RATIONAL EXPRESSIONS

A rational expression is a fraction, $\frac{P}{Q}$, where P and Q are polynomials, and $Q \neq 0$.

Examples: $\frac{3x^3+4x-8}{x-6}$, $\frac{5x^2+4}{7}$, $\frac{10}{x}$

A rational expression is undefined for any values of the variables that cause the denominator to be equal to zero.

Example: $\frac{3x^3+4x-8}{x-6}$

is undefined when x is 6 because $6-6=0$, and we can't divide by 0.

For what values of the variable are the following expressions undefined?

a) $\dfrac{9x^2+4x-7}{x-2}$

b) $\dfrac{x-3}{x^2-4}$

c) $\dfrac{9}{x^2+4}$

These values are called restrictions and must be considered when working with rational expressions.

Simplifying Rational Expressions

Sometimes a rational expression can be simplified if both the numerator and denominator have a common factor.

Example: $$\frac{x^2-3x}{x^2-9}$$

Factor the numerator and denominator, if possible.

$$\frac{x(x-3)}{(x+3)(x-3)}$$

We can cancel out the common factor in the numerator and denominator (the division equals 1), but we must state the restrictions that allow us to do so.

$$\frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \qquad x \neq \{-3, 3\}$$

$$\frac{x}{x+3} \text{ where } x \neq \{-3, 3\}$$

Simplify:

a) $\dfrac{a^2 - 1}{a + 1}$

b) $\dfrac{2x + 10}{x^2 + 7x + 10}$

c) $\dfrac{v^2 - 7v - 30}{v^2 - 5v - 24}$

d) $\dfrac{6m^3 + 42m^2}{2m^2 + 26m + 84}$

Rational Expressions and Arithmetic

To add, subtract, multiply & divide rational expressions, we are going to use the rules of arithmetic for fractions as well as our methods for factoring and simplifying.

1. Multiplication

Example: $$\left(\frac{3a-3b}{a}\right)\left(\frac{a^2}{a-b}\right)$$

* factor each polynomial, if possible

$$\left(\frac{3(a-b)}{a}\right)\left(\frac{a^2}{a-b}\right)$$

* state the restrictions, then multiply the expressions (canceling any common factors on top and bottom).

$$\frac{3(a-b)a^2}{a(a-b)}$$

$3a$ where $a \neq \{0, b\}$

Example: $\left(\dfrac{y^2+y}{y-2}\right)\left(\dfrac{1}{y+1}\right)$

Example: $$\frac{x^2-1}{x+3}\times\frac{x-3}{x^2-4x+3}$$

2. Division

Example: $\dfrac{2x-4}{x^2+6x+9} \div \dfrac{x^2-4}{x^2-9}$

* Factor each polynomial, if possible

$$\frac{2(x-2)}{(x+3)(x+3)} \div \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

* State the restrictions.
* Change divide to multiply and flip over the second fraction. State any new restriction(s) this creates.

$$\frac{2(x-2)}{(x+3)(x+3)} \times \frac{(x+3)(x-3)}{(x+2)(x-2)} \quad , \quad x \neq \{-3,-2,2,3\}$$

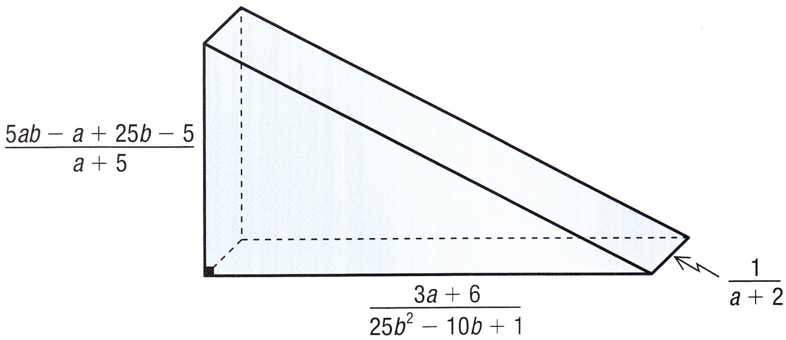
* multiply, canceling any common factors on top and bottom.

$$\frac{2(x-2)(x+3)(x-3)}{(x+3)(x+3)(x+2)(x-2)}$$

$$\frac{2(x-3)}{(x+3)(x+2)}\text{ , where } x \neq \{-3,-2,2,3\}$$

Example: Divide $\dfrac{x^2-1}{x+2} \div \dfrac{x-1}{3x+6}$

Determine the simplified algebraic expression that represents the volume of the adjacent right triangular-base prism.



