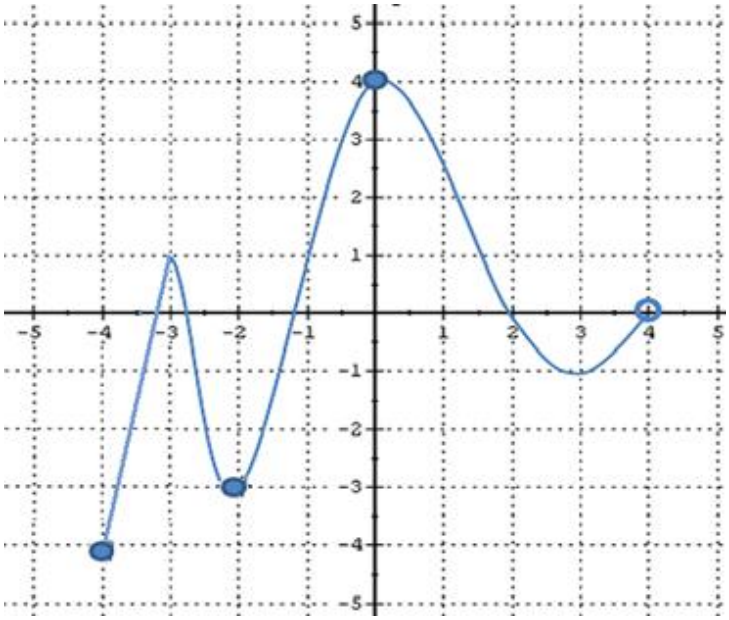
This is where the function crosses the x-axis ($y = 0$).

There may be an infinite number of x-intercepts.

Determine the intercepts of the function below.



Provide a study of the function below. This means to list all its properties .



Given $f(x) = 3\sqrt{(x+4)} - 12$ determine:

a) the y - intercept

b) $f(-3)$

c) the zero(s)

Given $g(x) = 5x^2 + 4x - 1$, determine the intercepts (if they exist).

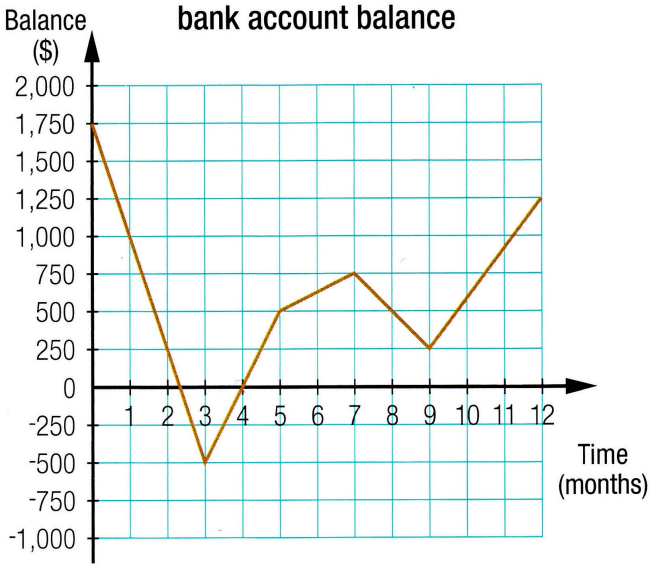
Work Book: Pages 50 - 58

Questions 7, 9, 13, 17 & 20

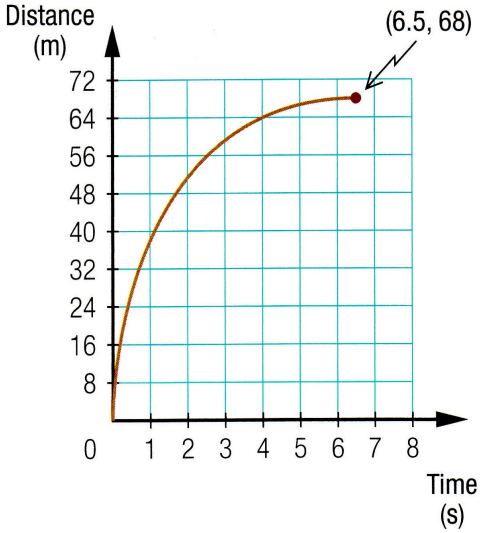
For each of the functions below, determine:

a) the domain and the range
b) the variation
c) the sign
d) the extrema
e) the initial value
f) the zero(s)

