Trigonometric Functions

Trigonometric functions are periodic. Their graphs have a pattern that repeats at regular intervals.

The period is the length (on the x - axis) of one cycle (the pattern that repeats itself).

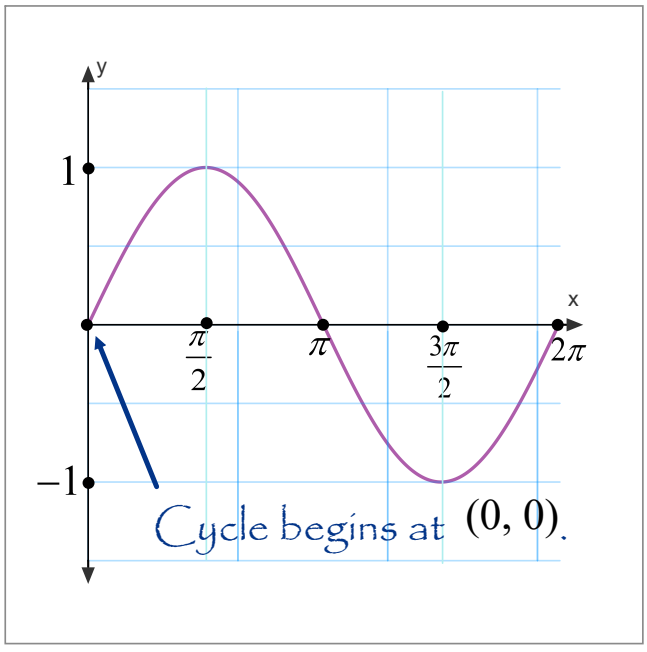
The frequency is the number of cycles that occur within a particular unit of measurement. It is the reciprocal of the period ($f = \frac{1}{p}$).

The Sine Function

https://giphy.com/gifs/mathematics-sin-pi-NKLdcqhwo2f8A/fullscreen

http://youtu.be/Ohp6Okk_tww

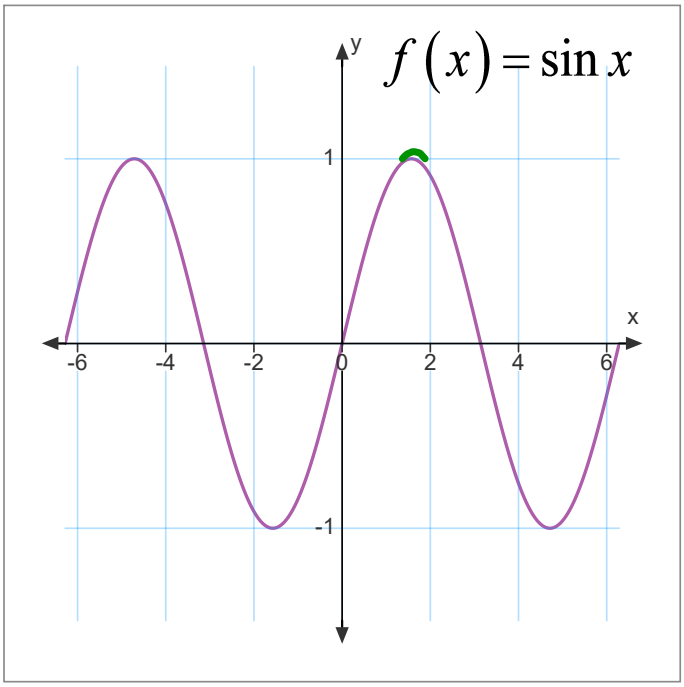
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$f(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0



Basic Sine Function

$$f(x) = \sin x$$

for $x \in [0, 2\pi]$



Properties

Dom: $\mathbb{R}$ Max: 1

Ran: $[-1, 1]$ Min: -1

Period: 2π

Inc: $\left[-\frac{\pi}{2} + 2\pi n, \frac{\pi}{2} + 2\pi n\right], n \in \mathbb{Z}$

Dec: $\left[\frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n\right], n \in \mathbb{Z}$

Pos: $[0 + 2\pi n, \pi + 2\pi n], n \in \mathbb{Z}$

Neg: $[\pi + 2\pi n, 2\pi + 2\pi n], n \in \mathbb{Z}$

Amplitude: The farthest (vertically) that the function goes from the middle axis.

