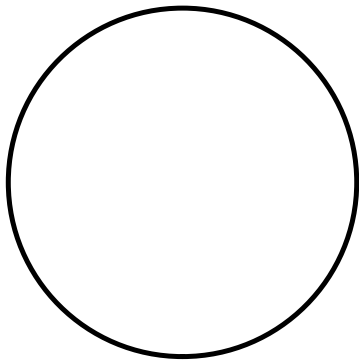


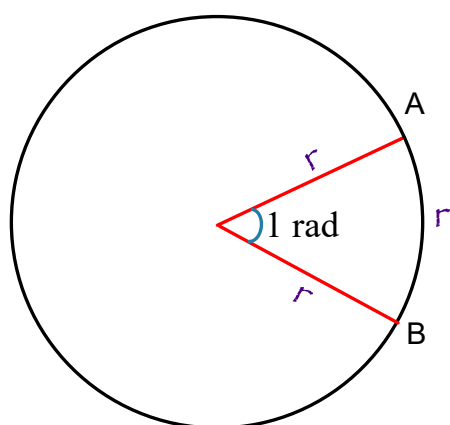
Radians

The radian (rad) is a unit of angle measure.



When a wheel completes a turn...

- 1) how many degrees has it moved?
- 2) how far has the rim of the wheel travelled?
- 3) How many times is the radius contained in the circumference?



When the distance covered on the circumference is equal to the length of the radius (i.e., $\widehat{AB} = r$), the measure of the central angle is equal to 1 radian.

- There are 2π radians in a circle.
- $360^\circ = 2\pi$ radians

$$180^\circ = \underline{\hspace{2cm}} \text{ rad}$$

$$90^\circ = \underline{\hspace{2cm}} \text{ rad}$$

$$135^\circ = \underline{\hspace{2cm}} \text{ rad}$$

We can convert between radians and degrees using the proportion

$$\frac{n^\circ}{360^\circ} = \frac{\theta \text{ rad}}{2\pi} \text{ or}$$

$$\frac{n^\circ}{180^\circ} = \frac{\theta \text{ rad}}{\pi}$$

Example: What is

- a) 50° in radians?
- b) 2.3 rad in degrees?
- c) 12π rad in degrees?
- d) 120° in radians?

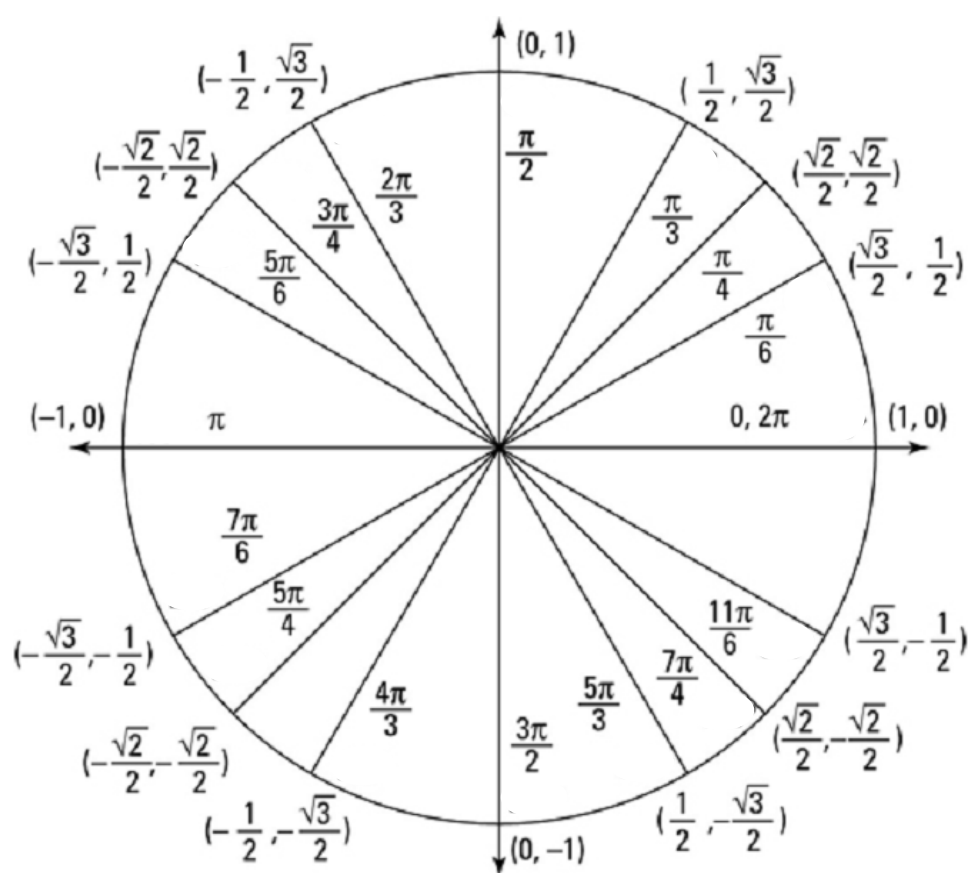
Determine the value of the "remarkable angles" in radians.

