

## Trigonometric Functions

Trigonometric functions are periodic. Their graphs have a pattern that repeats at regular intervals.

The period is the length (on the  $x$ -axis) of one cycle (the pattern that repeats itself).

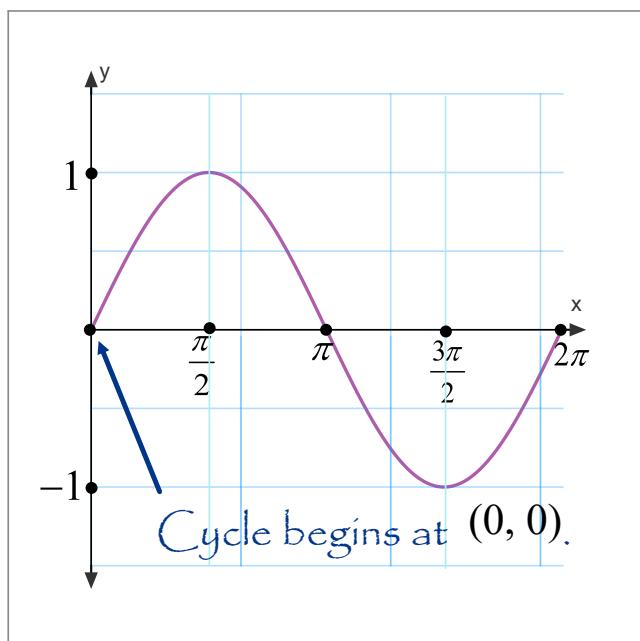
The frequency is the number of cycles that occur within a particular unit of measurement. It is the reciprocal of the period ( $f = \frac{1}{p}$  ).

## The Sine Function

<https://giphy.com/gifs/mathematics-sin-pi-NKLdcqhw02f8A/fullscreen>

[http://youtu.be/Ohp6Okk\\_tww](http://youtu.be/Ohp6Okk_tww)

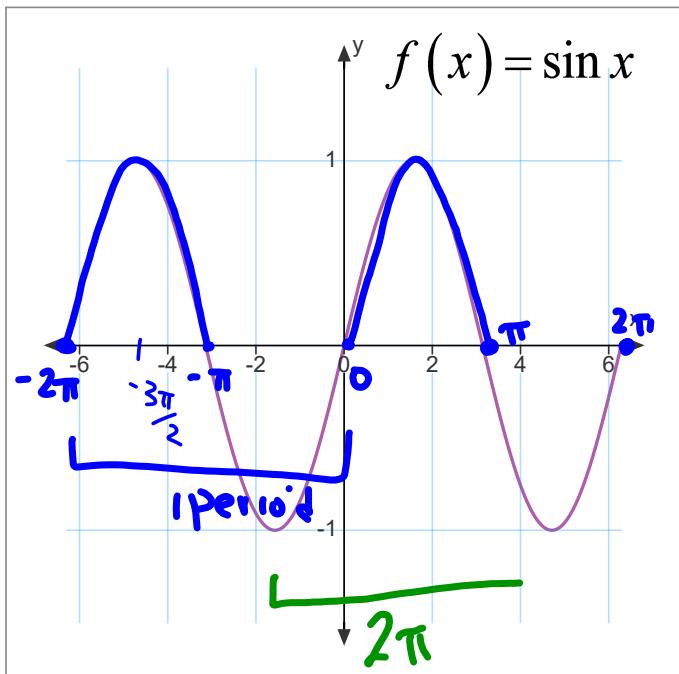
$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
$f(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0



Basic Sine Function

$$f(x) = \sin x$$

for  $x \in [0, 2\pi]$



### Properties

Dom:  $\mathbb{R}$  Max: 1

Ran:  $[-1, 1]$  Min: -1

Period:  $2\pi$

Inc:  $\left[ -\frac{\pi}{2} + 2\pi n, \frac{\pi}{2} + 2\pi n \right], n \in \mathbb{Z}$

Dec:  $\left[ \frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n \right], n \in \mathbb{Z}$