$$\text{Amplitude } (A) = \frac{\max - \min}{2}$$

$$= \frac{1-(-1)}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

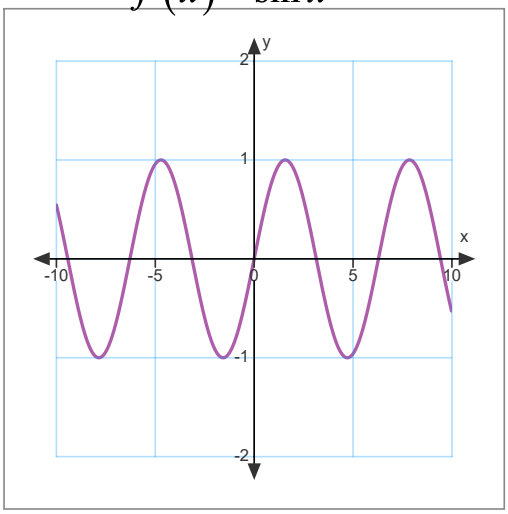
Transformed Sine Function

Standard Form: $f(x) = a\sin\overbrace{\left(b\left(x-h\right)\right)}^{\text{angle}}+k$

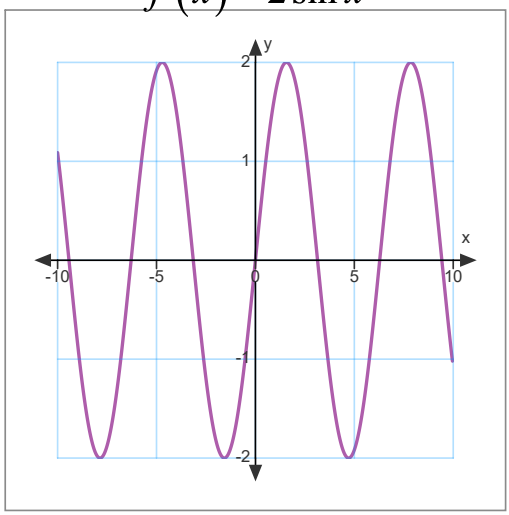
Recall: Changing parameter a results in a vertical stretch ($|a| > 1$) or compression ($|a| < 1$).

Changing the sign of parameter a results in a reflection about the x-axis.

