

Solving Second-Degree Equations

Zero Product Principle - a product of factors is equal to zero if and only if at least one of the factors is equal to zero.

Example: if $5 \times \blacksquare = 0$ $\blacksquare = 0$

We will use this and factoring techniques to solve equations.

Example: Solve $x^2 - 10x = 0$

factor the LHS $x(x - 10) = 0$

\therefore *either* x *or* $x - 10$ *must be 0.*

So $x = 0$ *or* $x - 10 = 0$
 $x = 10$

$$x = \{0, 10\}$$

Example: Solve $x^2 - 2x - 15 = 0$

factor the LHS

Example: Solve $-2x^2 - 5x + 3 = 0$

Factor the *LHS*

Solve: $2x^2 - x = 6$

Make the equation equal to 0, then factor the LHS.

Solve $4x^2 - 36 = 0$

Solve $2x^2 - 50 = 0$



Solve $5x^2 - 35 = 0$

$$5x^2 = 35$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

$$\therefore x = \{-\sqrt{7}, \sqrt{7}\}$$

Example: Solve $14x^2 + 28 = 0$



Example:

Solve $10x^2 - 4x - 7 = 4x^2 - 11x + 13$

Example: The length of a rectangle is 5cm longer than its width. If the area is equal to 150cm^2 , **what is the numerical value of the perimeter of the rectangle?**

width:

length:

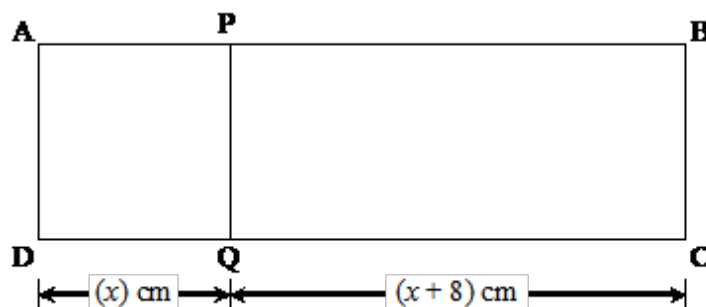
Area:



Perimeter = _____ *cm*

Example: In the figure, \overline{PQ} divides rectangle $ABCD$ into two quadrilaterals: square $APQD$ and rectangle $PBCQ$. The area of rectangle $ABCD$ is 120cm^2 . In addition, $m\overline{DQ} = (x)\text{cm}$ and $m\overline{QC} = (x + 8)\text{cm}$.

What is the numerical area of rectangle $PBCQ$?



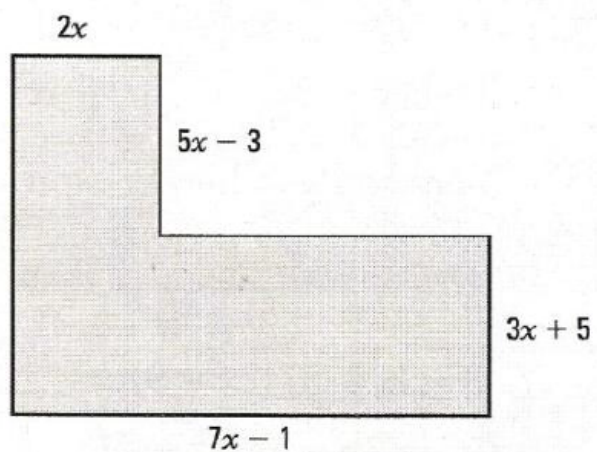
Today, a father is 2 years older than triple his son's age. Five years ago, the product of their ages was 420. How old is the father now?

Today, a mother's age is two years more than double her son's age. In ten years, the product of their ages will be 2040. How old are they today ?

The Quadratic Formula

The area of this figure is equal to 103.75cm^2 .

Determine the numerical length of each side .



$$2x(5x - 3) + (3x + 5)(7x - 1) = 103.75$$

$$10x^2 - 6x + 21x^2 - 3x + 35x - 5 = 103.75$$

$$31x^2 + 26x - 5 = 103.75$$

$$31x^2 + 26x - 108.75 = 0$$

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$$\begin{aligned}m \times n &= -3371.25 \\ m + n &= 26\end{aligned}$$