Pos:  $\left[ 0 + 2\pi n, \pi + 2\pi n \right], n \in \mathbb{Z}$

Neg:  $\left[ \pi + 2\pi n, 2\pi + 2\pi n \right], n \in \mathbb{Z}$

**Amplitude:** The farthest (vertically) that the function goes from the middle axis.

$$\begin{aligned}\text{Amplitude } (A) &= \frac{\max - \min}{2} \\ &= \frac{1 - (-1)}{2} \\ &= \frac{2}{2} \\ &= 1\end{aligned}$$

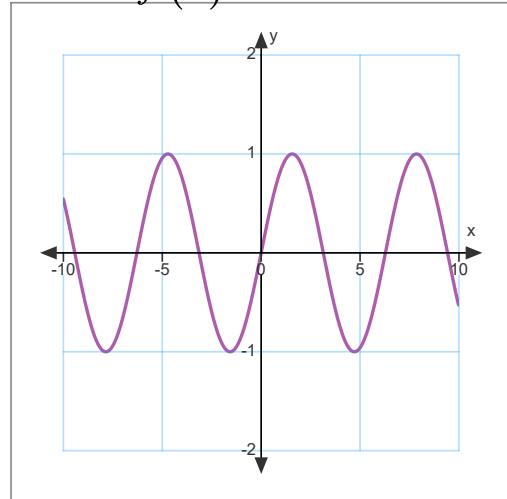
## Transformed Sine Function

Standard Form:  $f(x) = a \sin(\underbrace{b(x-h)}_{\text{angle}}) + k$

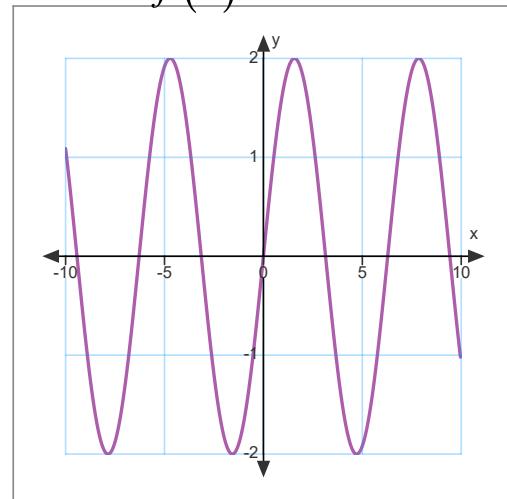
Recall: Changing parameter  $a$  results in a vertical stretch ( $|a| > 1$ ) or compression ( $|a| < 1$ ).

Changing the sign of parameter  $a$  results in a reflection about the  $x$ -axis.