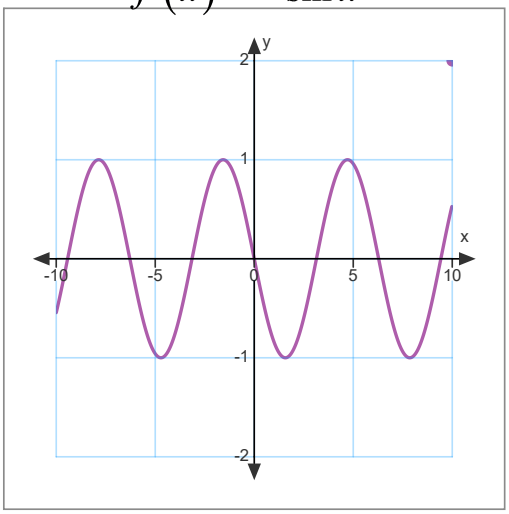
$f(x) = \sin x$



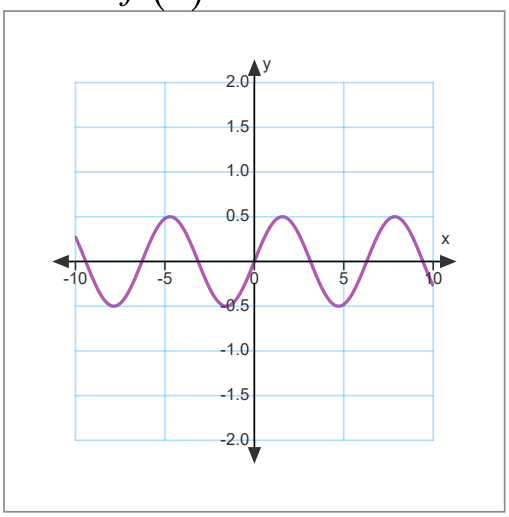
$f(x) = 2\sin x$



$f(x) = -\sin x$



$f(x) = 0.5\sin x$

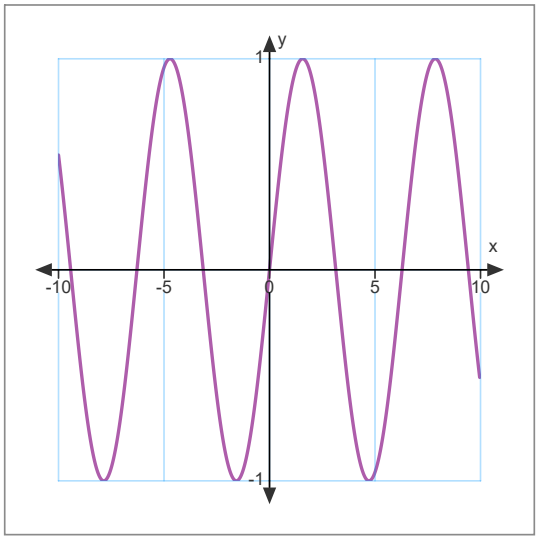


$$\therefore \text{ Amplitude} = |a|$$

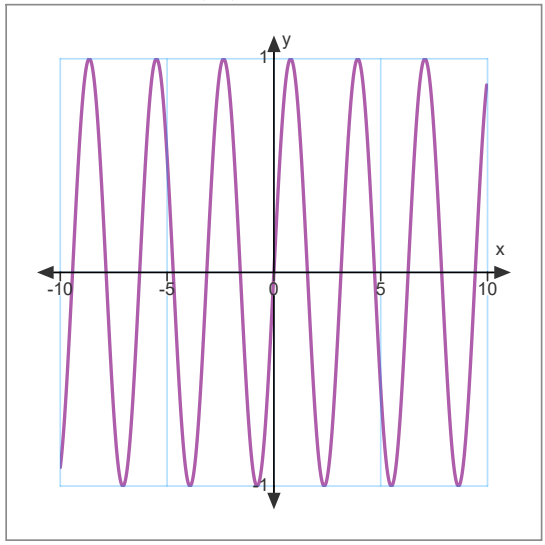
Changing parameter b results in a horizontal stretch ($|b|<1$) or compression ($|b|>1$).

Changing the sign of b results in a reflection about the y - axis.