Function ①
The variation in a bank account balance

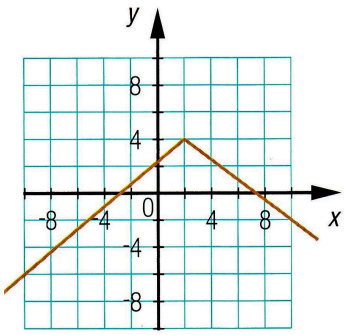


Function ②
The movement of a javelin

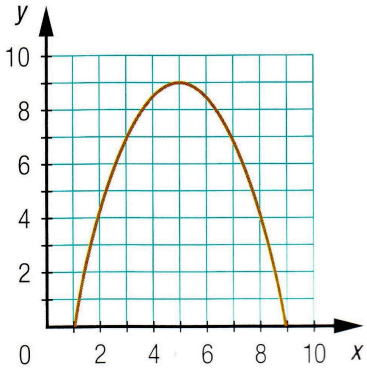


Provide a study for each function shown below.

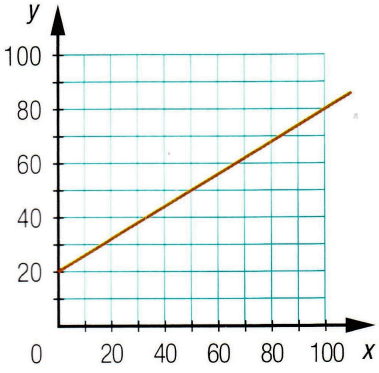
Graph ①



Graph ②



Graph ③



The adjacent graph displays the outside temperature recorded during the first 12 hours of an autumn day.

a) What was the lowest temperature?

b) What was the initial temperature?

c) At what time were the following recorded:

   1) the minimum temperature?

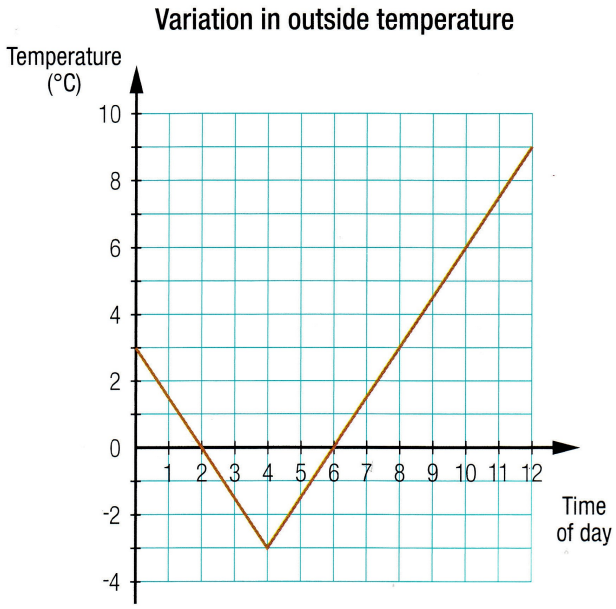
   2) the maximum temperature?

d) At what time(s) was the temperature 0°C?

e) Over what time interval(s) was the temperature:

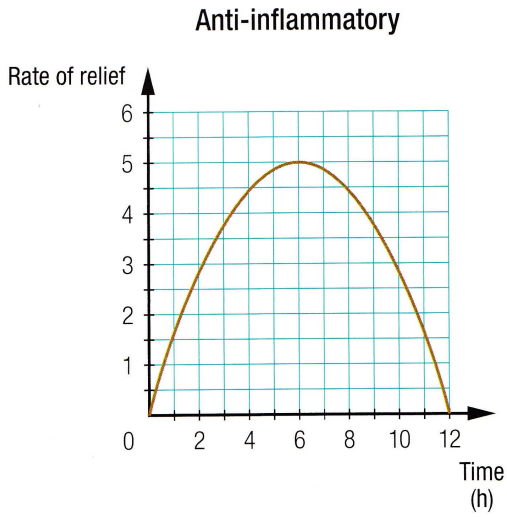
   1) negative?

