

The Cosine Function

https://youtu.be/jEcqQkl2_A4



Basic cosine function

$$f(x) = \cos x$$

Properties

Domain: \mathbb{R}

Range: $[-1, 1]$

Increasing: $[\pi, 2\pi] + 2\pi n, n \in \mathbb{Z}$

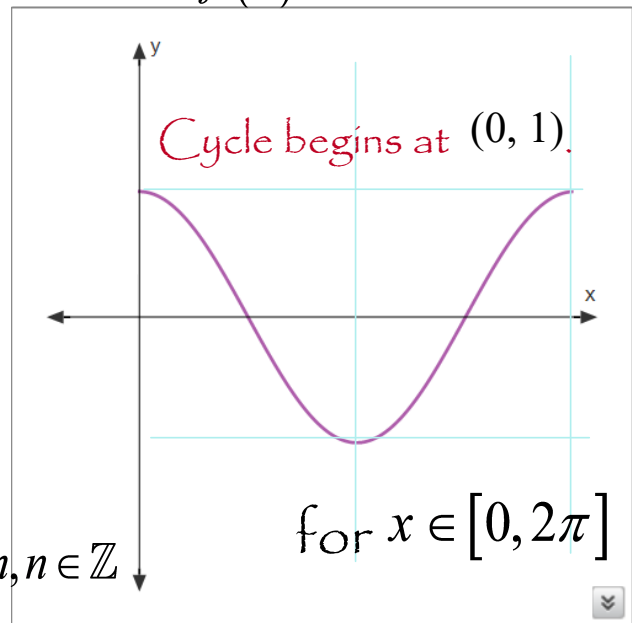
Decreasing: $[0, \pi] + 2\pi n, n \in \mathbb{Z}$

Positive: $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right] + 2\pi n, n \in \mathbb{Z}$

Negative: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] + 2\pi n, n \in \mathbb{Z}$

Maximum: 1

Minimum: -1



Period: 2π

Amplitude: 1

Notice that $y = \cos x$ is a translation of $y = \sin x$.
In fact, $f(x) = \cos x \iff f(x) = \sin\left(x + \frac{\pi}{2}\right)$.

For the transformed cosine function, a , b , h & k
will be the same as with the sine function

That is ... $A = |a|$

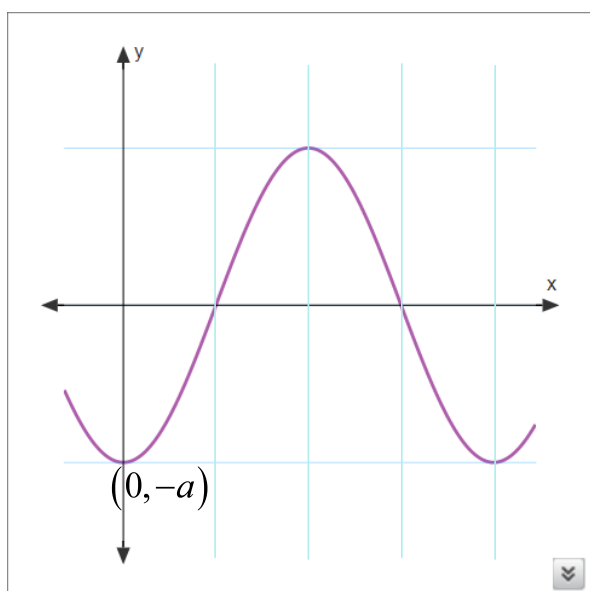
$$p = \frac{2\pi}{|b|}$$

k = middle axis (vertical translation)

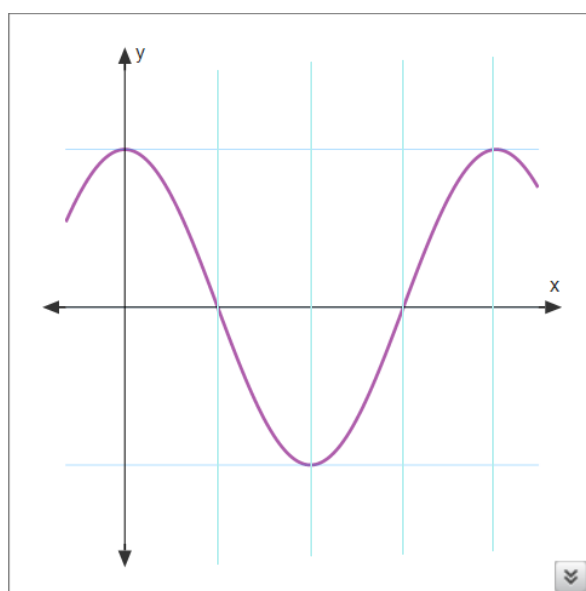
✦ h = horizontal translation

Except ...