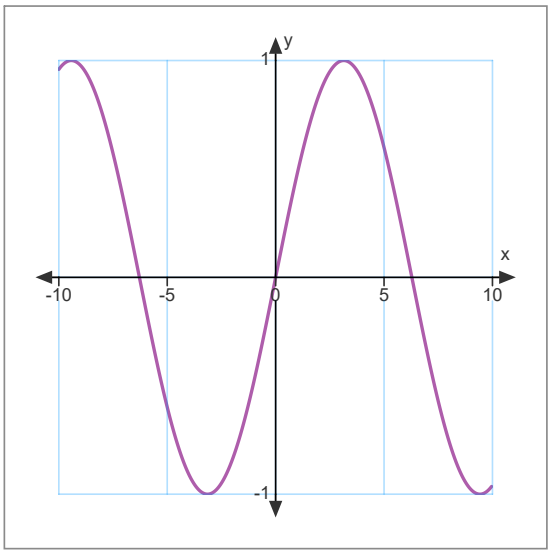
$f(x)=\sin x$



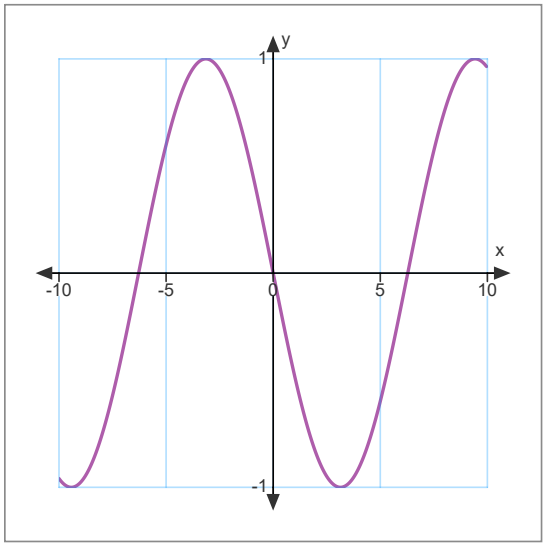
$f(x)=\sin 2x$



$$f(x) = \sin 0.5x$$



$$f(x) = \sin\left(-\frac{1}{2}x\right)$$



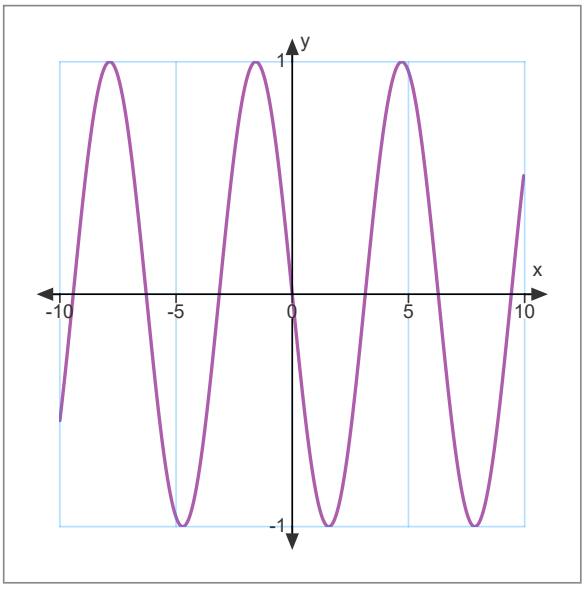
Parameter b affects the period.

Basic sine function: period $(p) = 2\pi$

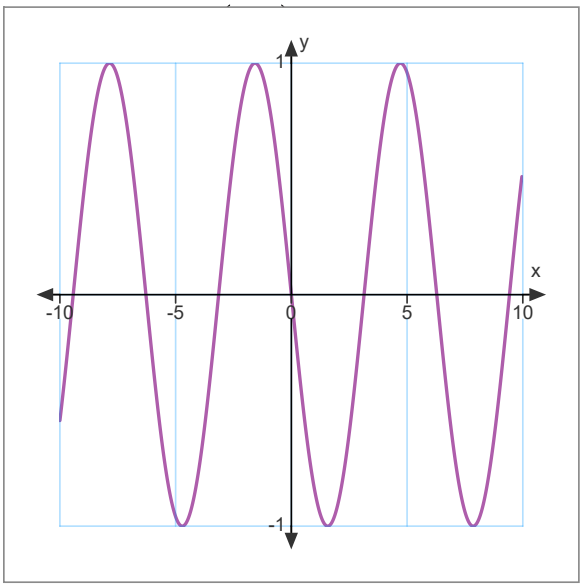
With b : $p = \frac{2\pi}{|b|}$

Notice that when a or b is negative, the reflections look the same.

$y = -\sin x$

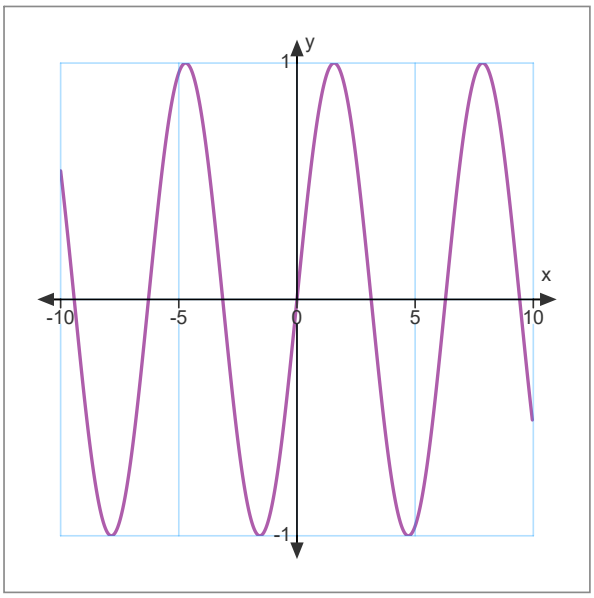


$y = \sin(-x)$

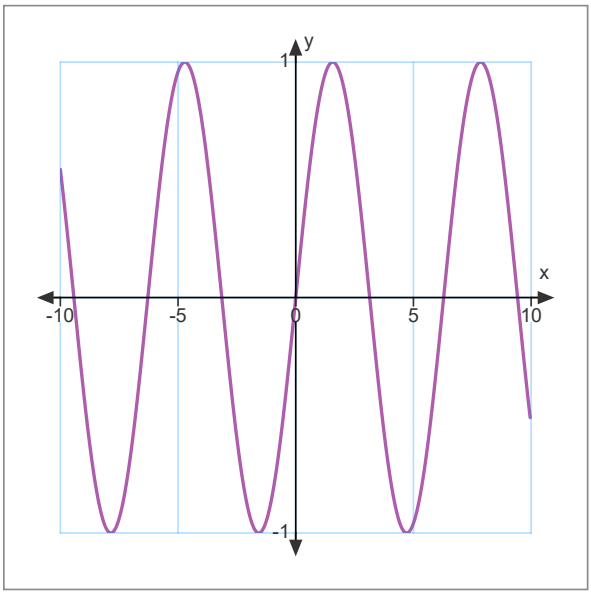


And when both a and b have the same sign, the graphs look the same (i.e. similar to the basic).