   2) positive?

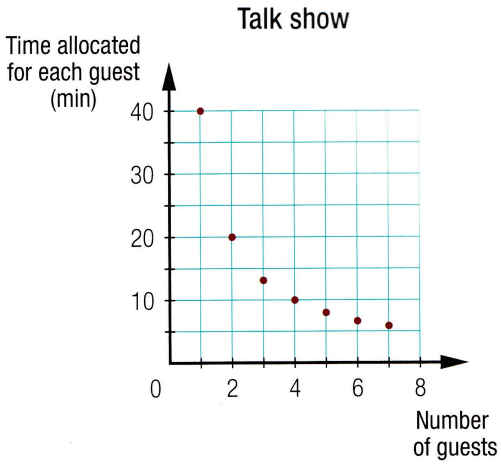


Provide a study for each function shown below.

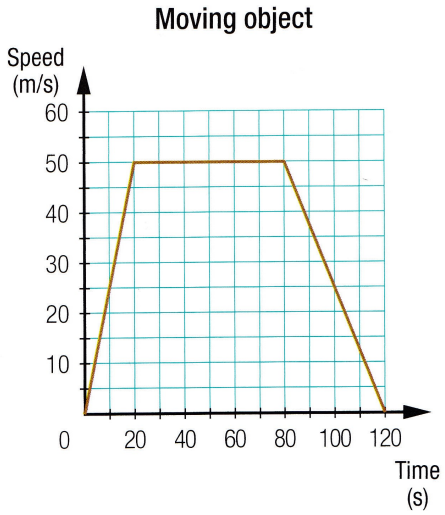
a)



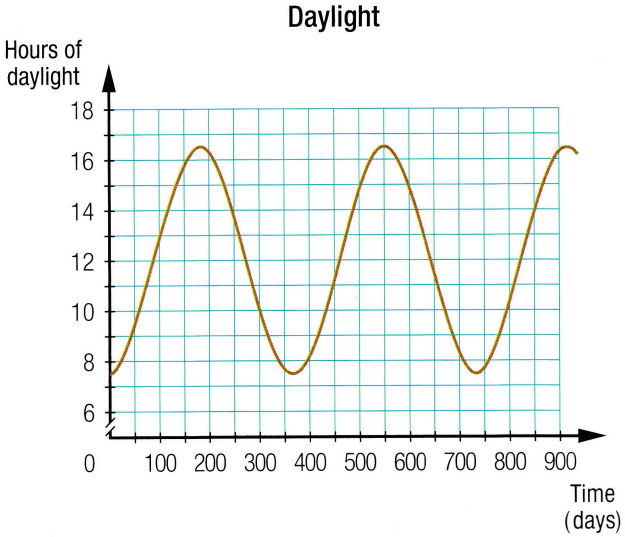
b)



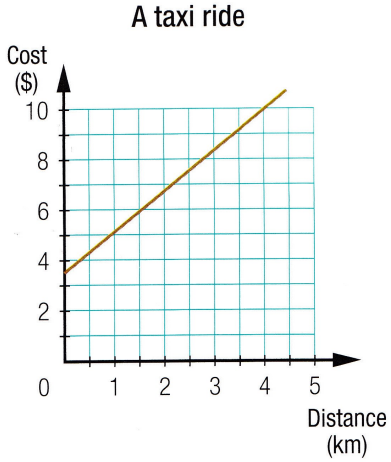
c)



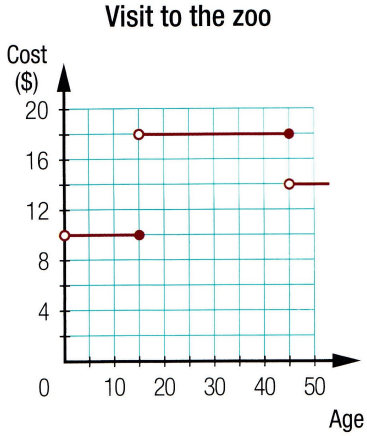
d)



e)



f)