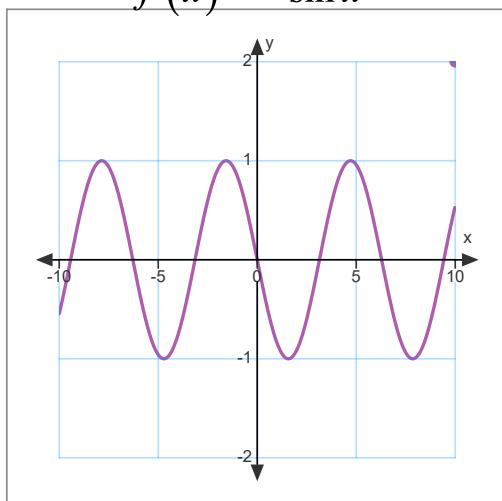
$$f(x) = \sin x$$



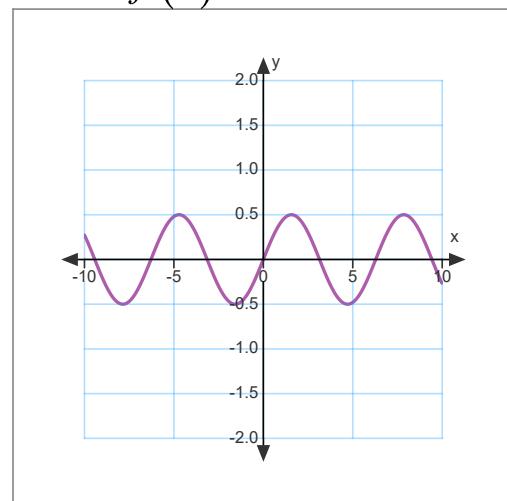
$$f(x) = 2 \sin x$$



$$f(x) = -\sin x$$



$$f(x) = 0.5 \sin x$$



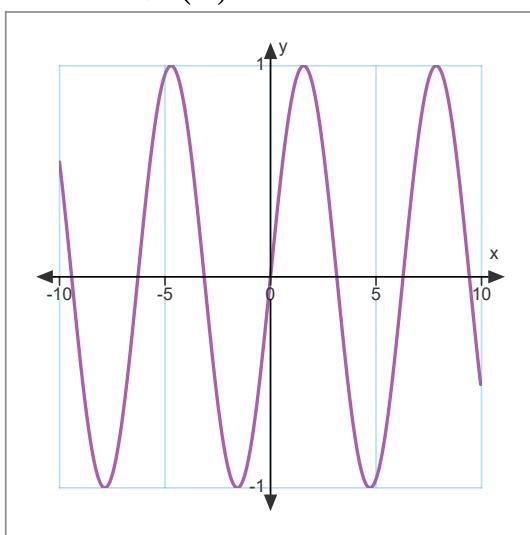
Parameter  $a$

$$\therefore \text{Amplitude} = |a|$$

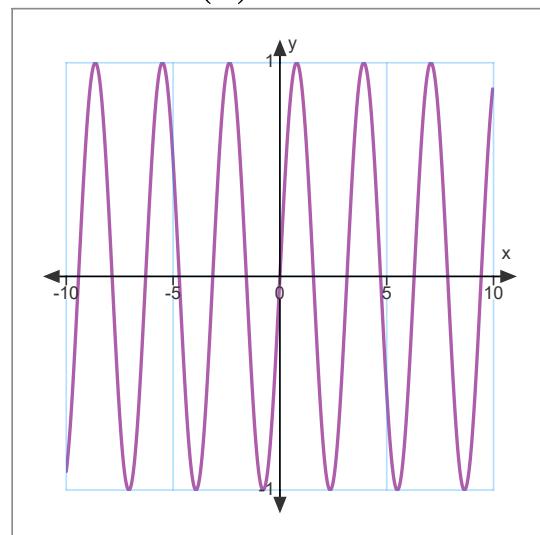
Changing parameter  $b$  results in a horizontal stretch ( $|b| < 1$ ) or compression ( $|b| > 1$ ).

Changing the sign of  $b$  results in a reflection about the  $y$ -axis.

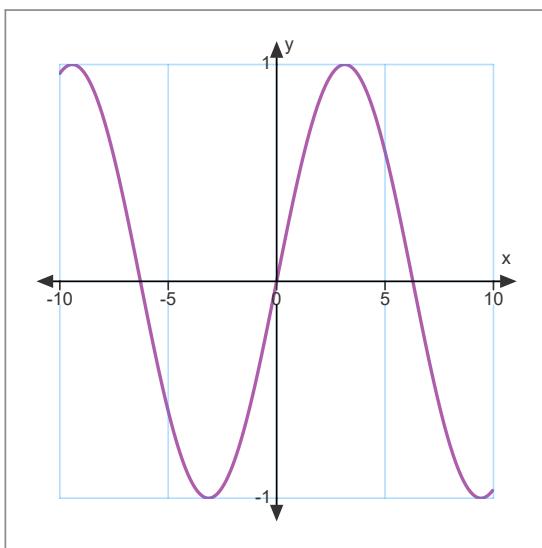
$$f(x) = \sin x$$



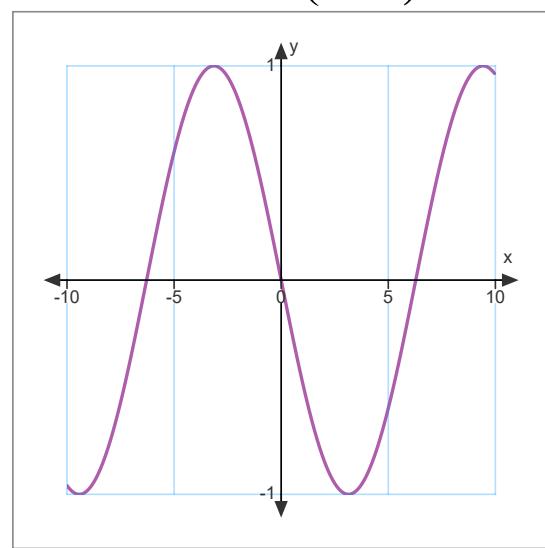
$$f(x) = \sin 2x$$



$$f(x) = \sin 0.5x$$



$$f(x) = \sin\left(-\frac{1}{2}x\right)$$



Parameter  $b$  affects the period.