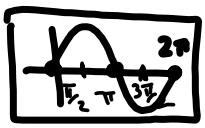
$y = -\sin(-x)$



$y = \sin x$



Graphing with a and b



1. $f(x) = 3\sin 2x$

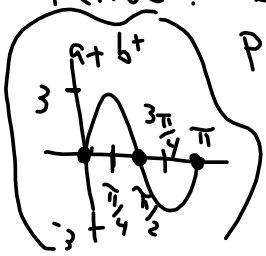
Amplitude : $a = 3$

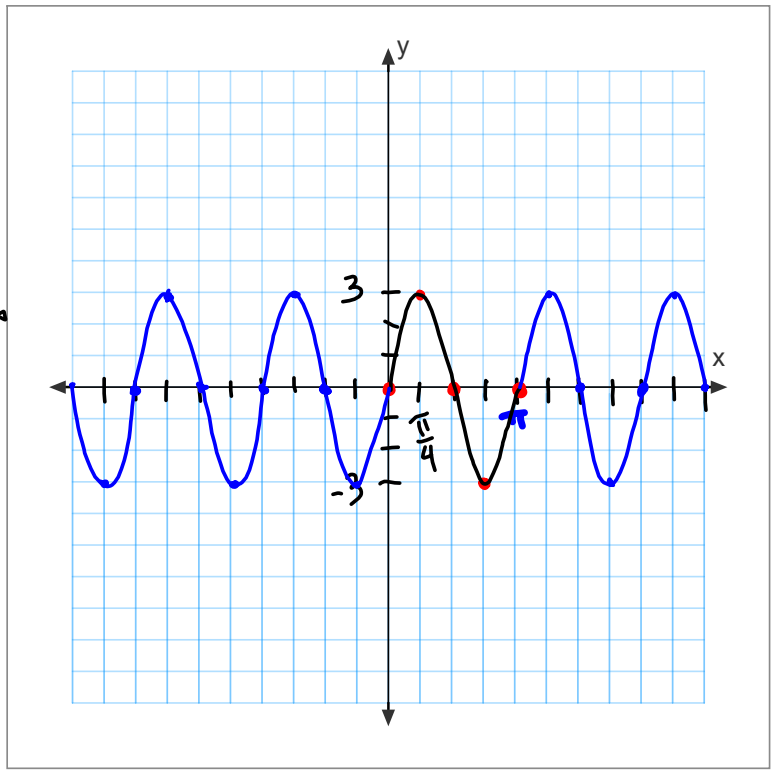
$\therefore A = 3$

max = 3 min = -3

Period : $b = 2$

$p = \frac{2\pi}{2} = \pi$





2. $f(x) = -\sin(-4x)$

$a = -1$

Amplitude = 1

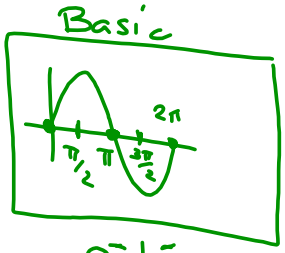
max = 1

min = -1

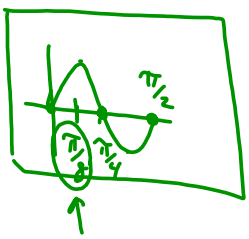
$b = -4$

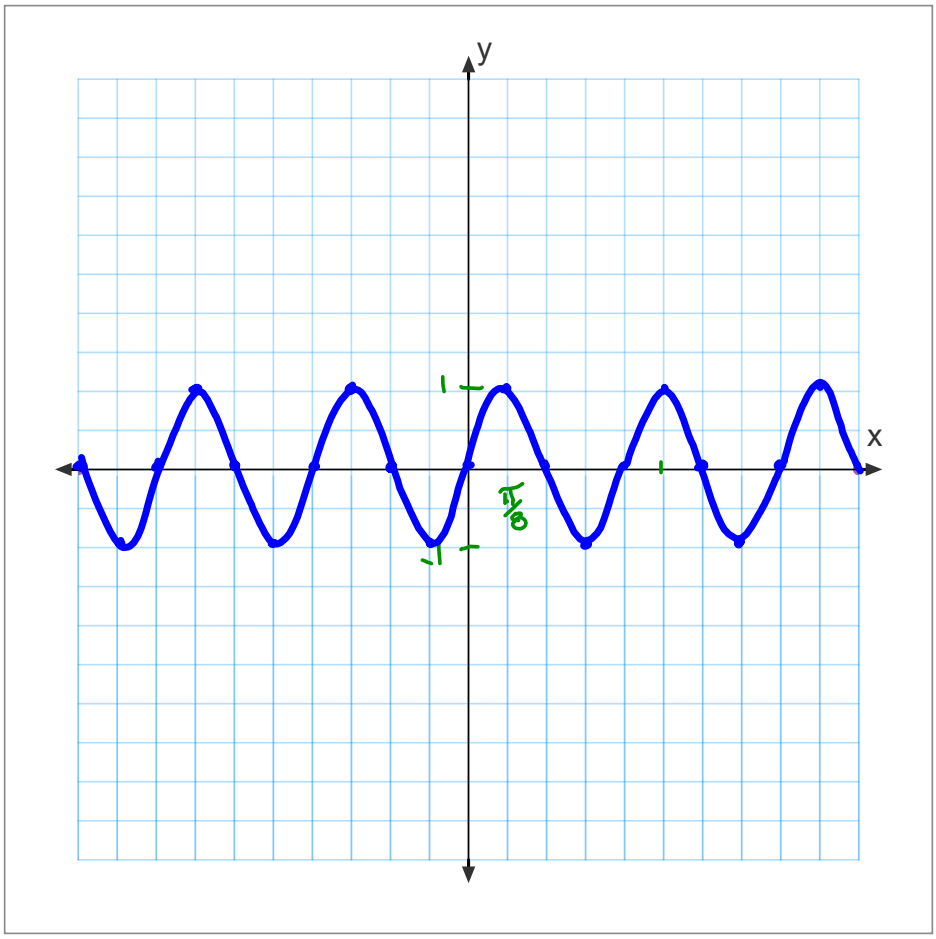
