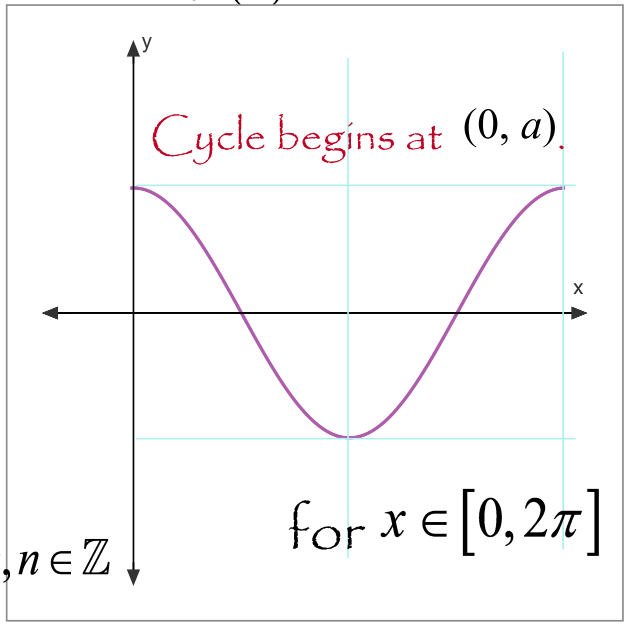
The Cosine Function

Basic cosine function

$f(x) = \cos x$



Properties

Domain: $\mathbb{R}$

Range: $[-1,1]$

Increasing: $[\pi, 2\pi] + 2\pi n, n \in \mathbb{Z}$

Decreasing: $[0, \pi] + 2\pi n, n \in \mathbb{Z}$

Positive: $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right] + 2\pi n, n \in \mathbb{Z}$

Negative: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] + 2\pi n, n \in \mathbb{Z}$

Maximum: 1

Minimum: -1

Period: 2π

Amplitude: 1

Notice that $y = \cos x$ is a translation of $y = \sin x$.

In fact, $f(x) = \cos x \iff f(x) = \sin\left(x + \frac{\pi}{2}\right)$.

For the transformed cosine function, a, b, h & k will be the same as with the sine function

That is ...

$$A = |a|$$

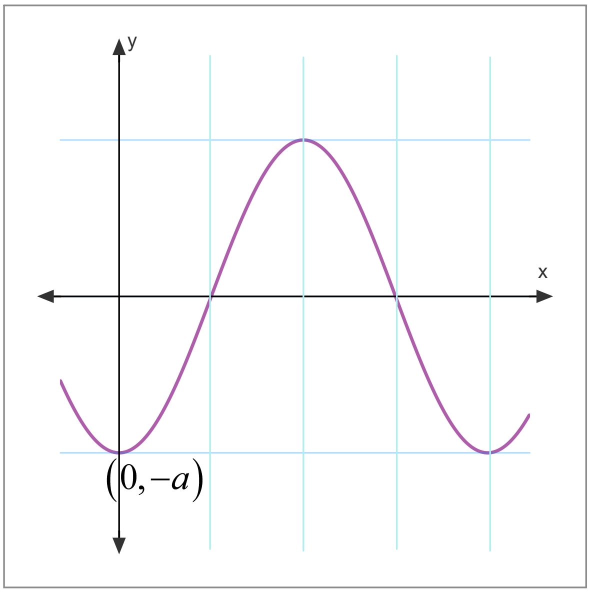
$$p = \frac{2\pi}{|b|}$$

k = middle axis (vertical translation)

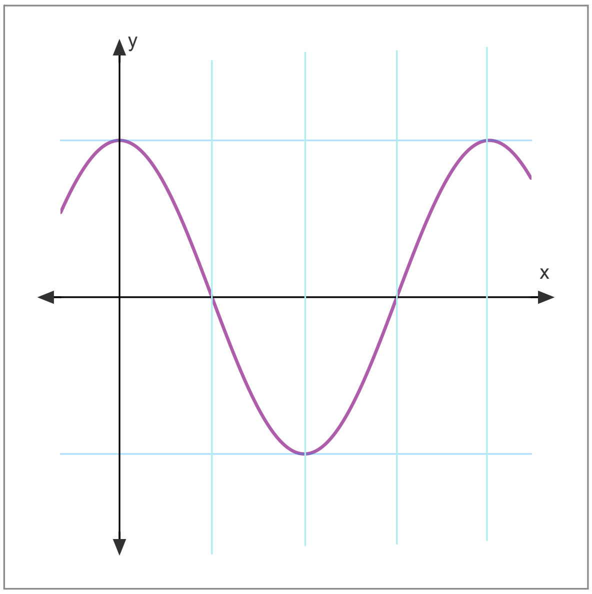
h = horizontal translation

Except ...