3. Addition and Subtraction

Example: $\dfrac{1}{x+1}+\dfrac{2}{x-1}$ [redacted]

To add fractions, we need a common denominator. When they don't have a common factor, the common denominator will be the product of the two denominators.

$$\left(\frac{x-1}{x-1}\right)\left(\frac{1}{x+1}\right)+\left(\frac{2}{x-1}\right)\left(\frac{x+1}{x+1}\right)$$

Multiply, then add the fractions; be sure to state any restrictions.

$$\frac{x-1}{(x-1)(x+1)}+\frac{2x+2}{(x-1)(x+1)}$$

$$\frac{3x+1}{(x-1)(x+1)} \quad \text{or} \quad \frac{3x+1}{x^2-1} \qquad x \neq \{-1,1\}$$

Example: $\frac{x+3}{x-5}-\frac{x-1}{x+7}$

1: Factor

2: Common denominator

no common factor ⇒ cd is their product

3: State restrictions

4: Do the 2 multiplications, then subtract the numerators; be sure to subtract each term of the second numerator !

Example: $$\frac{2x+3}{x+1}-\frac{x-4}{x+6}$$

Example: $\dfrac{a-b}{a^2-1}+\dfrac{b-1}{a-1}$ [redacted]

$$\frac{a-b}{(a+1)(a-1)}+\frac{b-1}{a-1}$$

Since there is a common factor in each denominator, we need to multiply one by the factor that will make both look the same.

$$\frac{a-b}{(a+1)(a-1)}+\frac{b-1}{a-1}\left(\frac{a+1}{a+1}\right)$$

State any restrictions.
Multiply, then add the numerators.

$$\frac{a-b}{(a+1)(a-1)}+\frac{ab+b-a-1}{(a+1)(a-1)}$$

$$\frac{ab-1}{(a+1)(a-1)}$$ or $$\frac{ab-1}{a^2-1}$$ $$a \neq \{-1,1\}$$

Example: $$\frac{x}{x^2-9}-\frac{1}{2x-6}$$

$$\frac{x}{(x+3)(x-3)}-\frac{1}{2(x-3)}$$

Denominators have a common factor

$$\left(\frac{2}{2}\right)\frac{x}{(x+3)(x-3)}-\frac{1}{2(x-3)}\left(\frac{x+3}{x+3}\right)$$

$$\frac{2x}{2(x+3)(x-3)}-\frac{x+3}{2(x-3)(x+3)}$$

$$= \frac{x-3}{2(x+3)(x-3)}$$

$$= \frac{1}{2(x+3)} \quad x \neq \{-3, 3\}$$

Example: $$\frac{x+1}{x^2-2x+1}-\frac{1}{x-1}$$

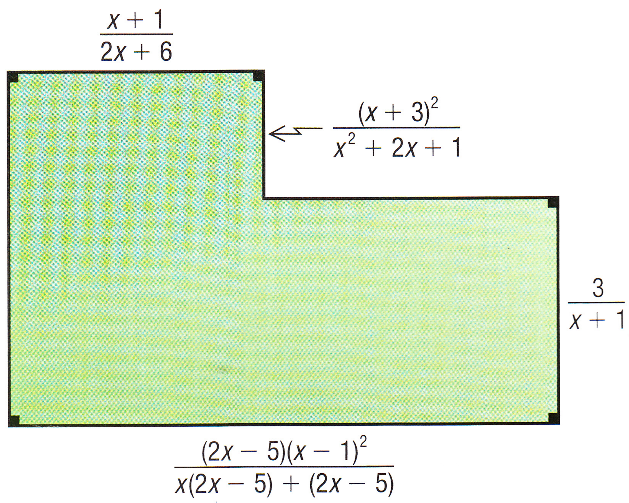
$$\frac{x+1}{(x-1)(x-1)}-\frac{1}{x-1}$$