The quadratic formula provides a solution to any quadratic (second-degree) equation.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$ax^2 + bx + c = 0$$

$$\underset{a}{31}x^2 + \underset{b}{26}x - \underset{c}{108.75} = 0$$

$$x = \frac{-26 \pm \sqrt{26^2 - 4(31)(-108.75)}}{2(31)}$$

$$x = \frac{-26 \pm \sqrt{14161}}{62}$$

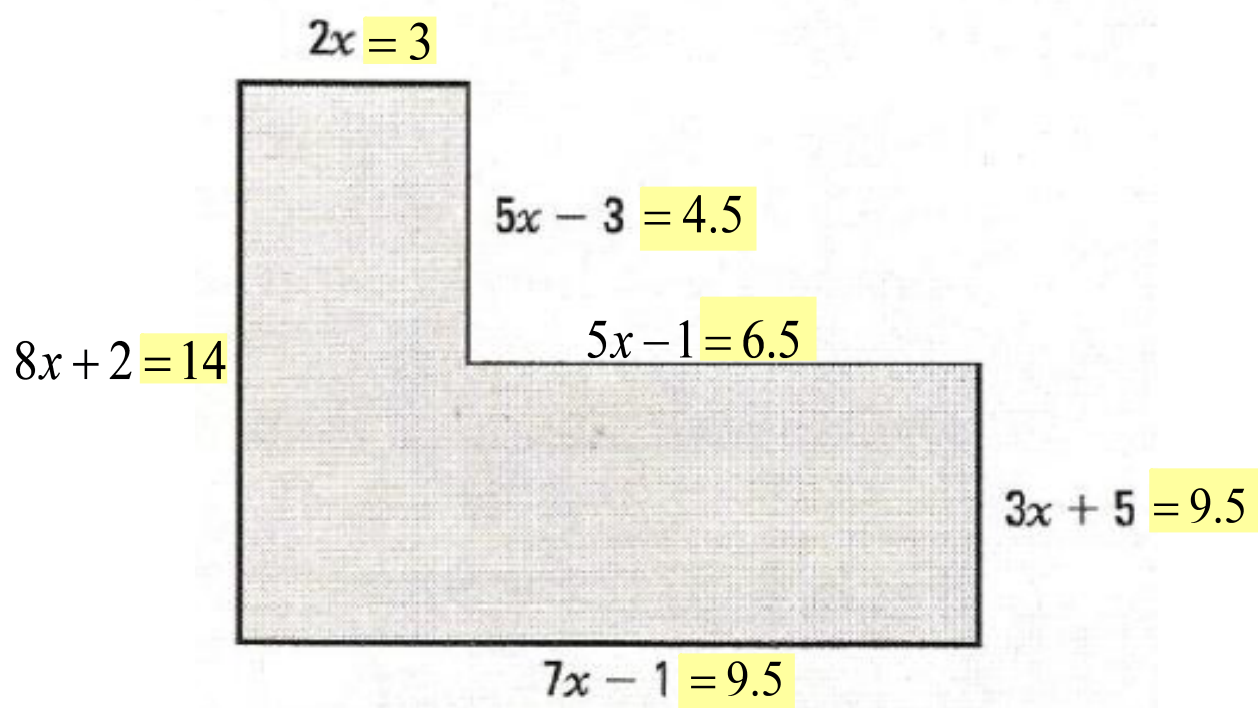
$$1 \quad x = \frac{-26 + 119}{62}$$

$$x = \frac{93}{62} = 1.5$$

$$2 \quad x = \frac{-26 - 119}{62}$$

$$x = \frac{-145}{62} \approx -2.34$$

$$\therefore x = 1.5$$



Example:

Solve $15x^2 - 2x - 8 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(15)(-8)}}{2(15)}$$

$$x = \frac{2 \pm \sqrt{4 + 480}}{30}$$

$$x = \frac{2 \pm \sqrt{484}}{30}$$

$$x = \frac{2 \pm 22}{30}$$

$$x = \frac{2 + 22}{30} = \frac{24}{30} = \frac{4}{5} \quad \text{or} \quad x = \frac{2 - 22}{30} = \frac{-20}{30} = -\frac{2}{3}$$

Solve

a) $x^2 - 6x - 91 = 0$

b) $9x^2 + 30x + 25 = 0$

$$c) \quad 5x^2 + 9x + 12 = 0$$

The Discriminant (Δ)

~ the portion under the root sign: $\Delta = b^2 - 4ac$

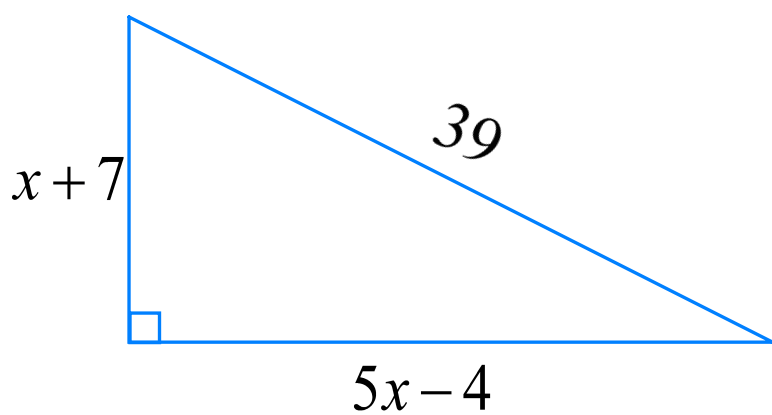
If ...