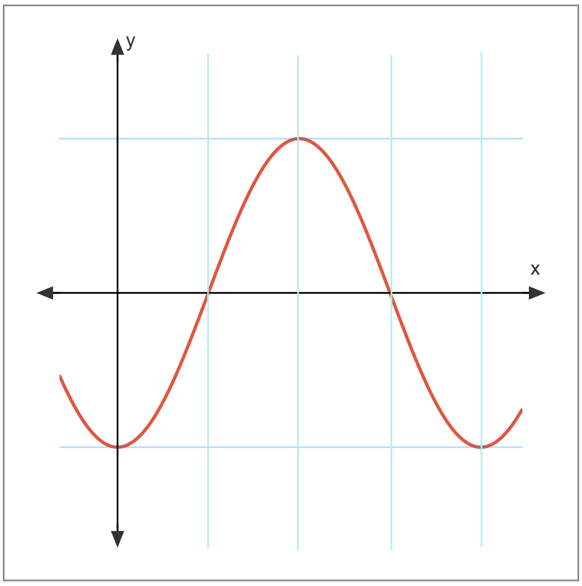
$y = -\cos x$



$y = \cos(-x)$



$$y = -\cos(-x)$$



Both a^-, b^+ and a^-, b^- result in a flip over the x-axis.

Therefore, if a is negative, the function flips.

Transformed Cosine Function

$$f(x) = a\cos(b(x-h)) + k$$

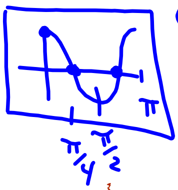
Graph $f(x) = \frac{1}{2}\cos(2(x-\pi)) + 3$

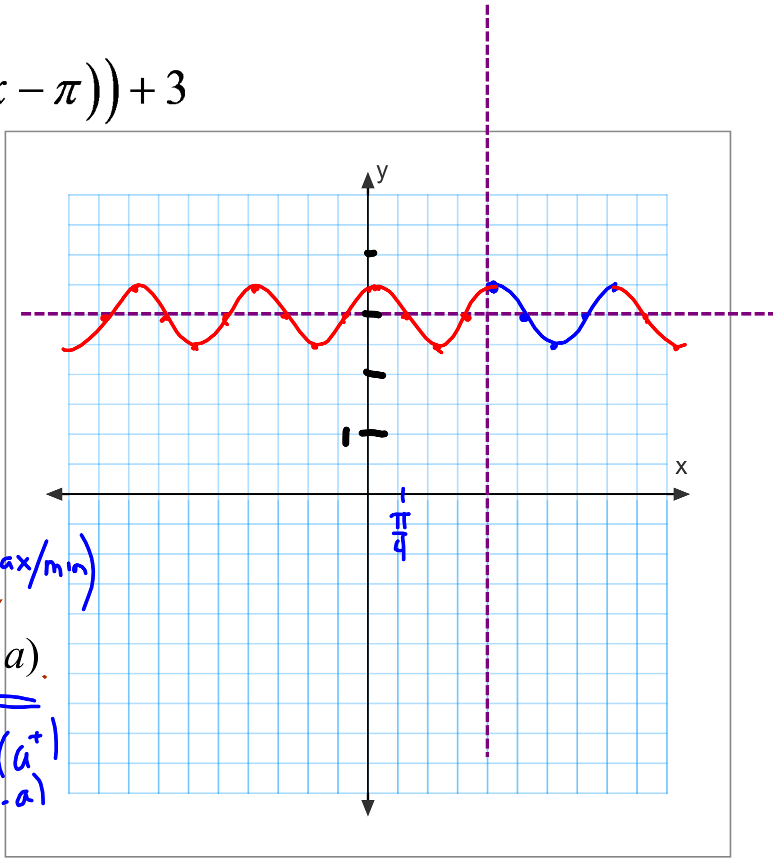
$A = \frac{1}{2} \Leftarrow a = 1/2$

$k = 3 \Rightarrow max = 3 + 0.5 = 3.5$

$min = 3 - 0.5 = 2.5$

$b = 2 \Rightarrow p = \frac{2\pi}{2} = \pi$





* Cycle now begins at $(h, k \pm a)$. or (h, max/min)

$h = \pi$ $k = 3$

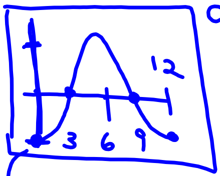
$k + a$ (a^+)
$k - a$ $(-a)$

Graph $f(x) = -4\cos\left(\frac{\pi}{6}(x-3)\right) - 2$

$a = -4 \Rightarrow A = 4$

$k = -2$ max = 2, min = -6

$b = \pi/6 \Rightarrow p = \frac{2\pi}{\pi/6} = 2\pi \cdot 6/\pi = 12$



starts at (h, min)

(3, -6)