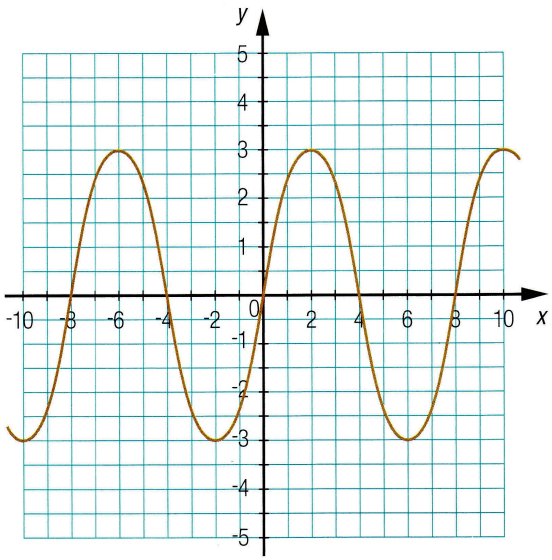


For each of the following graphs, do the following:

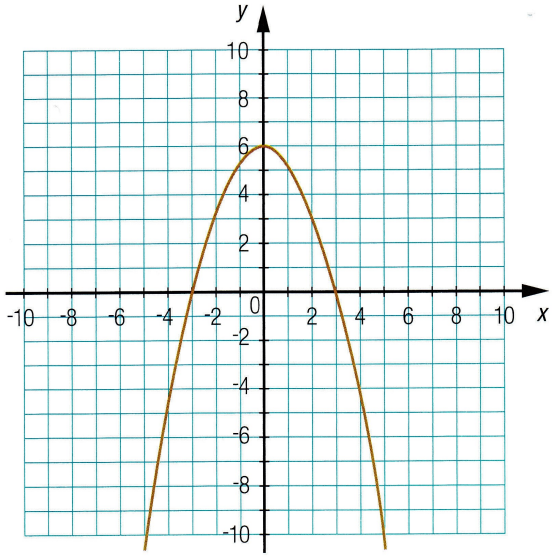
a) Determine the following properties:

1) the domain and the range
2) the zero(s)
3) the extrema
4) the initial value

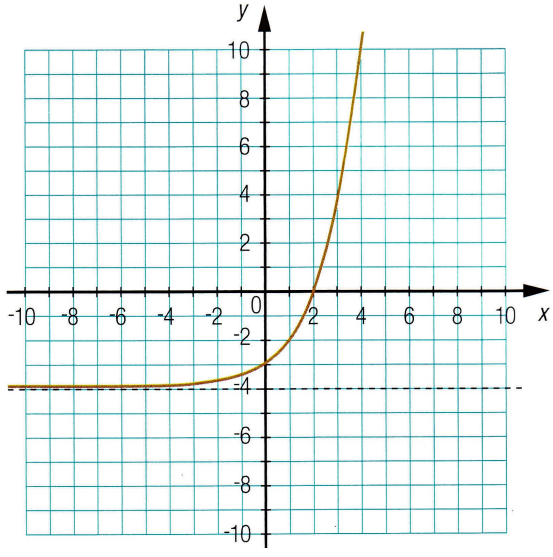
Graph ①



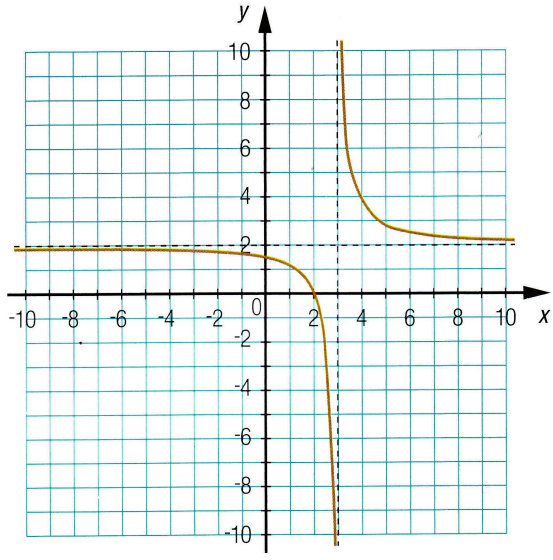
Graph ②



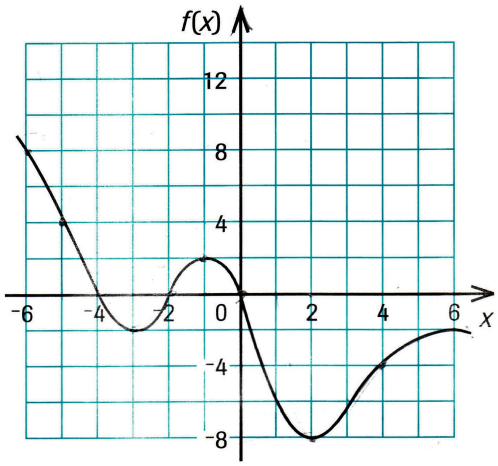
Graph ③



Graph ④



The graph of function f is shown. Use the graph to answer the following questions.



a) What is the value of $f(2)$?

b) For what value of x is $f(x) = 8$?

c) State the zeros of f.

d) Give an interval for which $f(x) > 0$.

e) State an interval for which $f(x) < -6$.

f) Give two intervals for which function f is increasing.

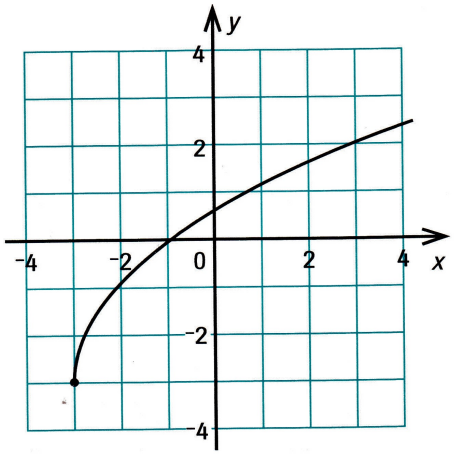
g) State a value for which $f(x) = x$, if any.

h) Evaluate $f(f(1))$.

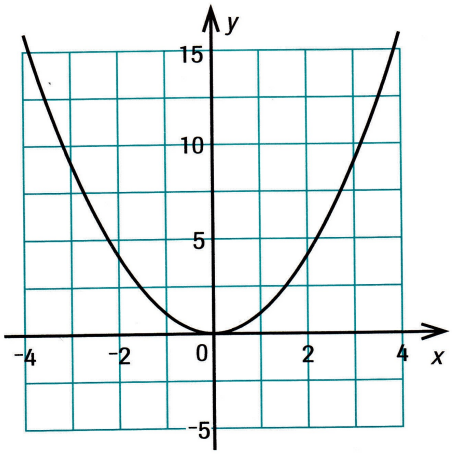
i) Supposing that the curve keeps the same look and the function decreases from $x = 6$, which value corresponds approximately to $f(10)$?

Provide a study for each function shown below.

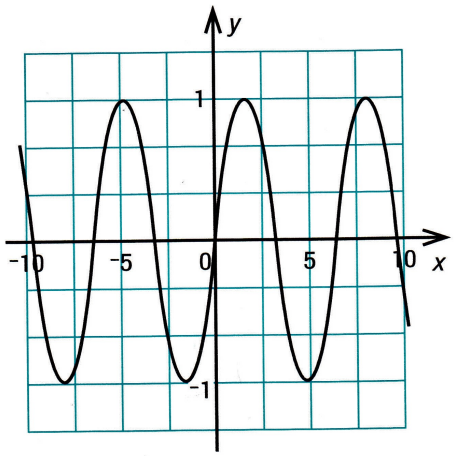
a)



b)



c)



d)