$P = \frac{2\pi}{4} = \frac{\pi}{2}$

Basic



a⁻, b⁻





3. $f(x) = -2\sin\left(\frac{\pi x}{2}\right)$

Basic

$a = -2$

Amplitude = 2

max = 2

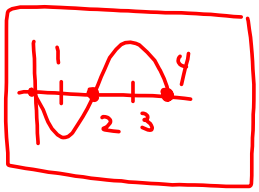
min = -2

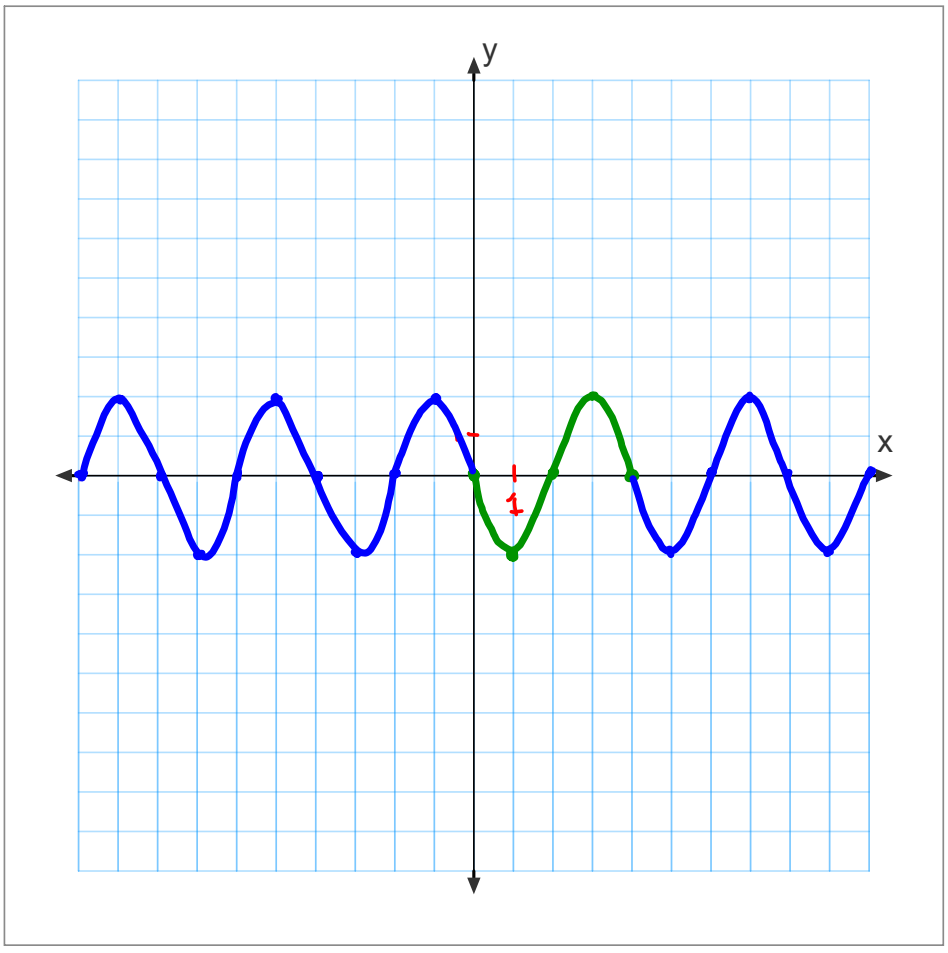
$b = \pi/2$

$p = 2\pi \div \pi/2$

$p = 2\pi \cdot \frac{2}{\pi} = 4$

a^-, b^+





Parameters h and k result in a translation of the function h units horizontally and k units vertically.

$f(x) = 3\sin(2(x-1))-1$

$h = 1$

$k = -1$

a = 3

Amplitude = 3

max = k + A

