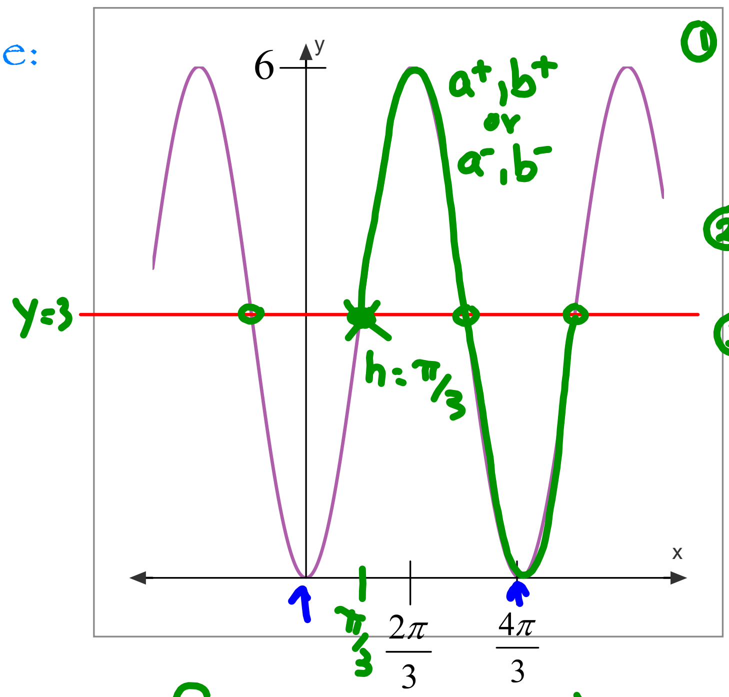
Example:



① $A = \frac{6-0}{2} = 3$

$a = \pm 3$

② $k = \frac{6+0}{2} = 3$

③ $P = \frac{4\pi}{3}$

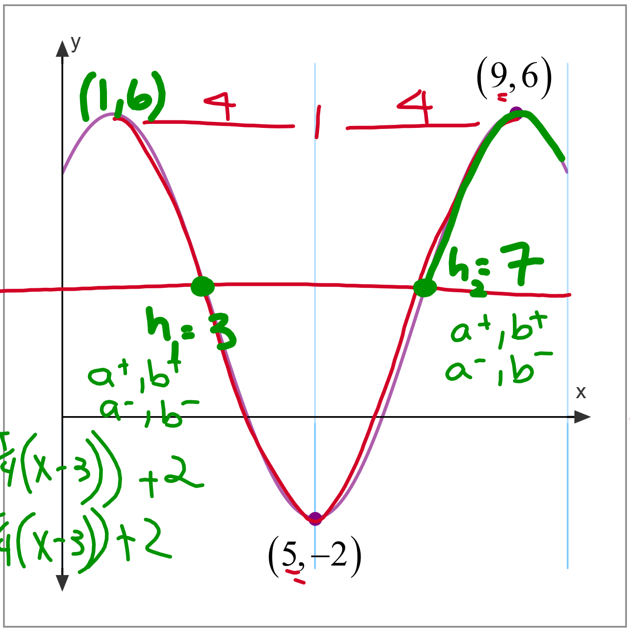
$b = 2\pi \cdot \frac{3}{4\pi} = \frac{3}{2}$ 1.5

$\therefore b = \pm \frac{3}{2}$

$= \pm 1.5$

$$f(x) = 3\sin\left(\frac{3}{2}\left(x - \frac{\pi}{3}\right)\right) + 3$$

Example:



$y = 2$

$\boxed{h = 3}$

① $f(x) = 4\sin\left(-\frac{\pi}{4}(x-3)\right) + 2$

② $f(x) = -4\sin\left(\frac{\pi}{4}(x-3)\right) + 2$

$\boxed{h = 7}$

① $f(x) = 4\sin\left(\frac{\pi}{4}(x-7)\right) + 2$

② $f(x) = -4\sin\left(-\frac{\pi}{4}(x-7)\right) + 2$

$\text{Amp} = \frac{6-(-2)}{2} = \frac{8}{2} = 4$

$a = \pm 4$

$k = 6 - 4 = 2$

$k = -2 + 4 = 2$

$k = \frac{6 + -2}{2} = \frac{4}{2} = 2$

period = 8

$b = \pm\frac{2\pi}{8} = \pm\frac{\pi}{4}$

$h = 3$ or 7

$A = \frac{-1.5-(-4.5)}{2} = \frac{3}{2}$

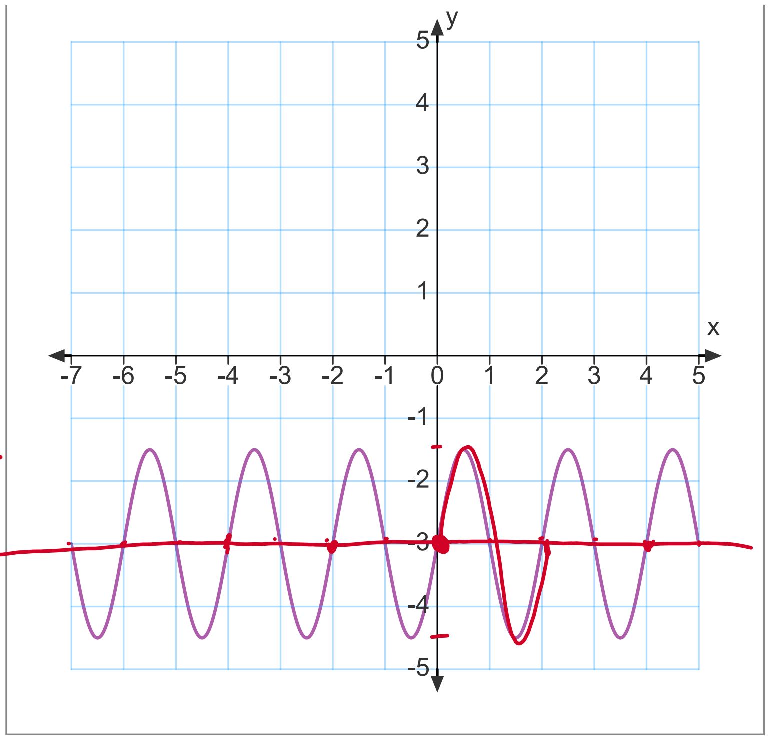
$a = \pm \frac{3}{2}$

$k = -1.5 - 1.5 = -3$

$p = 2 \quad b = \pm \frac{2\pi}{2} = \pm\pi$

$h = 0 \quad a^+, b^+ \quad a^-, b^-$

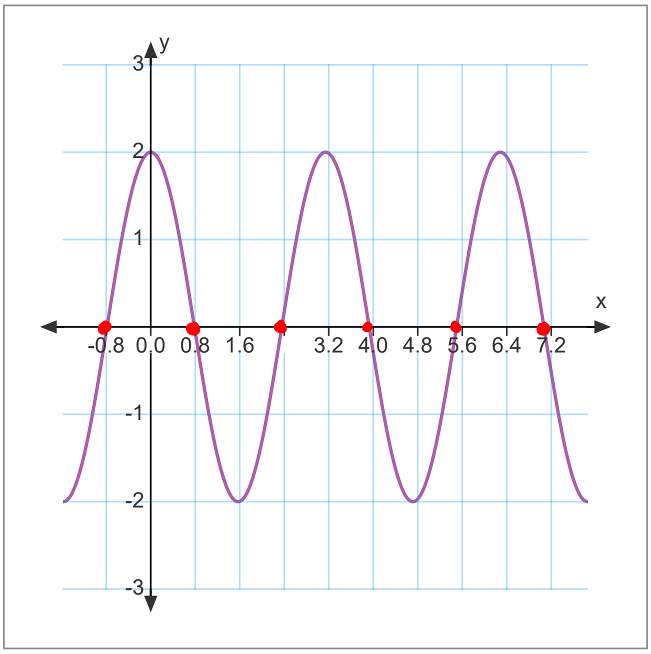
$y = \frac{3}{2}\sin(\pi x) - 3$



Finding the Zeros & Solving Equations

1) by sketching the graph

Example: $f(x) = -2\sin 2\left(x - \frac{\pi}{4}\right)$

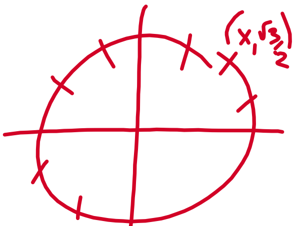


2) Algebraically

Example: $f(x) = 2\sin x - \sqrt{3}$

$a = 2$ $\quad \underbrace{b = 1} \quad h = 0 \quad k = -\sqrt{3}$

let $y = 0$



Trig Circle

isolate

$0 = 2\underline{\sin x} - \sqrt{3}$

$+\sqrt{3} \qquad +\sqrt{3}$

$\sqrt{3} = 2\sin x$

$\div 2 \qquad \div 2$

$\frac{\sqrt{3}}{2} = \sin x$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$x = \left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ only for 1 cycle

$p = ?$ $\quad b = 1 \Rightarrow p = \frac{2\pi}{1} = 2\pi$

$\therefore$ zeros are: $\left\{\frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n\right\}, n \in \mathbb{Z}$

$a=1 \quad b=3 \quad h=0 \quad k=\frac{1}{2}$

