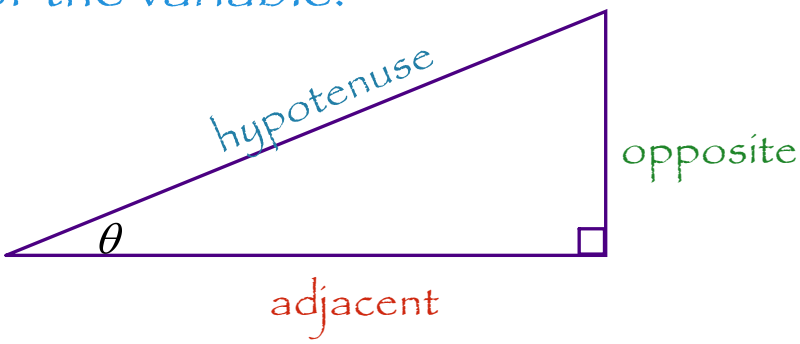
Trigonometric Identities

An identity is a statement that is true for all possible values of the variable.

Recall:



$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

Secant: $\sec\theta = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent}$

Cosecant: $\csc\theta = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$

(cosec θ)

Cotangent: $\cot\theta = \frac{1}{\tan\theta} = \frac{adjacent}{opposite}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

* $\cos^2\theta + \sin^2\theta = 1$

i) $\cos^2\theta = 1 - \sin^2\theta$

ii) $\sin^2\theta = 1 - \cos^2\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

* $1 + \tan^2\theta = \sec^2\theta$

  i) $\tan^2\theta = \sec^2\theta - 1$

  ii) $\sec^2\theta - \tan^2\theta = 1$

$$\cos^2\theta + \sin^2\theta = 1$$

* $\cot^2\theta + 1 = \csc^2\theta$

  i) $\cot^2\theta = \csc^2\theta - 1$

  ii) $\csc^2\theta - \cot^2\theta = 1$

We use identities to simplify statements or in proofs by using substitution .

Examples: Simplify each of the following expressions.

a) $\sin x \sec x \cot x$

b) $\dfrac{\tan^2 \theta}{1 + \tan^2 \theta}$

c) $\dfrac{\cos\theta + \sin^2\theta \sec\theta}{\sec\theta}$

d) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}$

Examples: Prove each of the following identities.

a) $\left(\cos\theta - \sin^2\theta\cos\theta\right)\left(1+\tan^2\theta\right) = \cos\theta$

Simplify the left side.

b) $\dfrac{\sec\theta\csc\theta}{\tan\theta+\cot\theta}=1$

More trigonometric identities:

a) $\left(\sec\phi - \tan\phi\right)^2 = \dfrac{1-\sin\phi}{1+\sin\phi}$

b) $\sec t - \dfrac{\cos t}{1+\sin t} = \tan t$

c) $\dfrac{\sec^2 x \cot x}{\csc^2 x} = \tan x$

d) $\dfrac{\tan x}{\sec x - 1} + \dfrac{\tan x}{\sec x + 1} = 2\csc x$

e) $\sec^4\theta - 1 = 2\tan^2\theta + \tan^4\theta$

f) $$\frac{\sin^2\varphi\sec^2\varphi+\cos^2\varphi\left(\tan^2\varphi+1\right)-1}{\sec^2\varphi}=\sin^2\varphi$$

Prove the following identities:

a) $\dfrac{1}{\sin\delta\cos\delta} - \dfrac{\cos\delta}{\sin\delta} - \tan\delta = 0$

b) $\dfrac{\cos\beta}{\sec\beta - 1} - \dfrac{\cos\beta}{\tan^2\beta} = \cot^2\beta$