$b^2 - 4ac > 0$ there are 2 real answers

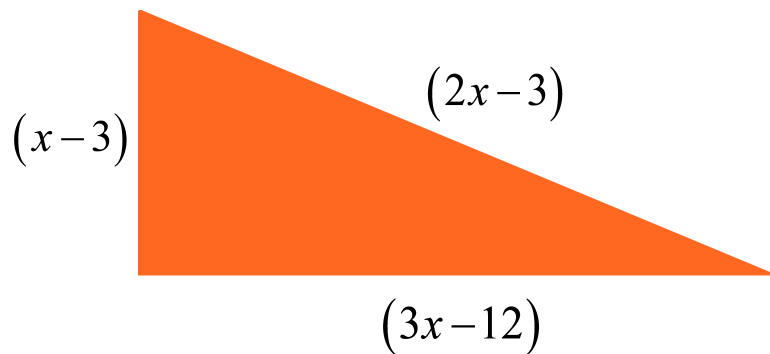
$b^2 - 4ac = 0$ there is one real answer

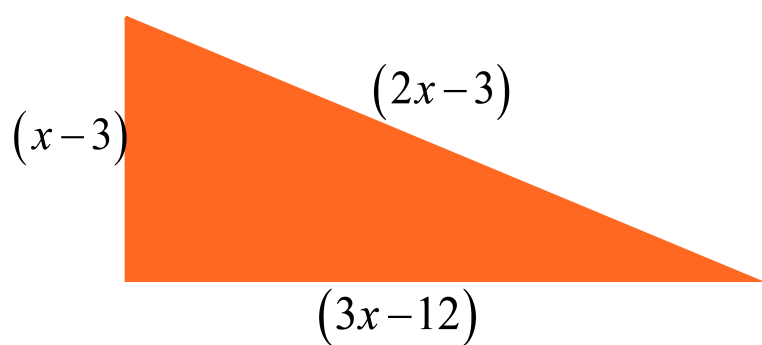
$b^2 - 4ac < 0$ there are no real answers

Determine the value of x

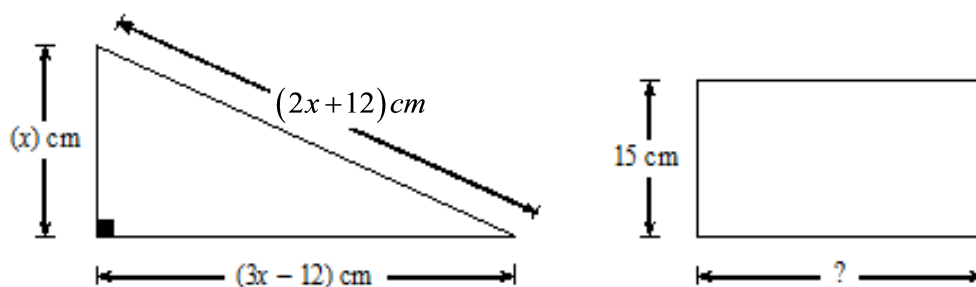


Determine the numerical perimeter of the right triangle shown below. All measurements are in centimetres.

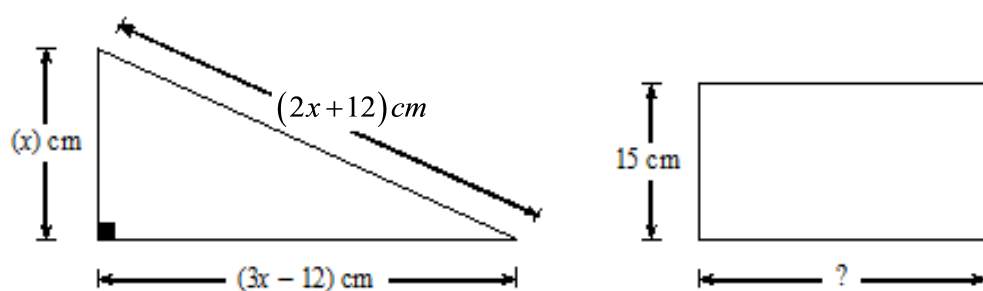




The right triangle and the rectangle given below are equivalent. The sides of the right angle of the triangle measure (x) cm and $(3x - 12)$ cm respectively and the hypotenuse measures $(2x + 12)$ cm. The height of the rectangle is 15 cm.



What is the numerical length of the base of the rectangle?



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