$$30^\circ = \underline{\hspace{2cm}} \text{ rad}$$

$$45^\circ = \underline{\hspace{2cm}} \text{ rad}$$

$$60^\circ = \underline{\hspace{2cm}} \text{ rad}$$

Trigonometric Circle (in Radians)



Example: Determine the exact values of the coordinates of the following points.

Recall: Coterminal angles

a) $P\left(\frac{26\pi}{3}\right)$

b) $P\left(\frac{47\pi}{6}\right)$

c) $P\left(-\frac{43\pi}{4}\right)$

Example: Determine the values of the coordinates of the following points.

Recall: $P(\theta) = (\cos \theta, \sin \theta)$

$$P\left(\frac{2\pi}{5}\right)$$

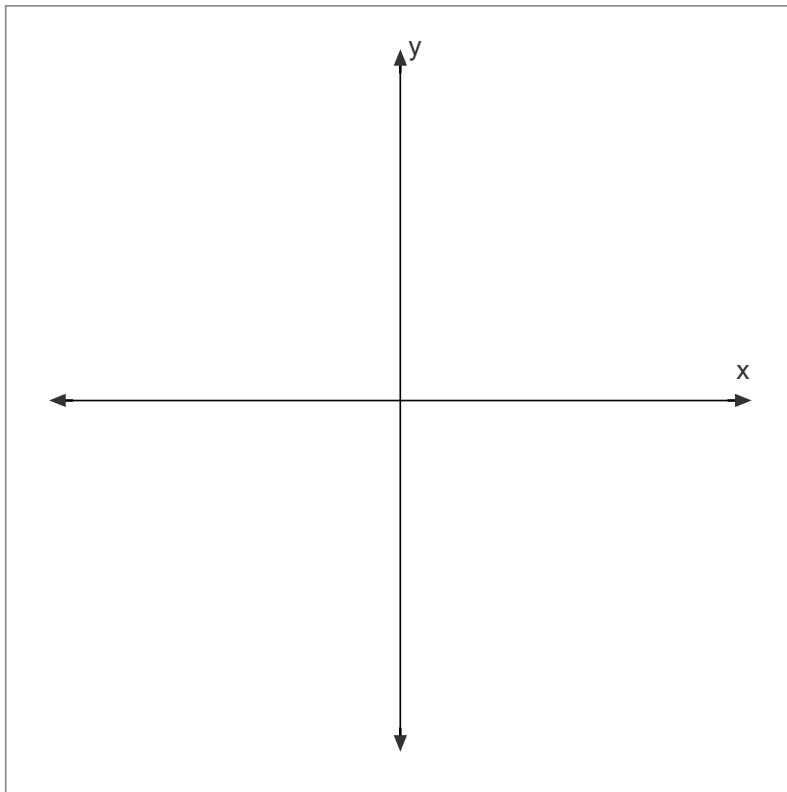
$$P\left(\frac{7\pi}{9}\right)$$

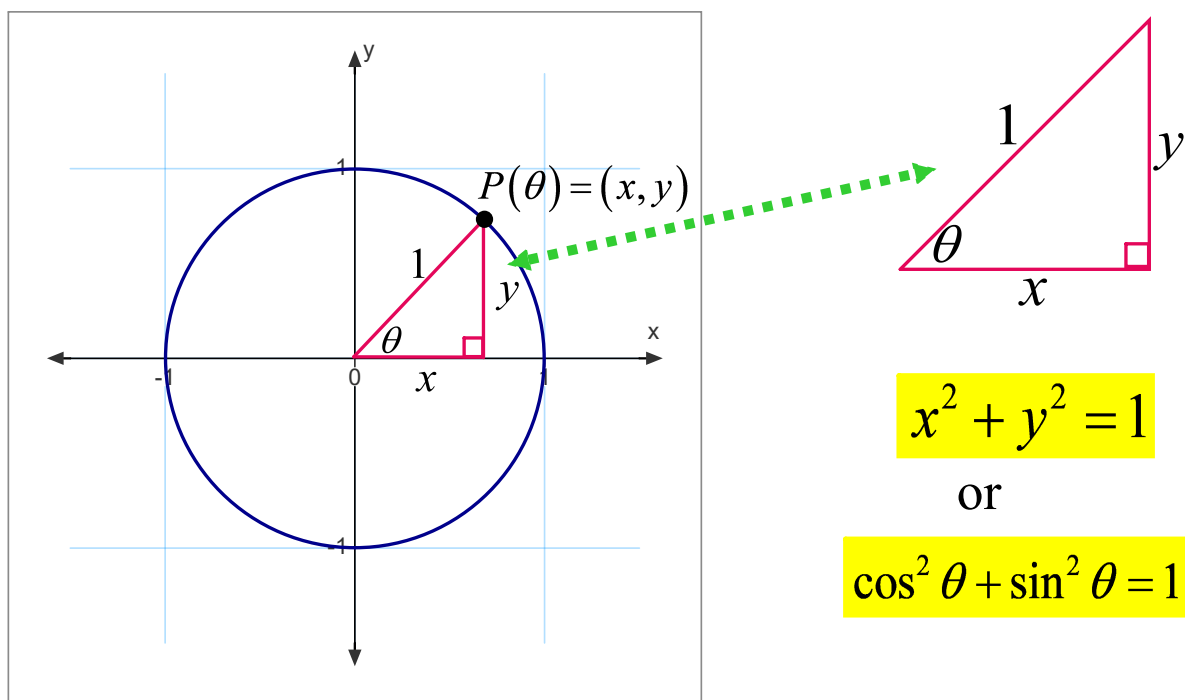
Determine the exact value of...

$$\tan\left(\frac{15\pi}{4}\right)$$

$$\csc\left(\frac{31\pi}{6}\right)$$

Determine the sign of $\sin \theta$, $\cos \theta$ and $\tan \theta$ in each of the four quadrants.





Determine the missing value (exact) of a trigonometric point in quadrant 2, if $P(\theta) = \left(x, \frac{5}{13}\right)$.

If $\sin \theta = \frac{5}{13}$, where $\frac{\pi}{2} \leq \theta \leq \pi$, find the exact values of the other 5 ratios.

If $\cos \theta = \frac{4}{5}$, where $\frac{3\pi}{2} \leq \theta \leq 2\pi$, find the exact values of the other 5 ratios.

