

5. $\sec 165^\circ = ?$ (exact) $\sec \theta = \frac{1}{\cos \theta}$

$\sec(45^\circ + 120^\circ)$

$$\begin{aligned}\cos(45^\circ + 120^\circ) &= \cos 45^\circ \cdot \cos 120^\circ - \sin 45^\circ \sin 120^\circ \\ &= \left(\frac{\sqrt{2}}{2} \cdot -\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) \\ &= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\ \cos(165^\circ) &= -\frac{\sqrt{2} - \sqrt{6}}{4} \\ \therefore \sec(165^\circ) &= \frac{4}{-\sqrt{2} - \sqrt{6}} \left(\frac{-\sqrt{2} + \sqrt{6}}{-\sqrt{2} + \sqrt{6}} \right) = \frac{-4\sqrt{2} + 4\sqrt{6}}{2 - 6} ; \frac{-4\sqrt{2} + 4\sqrt{6}}{-4} \\ &= \underline{\underline{\sqrt{2} - \sqrt{6}}}\end{aligned}$$

6. Determine the exact value of $\tan\left(\frac{17\pi}{12}\right)$.

$$\begin{aligned} \tan\left(\frac{8\pi}{12} + \frac{9\pi}{12}\right) &= \tan\left(\frac{2\pi}{3} + \frac{3\pi}{4}\right) \\ &= \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \\ \tan \frac{2\pi}{3} &= \frac{\sqrt{3}}{2} : -\frac{1}{2} \\ &= \frac{\sqrt{3}}{2} \cdot -\frac{2}{1} : -\sqrt{3} \\ \tan \frac{3\pi}{4} &= \frac{\sqrt{2}}{2} : -\frac{\sqrt{2}}{2} \\ &= -1 \end{aligned}$$

What are the exact coordinates of the trigonometric point $P(195^\circ)$?

$$\begin{aligned} & \left(\cos 195^\circ, \sin 195^\circ \right) = \left(\frac{-\sqrt{6}-\sqrt{2}}{4}, \frac{-\sqrt{6}+\sqrt{2}}{4} \right) \\ & \cos(150^\circ + 45^\circ) \\ & \cos 150^\circ \cdot \cos 45^\circ - \sin 150^\circ \cdot \sin 45^\circ \\ & -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ & -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4} \quad \left| \begin{array}{l} \sin(225^\circ - 30^\circ) \\ \sin 225^\circ \cdot \cos 30^\circ - \sin 30^\circ \cdot \cos 225^\circ \\ -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{6}}{4} - \frac{-\sqrt{2}}{4} = \frac{-\sqrt{6}+\sqrt{2}}{4} \end{array} \right. \end{aligned}$$

Knowing that $\cos a = \frac{-21}{29}$ and $\cos b = \frac{7}{25}$, and that

$$\frac{\pi}{2} \leq a \leq \pi \quad Q_2$$

$$0 \leq b \leq \frac{\pi}{2} \quad Q_1$$

calculate $\cos(a+b)$.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(a) \sin^2 a + \left(\frac{-21}{29}\right)^2 = 1$$

$$\sin^2 a + \frac{991}{841} = \frac{841}{841}$$

$$\sin^2 a = \frac{841 - 991}{841}$$

$$\sin a = \frac{20}{29}$$

$$\sin^2 a = \frac{400}{841}$$

$$\sin a = \pm \frac{20}{29}$$

$$\begin{aligned} & \underline{\cos a} \cdot \underline{\cos b} - \sin a \cdot \sin b \\ &= \frac{-21}{29} \cdot \frac{7}{25} - \frac{20}{29} \cdot \frac{24}{25} \\ &= \frac{-147}{725} - \frac{480}{725} = \boxed{\frac{-627}{725}} \end{aligned}$$

$$(b) \sin^2 b = \frac{625}{625} - \frac{49}{625}$$

$$\sin^2 b = \frac{576}{625}$$

$$\sin b = \pm \frac{24}{25} \Rightarrow \sin b = \frac{24}{25}$$

Knowing that $\sin a = -\frac{15}{17}$ and $\sin b = \frac{12}{13}$, and that
 $\frac{3\pi}{2} \leq a \leq 2\pi$ and $\frac{\pi}{2} \leq b \leq \pi$ calculate...

$$\cos^2 b = 1 - \sin^2 b \\ = \frac{169}{169} - \frac{144}{169} \\ = \frac{25}{169}$$

$$\cos^2 a = 1 - \sin^2 a \\ = \frac{289}{289} - \frac{225}{289} \\ = \frac{64}{289}$$

$$\begin{aligned}\sin(a+b) &= \sin a \cos b + \sin b \cos a \\ &= \left(-\frac{15}{17}\right)\left(-\frac{5}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{8}{17}\right) \\ &= \frac{75}{221} + \frac{96}{221} = \frac{171}{221}\end{aligned}$$

$$\cos a = -\frac{8}{17}$$

$$\cos b = \pm \frac{5}{13}$$

$$\cos(a-b)$$

Can you find an expression that corresponds to ...

$$\begin{aligned}1. \quad \sin(2\theta) &= \sin(\theta + \theta) \\&= \sin\theta \cdot \cos\theta + \sin\theta \cdot \cos\theta \\&= 2\sin\theta\cos\theta\end{aligned}$$

$$\begin{aligned}2. \quad \cos 2\theta &= \cos(\theta + \theta) = \cos\theta\cos\theta - \sin\theta\sin\theta \\&= \cos^2\theta - \sin^2\theta\end{aligned}$$

These expressions are also rules and are known as the double angle formulas.