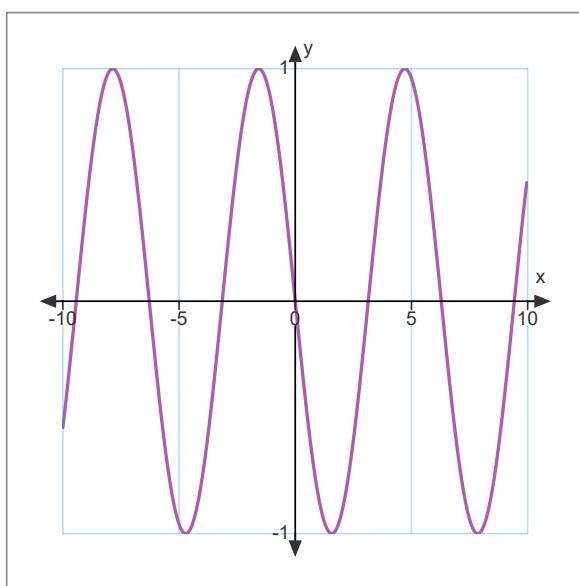
Basic sine function: period ( $p$ ) =  $2\pi$

With  $b$ :

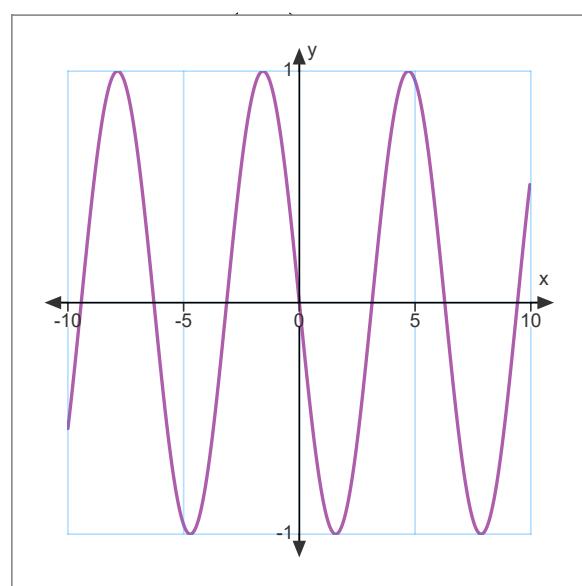
$$p = \frac{2\pi}{|b|}$$

Notice that when  $a$  or  $b$  is negative, the reflections look the same.

$$y = -\sin x$$

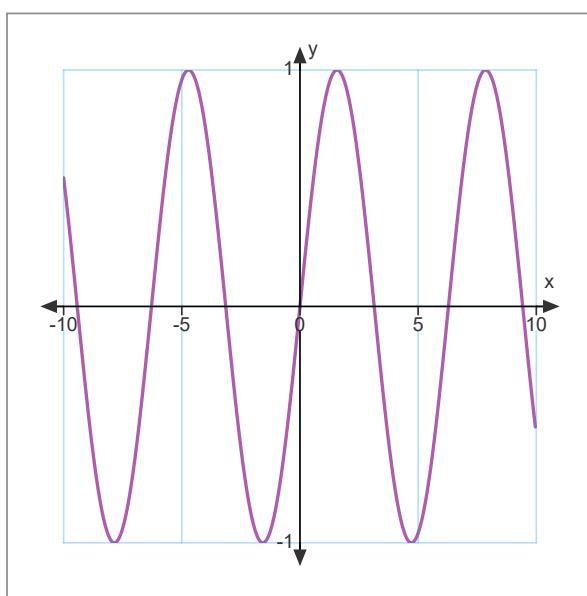


$$y = \sin(-x)$$



And when both  $a$  and  $b$  have the same sign, the graphs look the same (i.e. similar to the basic).

$$y = -\sin(-x)$$



$$y = \sin x$$

