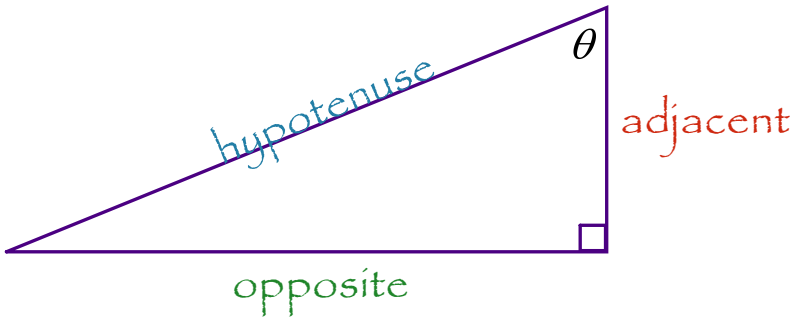
Trigonometry

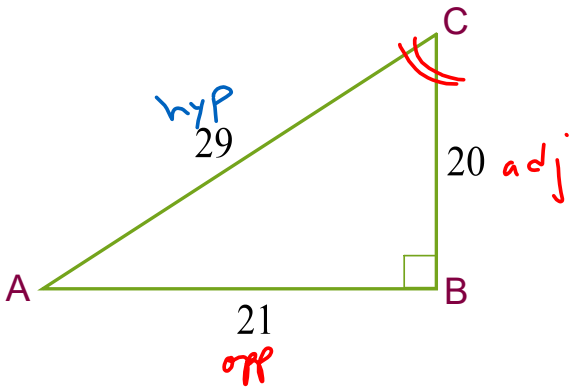
Recall:



$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

Given the following triangle,



determine ...