If $\tan \theta = -\frac{8}{15}$, where $\frac{\pi}{2} \leq \theta \leq \pi$, find the exact values of the other 5 ratios.

Arc Length

Recall: $\frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}$

Replacing with radians: $\frac{\theta}{2\pi} = \frac{\text{arc}}{2\pi r}$

$$\frac{2\pi r\theta}{2\pi} = \text{arc}$$

Arc Length: $L = \theta r$

Note: θ must be in radians.

Examples:

1. Determine the length of the arc, given
 $r = 12\text{cm}$ and $\theta = \frac{2\pi}{3}\text{rad}$.

$$L = \theta r$$

$$L = \frac{2\pi}{3} \times 12$$

$$L = \frac{24\pi}{3}$$

$$L = 8\pi\text{cm} \text{ or } 25.13\text{cm}$$

2. Determine the diameter of the circle if

$$\theta = \frac{5\pi}{6} \text{ rad and } L = 18 \text{ cm.}$$

$$L = \theta r$$

$$18 = \frac{5\pi}{6} r$$

$$108 = 5\pi r$$

$$\frac{108}{5\pi} \text{ cm} = r$$

$$\therefore d = \frac{216}{5\pi} \text{ cm}$$

3. Determine the measure of the central angle, if $L = 36\text{cm}$ and $r = 18\text{cm}$.

$$L = \theta r$$

$$36 = 18\theta$$

$$2\text{rad} = \theta$$

4. Determine the length of the arc (using $L = \theta r$)
if $r = 5m$ and $\theta = 400^\circ$.

$$\frac{400}{180} = \frac{\theta}{\pi}$$

$$\frac{400\pi}{180} = \theta$$

$$\frac{20\pi}{9} = \theta$$

$$L = \theta r$$

$$L = \frac{20\pi}{9} \times 5$$

$$L = \frac{100\pi}{9} m \text{ or } 34.91m$$