$$\frac{x+1}{(x-1)(x-1)}-\frac{1}{x-1}\left(\frac{x-1}{x-1}\right)$$

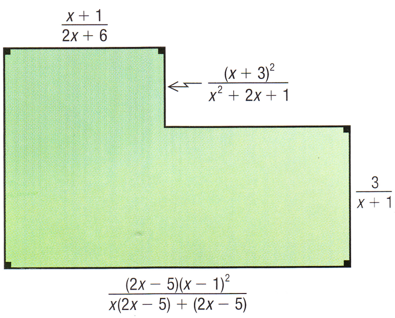
$$\frac{x+1}{(x-1)(x-1)}-\frac{x-1}{(x-1)(x-1)},x\neq 1$$

$$= \frac{2}{(x-1)(x-1)} \quad \text{or} \quad \frac{2}{x^2-2x+1}, \quad x \neq 1$$

Determine the simplified algebraic expression that represents the area of the figure illustrated below.



Determine the simplified algebraic expression that represents the area of the figure illustrated below.



Solving Rational Equations

To solve an equation means to determine the possible value(s) of the variables.

To solve rational equations we are going to use all our knowledge about rational expressions and equal fractions (proportions).

Example: Solve for x.

$$\frac{x+5}{x-2}=\frac{3}{8}$$

Example: Solve for x.

$$\frac{x}{x+2} - \frac{3}{x-2} = 1$$

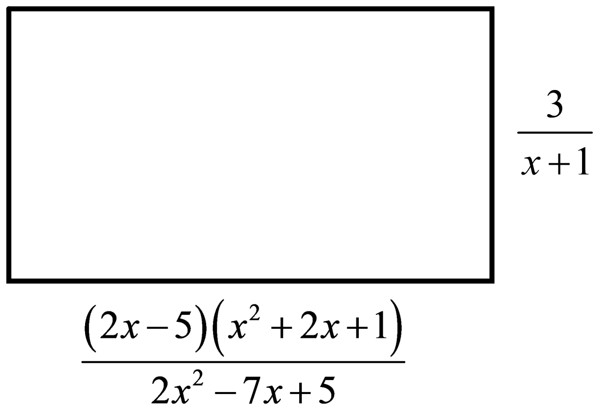
Example: Solve for m.

$$\frac{m-1}{3}-\frac{m+1}{8}=\frac{3m+1}{24}$$

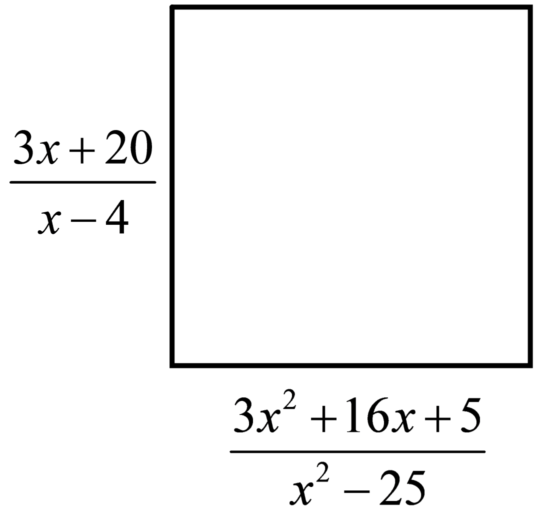
Example: Solve for a.

$$\frac{1}{a-5}+\frac{1}{a^2-11a+30}=\frac{7}{a-5}$$

Determine the value of x in the diagram below, given that the area of the rectangle is $4\ \text{dm}^2$.



Find the numerical value for the perimeter and the area of this square.



The perimeter of the right triangle below is $2a+5$. Find the length of the missing side.