$\sin A = \frac{20}{29}$

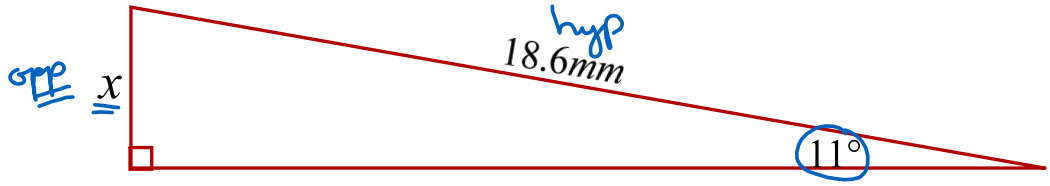
$\cos A = \frac{21}{29}$

$\tan A = \frac{20}{21}$

$\cos C = \frac{20}{29}$

$\tan C = \frac{21}{20}$

Finding a Missing Side

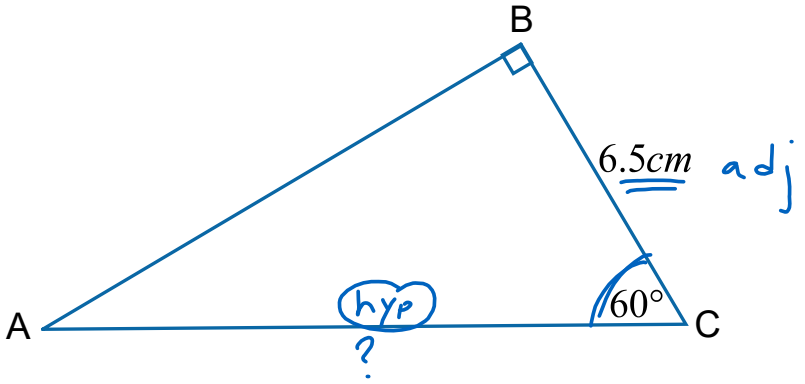


$\sin 11° = \frac{x}{18.6}$

$x = 18.6 \cdot \sin 11°$

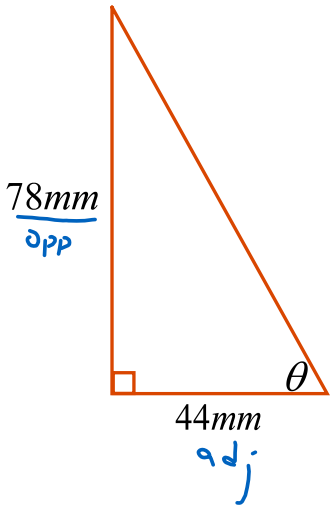
$x = 3.55\,mm$

$m\overline{AC} = ?$



$\cos 60° = \frac{6.5}{y} = \frac{6.5}{0.5} = 13\,cm$

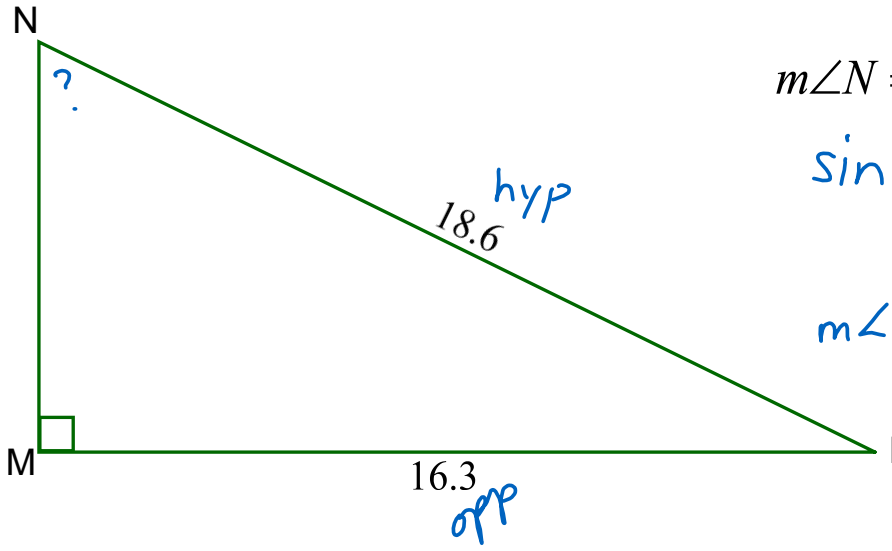
Finding a Missing Angle



$\tan\theta = \frac{78}{44}$

$\angle\theta = \tan^{-1}\left(\frac{78}{44}\right)$

$\angle\theta = 60.57°$

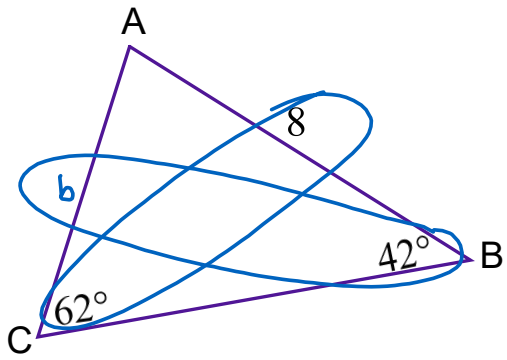


$m\angle N = ?$

$\sin N = \frac{16.3}{18.6}$

$m\angle N = \sin^{-1}\left(\frac{16.3}{18.6}\right)$

$\approx 61.2^\circ$



$b = 6.06$ u

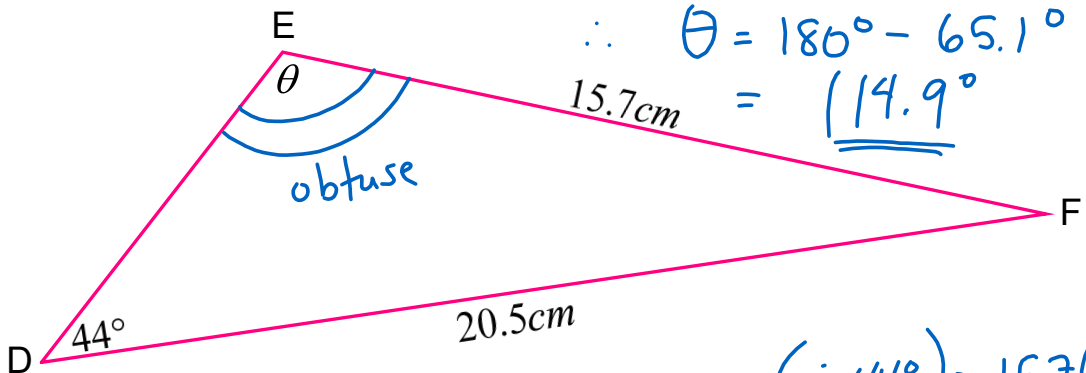
Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{8}{\sin 62^\circ} = \frac{b}{\sin 42^\circ}$$

$$8 \cdot \sin 42^\circ = b \sin 62^\circ$$

$$\frac{8 \cdot \sin 42^\circ}{\sin 62^\circ} = b$$



$\therefore \quad \theta = 180° - 65.1°$

$= 114.9°$

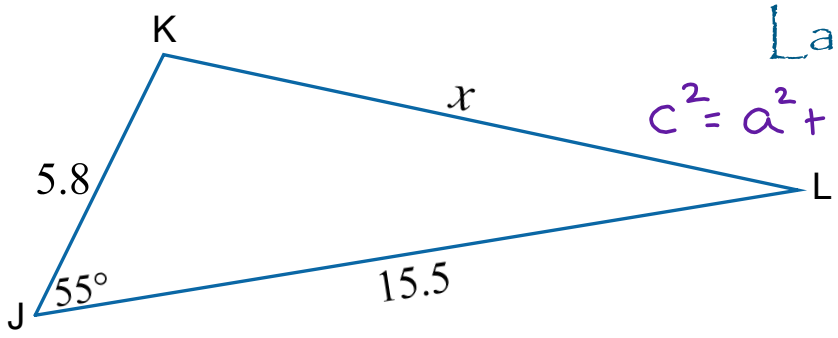
$\frac{15.7}{\sin 44°} = \frac{20.5}{\sin\theta} \Rightarrow 20.5(\sin 44°) = 15.7(\sin\theta)$