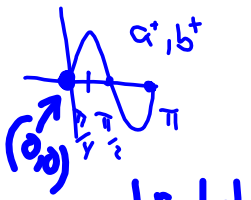
min = k − A

∴ max = −1 + 3 = 2

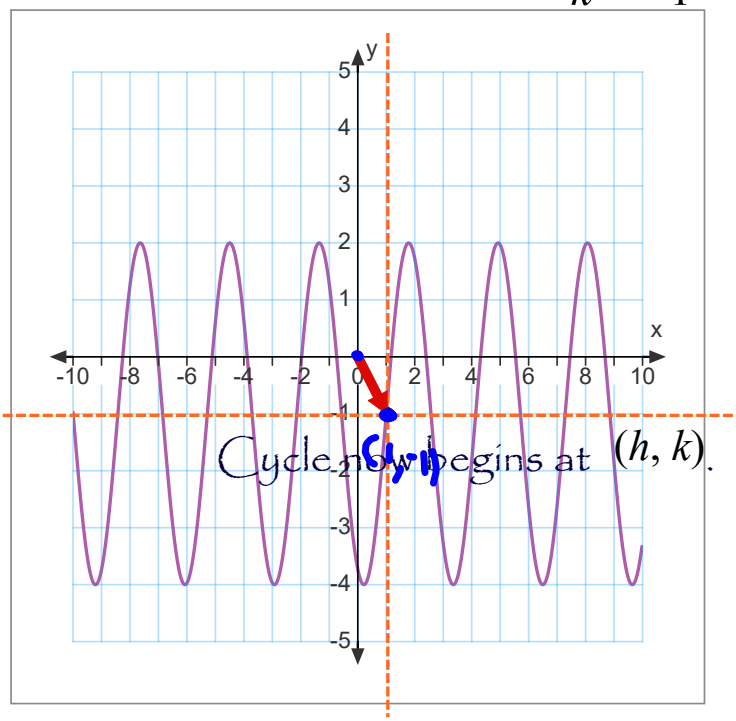
min = −1 − 3 = −4

b = 2

$P = \frac{2\pi}{2} = \pi$



Ir, Id



middle axis: $y = k$

$\max = k + A$

$\min = k - A$

Graphing with a, b, h and k

1. $f(x) = 3\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 2$

$a = 3$

Amp = 3

$k = 2$ (middle)

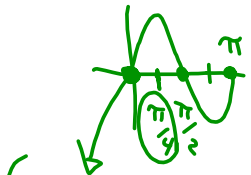
max = 2 + 3 = 5

min = 2 − 3 = −1

$b = 2$

$P = \frac{2\pi}{2} = \pi$

a^+, b^+



(h, k)

$\left(\frac{\pi}{2}, 2\right)$