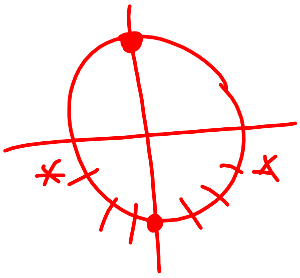
Example: Determine the zeros of $f(x)=\sin 3x+\frac{1}{2}$.

Note: $b(x-h)$ represents the angle.

let $y=0$

$0=\sin 3x+\frac{1}{2}$

$-\frac{1}{2}=\sin 3x$



$\sin^{-1}\left(-\frac{1}{2}\right)=3x$ (on circle)

$\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}=3x$

$\left\{\frac{7\pi}{18}, \frac{11\pi}{18}\right\}=x$

$b=3 \Rightarrow p=\frac{2\pi}{3}$

$$\boxed{\text{Zeros} \quad \{z_1+pn, z_2+pn\} \quad n\in\mathbb{Z}}$$

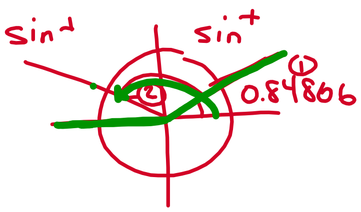
$$\therefore \; x=\left\{\frac{7\pi}{18}+\frac{2\pi n}{3}, \frac{11\pi}{18}+\frac{2\pi n}{3}\right\} \quad n\in\mathbb{Z}$$

Example: Determine the zeros of $f(x) = 4\sin \pi(x+1) - 3$.

$0 = 4\sin\pi(x+1) - 3$

$3 = 4\sin\pi(x+1)$

$\frac{3}{4} = \sin \underbrace{\pi(x+1)}_{\theta}$



$\pi(x+1) = \sin^{-1}\left(\frac{3}{4}\right)$ — not a special angle. calculator in Rad mode

① $\pi(x+1) = 0.84806$

② $\pi(x+1) = \pi - 0.84806$

$\pi(x+1) = 2.2935$

$b = \pi$

$P = \frac{2\pi}{\pi}$

$= 2$

①	②
$\pi(x+1) = 0.84806$ $\div\pi \quad \div\pi$	$\pi(x+1) = 2.2935$
$x+1 = 0.2699$	$x+1 = 0.7301$
$x = -0.7301$	$x = -0.2699$

one cycle

zeros: $\left\{-0.7301 + 2n,\ -0.2699 + 2n\right\},\ n \in \mathbb{Z}$

Example: Determine the zeros of $f(x) = 6\sin\left(\frac{\pi}{20}(x-6)\right) + 8$.

$$0 = 6\sin\left(\tfrac{\pi}{20}(x-6)\right) + 8$$

$$-8 = 6\sin\left(\tfrac{\pi}{20}(x-6)\right)$$

$\sin\theta = -1.\overline{3}$ $\Leftrightarrow$ $-1.\overline{3} = \sin\left(\tfrac{\pi}{20}(x-6)\right)$

$$\sin^{-1}(-1.\overline{3}) = \left(\tfrac{\pi}{20}(x-6)\right)$$

No Solution

If $|k| > |a|$ then no zeros!