$\frac{20.5(\sin 44°)}{15.7} = \sin\theta$

$0.907038 = \sin\theta$

$\sin^{-1}(0.907038) = m\angle\theta$

$65.1° = m\angle\theta$



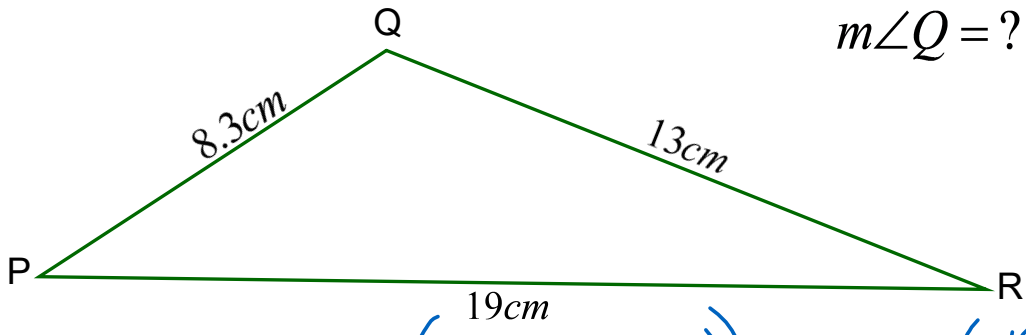
Law of Cosines

$c^2 = a^2 + b^2 - 2ab(\cos C)$

$x^2 = 5.8^2 + 15.5^2 - 2(5.8)(15.5)\cos 55°$

$x^2 = 170.761$

$x = \sqrt{170.761} = 13.07$ m



$m\angle Q = \cos^{-1}\left(\frac{(19^2 - 8.3^2 - 13^2)}{(-2(8.3)(13))}\right) = \cos^{-1}\left(\frac{123.11}{-215.8}\right)$

$= 124,78^\circ$

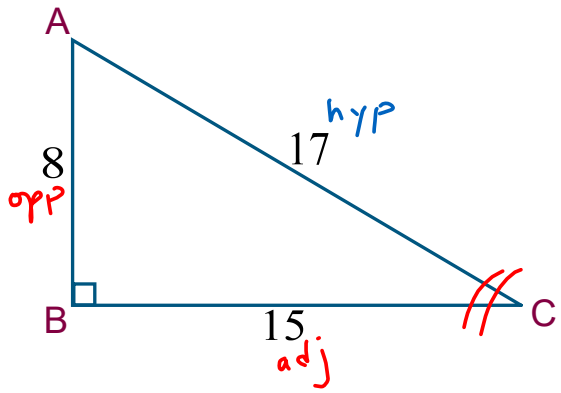
There are 3 other ratios: the reciprocals of sine, cosine and tangent.

Secant: $\sec\theta = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent}$

Cosecant: $\csc\theta = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$
(cosec θ)

Cotangent: $\cot\theta = \frac{1}{\tan\theta} = \frac{adjacent}{opposite}$

Given the following triangle,



determine ...

$\csc A = \frac{hyp}{opp} = \frac{17}{15}$

$\sec A = \frac{hyp}{adj} = \frac{17}{8}$

$\cot C = \frac{adj}{opp} = \frac{15}{8}$

Example: Determine the value of x.

a) $\csc 20° = \frac{x}{2}$ $\Rightarrow$ $x = 2\csc 20°$

$x = 2\left(\frac{1}{\sin 20°}\right)$

$\therefore x = 2/\sin 20° = 5.85$

b) $\cot 50° = \frac{24}{x}$

$\frac{1}{\tan 50°} = \frac{24}{x}$

$x = 24\tan 50°$

$= 28.6$

c) $\sec x = 2$

$\frac{1}{\cos x} = 2$

$2\cos x = 1$

$\cos x = 1/2$

$x = \cos^{-1}(1/2) = 60°$

d) $\csc x = \sqrt{2}$

$\frac{1}{\sin x} = \sqrt{2}$

$\sqrt{2}\sin x = 1$

$\sin x = \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \{45°, 135°\}$

If $\sin\theta = \frac{opp}{hyp}$ and $\cos\theta = \frac{adj}{hyp}$

what is $\frac{\sin\theta}{\cos\theta}$?

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj} = \frac{opp}{adj} = \tan\theta$$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}} \quad \underline{\text{and}} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$