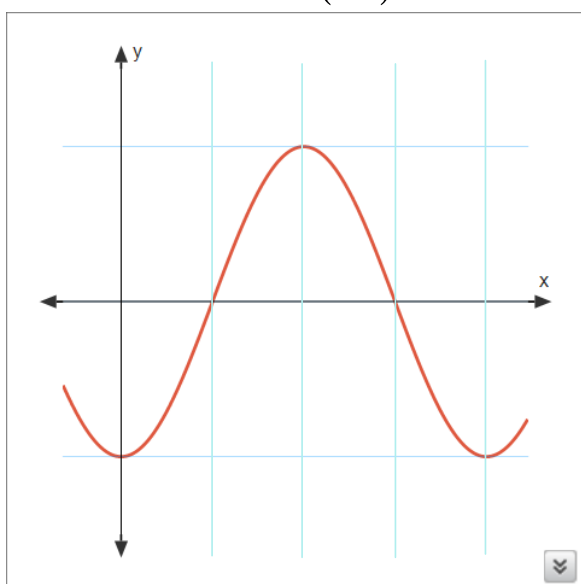
$$y = -\cos x$$



$$y = \cos(-x)$$



$$y = -\cos(-x)$$



Both a^- , b^+ and a^- , b^- result in a flip over the x -axis.
Therefore, if a is negative, the function flips.

Transformed Cosine Function

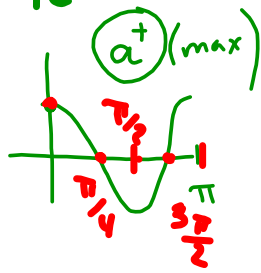
$$f(x) = a \cos(b(x-h)) + k$$

Graph $f(x) = \frac{1}{2} \cos(2(x-\pi)) + 3$

$a = \frac{1}{2} \Rightarrow A = \frac{1}{2}$

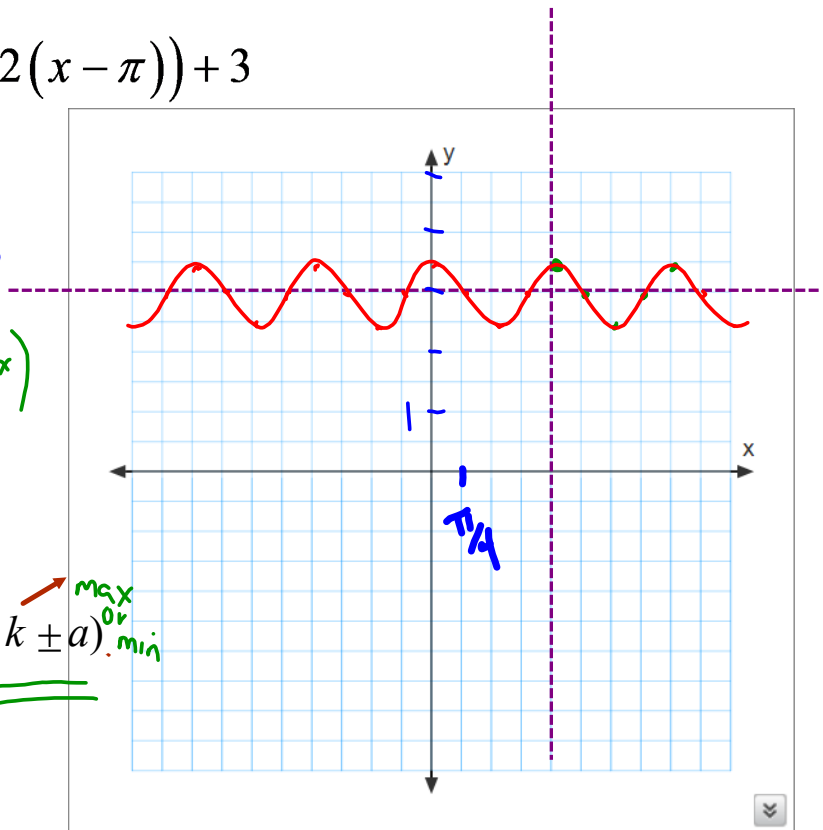
$k = 3 \Rightarrow \begin{matrix} \text{max} = 3.5 \\ \text{min} = 2.5 \end{matrix}$

