Arc Length

Recall: $$\frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}} = 2\pi r$$

Replacing with radians: $$\frac{\theta}{2\pi} = \frac{arc}{2\pi r}$$

$$\frac{\cancel{2\pi} r\theta}{\cancel{2\pi}} = arc$$

$$\boxed{\text{Arc Length: } L = \theta r}$$

Theta

Note: θ must be in radians.

Examples:

1. Determine the length of the arc, given $r = 12cm$ and $\theta = \frac{2\pi}{3} rad$.

$$L = \theta r$$

$$L = \frac{2\pi}{3} \times 12$$

$$L = \frac{24\pi}{3}$$

$L = 8\pi cm$ or $25.13cm$

2. Determine the diameter of the circle if

$\theta = \frac{5\pi}{6} rad$ and $L = 18cm$.

$$L = \theta r \quad \overset{\text{or}}{\Rightarrow} \quad r = L/\theta$$

$$18 = \frac{5\pi}{6} r$$

$$108 = 5\pi r$$

6.875 or $\frac{108}{5\pi} cm = r$

$\frac{108}{5\pi} \times \frac{2}{1} = \frac{216}{5\pi}$

$\times 2$

$$\therefore d = \frac{216}{5\pi} cm$$

3. Determine the measure of the central angle, if $L = 36cm$ and $r = 18cm$.

$\theta = L/r$ $\leftarrow$ $L = \theta r$

$36 = 18\theta$

$2rad = \theta$

4. Determine the length of the arc (using $L = \theta r$) if $r = 5m$ and $\theta = 400°$.

convert to rad

$$\Rightarrow \frac{400}{180} = \frac{\theta}{\pi}$$

$$\frac{400\pi}{180} = \theta$$

$$\frac{20\pi}{9} = \theta$$

$$\boxed{\frac{n°}{360°} = \frac{x\,\text{rad}}{2\pi} \text{ or } \frac{n°}{180°} = \frac{x\,\text{rad}}{\pi}}$$

$$L = \theta r$$

$$L = \frac{20\pi}{9} \times 5$$

$$L = \frac{100\pi}{9} m \quad \text{or} \quad 34.91m$$

Work Book: Pages 194 - 196

Questions 1, 2, 3, 9, 10 & 11

Pages 202 & 207

Questions 10 c, d, 23 & 24

P 196

10. a) $\underbrace{2h\ 15m}_{\Downarrow}$ Arc Length = ? $r = 0.8cm$

$\theta = 9\pi/2$ 4.5π $L = 4.5\pi \cdot 0.8$

$\boxed{L = 11.31cm}$

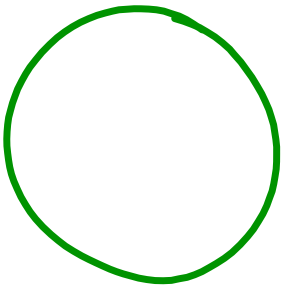
b) 1day

$\theta = 48\pi rad \Rightarrow L = 48\pi \cdot 0.8 = 120.64cm$

c) 1 yr.

$120.64 \times 365 = 44032.56$ cm

11.



speed = $\frac{150 \text{ turns}}{\text{minute}}$

$\frac{150 \cdot 2\pi}{60 \text{ sec}}$

$\frac{300\pi \text{ rad}}{60 \text{ sec}}$

a) $\frac{\text{rad}}{\text{sec}} =$

$\boxed{\frac{5\pi \text{ rad}}{\text{sec}}}$

b) $r = 40\text{cm}$

$\boxed{30 \text{ min}}$

$30 \times 60 = 1800 \text{ sec}$

OR

$30 \times 150 = 4500 \text{ turns}$

① $\theta = 1800 \cdot 5\pi \text{ rad}$

$= 9000\pi$

$\Rightarrow \theta = 4500 \cdot 2\pi \text{ rad}$

$L = \theta \cdot r$

$L = 9000\pi \cdot 40\text{cm} = 360\,000\pi \text{ cm}$

or

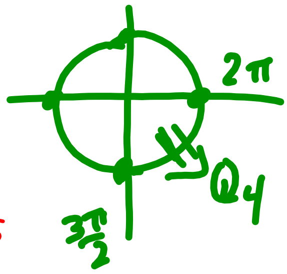
$1\,130\,973.36 \text{ m}$

$3600\pi \text{ m}$

$3.6\pi \text{ km}$

If $\cos\theta = \frac{4}{5}$, where $\boxed{\frac{3\pi}{2} \le \theta \le 2\pi}$, find the exact values of the other 5 ratios.

$Q_4 \Rightarrow$ $\sin\theta = (-)$
$\tan\theta = (-)$



$$x^2 + y^2 = 1$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$\left(\frac{4}{5}\right)^2 + y^2 = 1$$
$$\frac{16}{25} + y^2 = 1$$
$$y^2 = \frac{9}{25}$$
$$y = \pm\frac{3}{5}$$
$$\therefore y = \sin\theta = \frac{-3}{5}$$

$\boxed{\cos\theta = \frac{4}{5}}$ $\sec\theta = \frac{5}{4}$

$\sin\theta = -\frac{3}{5}$ $\csc\theta = -\frac{5}{3}$

$\tan\theta = -\frac{3}{4}$ $\cot\theta = -4/3$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{3}{5}}{\frac{4}{5}} = \frac{-3}{5}\cdot\frac{5}{4} = -\frac{3}{4}$$

If $\tan\theta = -\frac{8}{15}$, where $\frac{\pi}{2} \le \theta \le \pi$, find the exact values of the other 5 ratios.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{c}$

$a = 8 \qquad c = 15$

$\therefore \cos\theta = -\frac{15}{b}$

$\sin\theta = \frac{8}{b}$

$\left(-\frac{15}{b}\right)^2 + \left(\frac{8}{b}\right)^2 = 1$

$\frac{225}{b^2} + \frac{64}{b^2} = 1$

$\frac{289}{b^2} = 1$

$\therefore b^2 = 289$

$b = 17$

$\therefore \sin\theta = \frac{8}{17} \qquad \cos\theta = \frac{-15}{17}$

$\csc\theta = \frac{17}{8} \qquad \sec\theta = -\frac{17}{15}$

$\cot = -\frac{15}{8}$