Sum and Difference of Two Angles

There are formulae that allow us to determine the sine, cosine and tangent of the sum or difference of two angles.

	Sum	Difference
Sine	$\sin(a+b)=\sin a\cos b+\sin b\cos a$	$\sin(a-b)=\sin a\cos b-\sin b\cos a$
Cosine	$\cos(a+b)=\cos a\cos b-\sin a\sin b$	$\cos(a-b)=\cos a\cos b+\sin a\sin b$
Tangent	$\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$	$\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

We can use these rules to determine the exact values of some angles not found on the trig circle.

Examples: Determine the exact values of ...

Find 2 "special" angles that sum or differ to 105°

1. $\sin(105^\circ)$

$= \sin(60^\circ + 45^\circ) = \sin(a+b)$

$= \sin a \cos b + \sin b \cos a$

$= \sin 60^\circ \cdot \cos 45^\circ + \sin 45^\circ \cdot \cos 60^\circ$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

OR $\frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin \frac{23\pi}{12}$

2. $\cos 15° = \cos(45° - 30°) \Rightarrow \cos(a-b)$

$= \cos a \cdot \cos b + \sin a \cdot \sin b$

$= \cos 45° \cdot \cos 30° + \sin 45 \cdot \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

3. $\sin\frac{23\pi}{12}$

split into two angles (sum or diff) that will reduce to "special" angles.

$$\sin\left(\frac{15\pi}{12}+\frac{8}{12}\pi\right)$$

$$\sin\left(\frac{5\pi}{4}+\frac{2}{3}\pi\right) \Rightarrow \sin(a+b)$$

$$\sin\frac{5\pi}{4}\cdot\cos\frac{2\pi}{3}+\sin\frac{2\pi}{3}\cdot\cos\frac{5\pi}{4}$$

$$-\frac{\sqrt{2}}{2}\cdot\frac{-1}{2}+\frac{\sqrt{3}}{2}\cdot-\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{4}+\frac{-\sqrt{6}}{4} \quad \text{OR} \quad \frac{\sqrt{2}-\sqrt{6}}{4}$$

4. $\cos\left(-\frac{\pi}{12}\right)$

$$= \cos\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$$

$$= \cos\left(\pi/4 - \frac{\pi}{3}\right)$$

$$= \cos \pi/4 \cdot \cos \pi/3 + \sin \pi/4 \cdot \sin \pi/3$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$