$P = \frac{2\pi}{2} = \pi$



Cycle now begins at $(h, k \pm a)$ ^{max or min}

$(\pi, 3.5)$

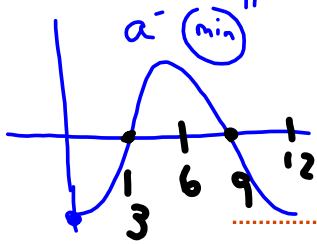


Graph $f(x) = -4 \cos\left(\frac{\pi}{6}(x-3)\right) - 2$

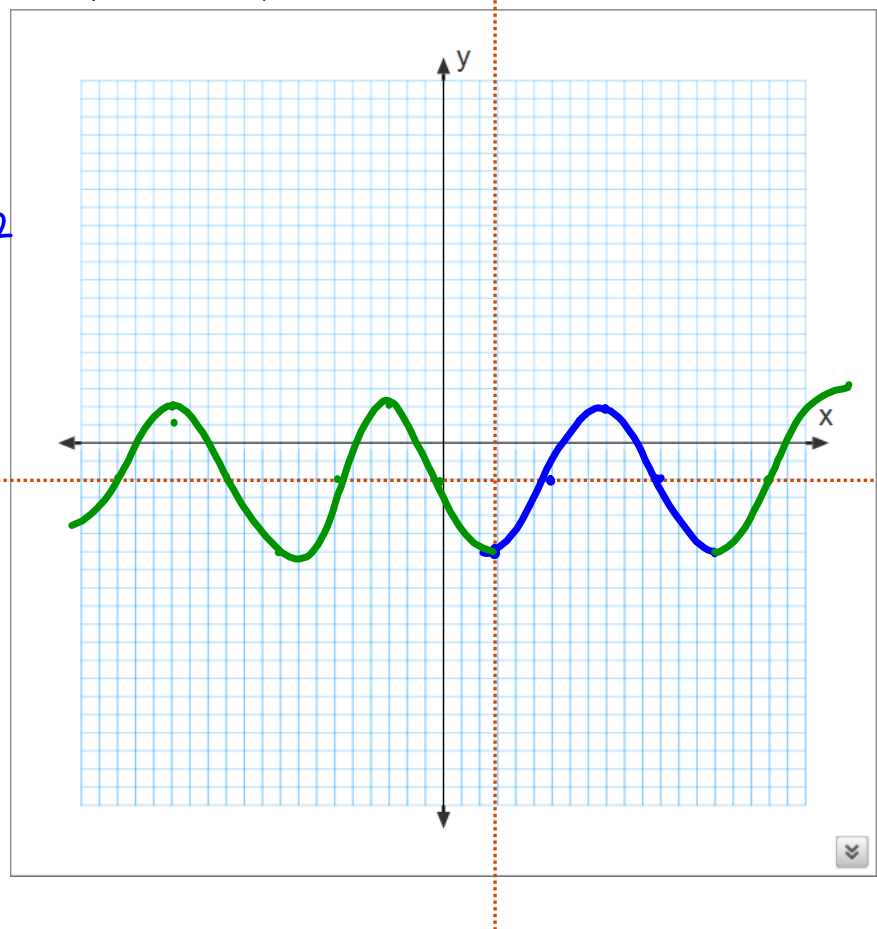
$A = 4$

$k = 2 \begin{cases} \text{max} = 2 \\ \text{min} = -6 \end{cases}$

$b = \frac{\pi}{6} \Rightarrow 2\pi \div \frac{\pi}{6}$
 $2\pi \times \frac{6}{\pi} = 12$



start: $(3, -6)$



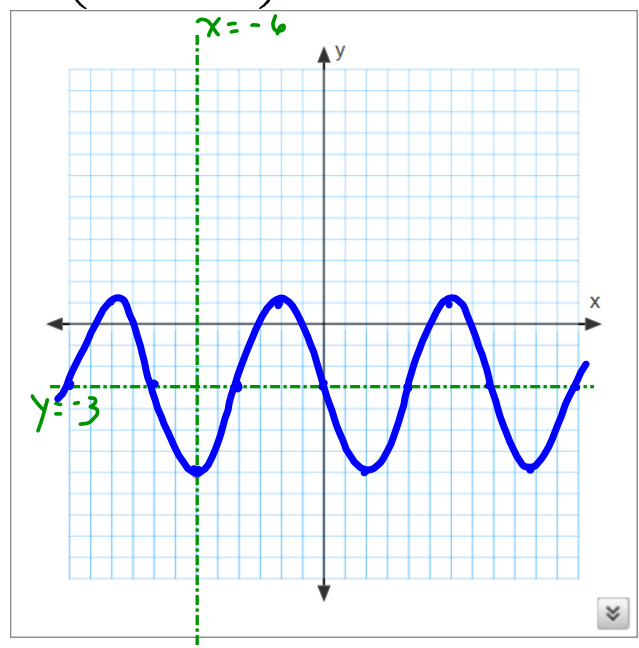
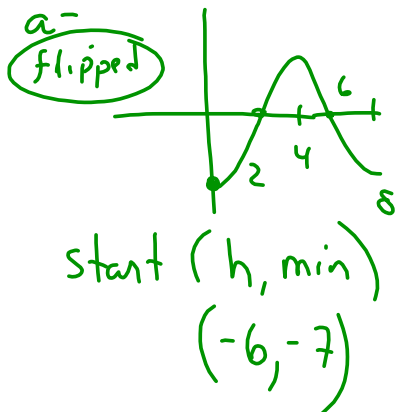
Graph the function $f(x) = -4\cos\left(\frac{\pi}{4}(x+6)\right) - 3$

$$A = 4 \quad k = -3$$

$$\text{max} = 1$$

$$\text{min} = -7$$

$$b = \frac{\pi}{4} \Rightarrow p = 8$$



Finding the Rule

Examples

$$A = \frac{\text{max} - \text{min}}{2} = \frac{3 - (-5)}{2} = \frac{8}{2} = 4$$

$$\therefore a = \pm 4$$

$$k = 3 - 4 = -1$$

$$\frac{1}{2}p = 1 \Rightarrow p = 2$$

$$\therefore b = \pm \frac{2\pi}{2} = \pm \pi$$

$$h = 0 \text{ (max)}$$

$$\therefore a^+$$

