

Trigonometric Circle

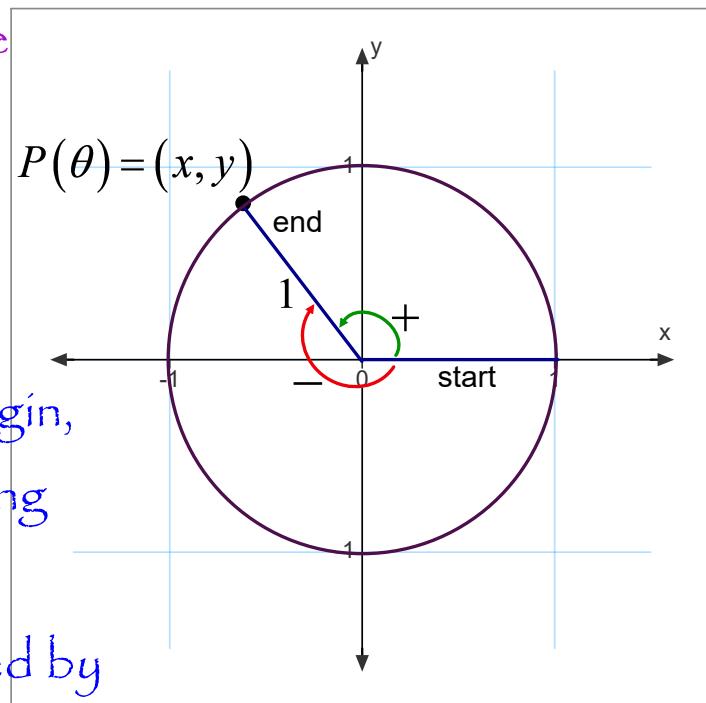
The trigonometric circle is a unit circle centred at the origin.

Any point on the circle is called a trigonometric point.

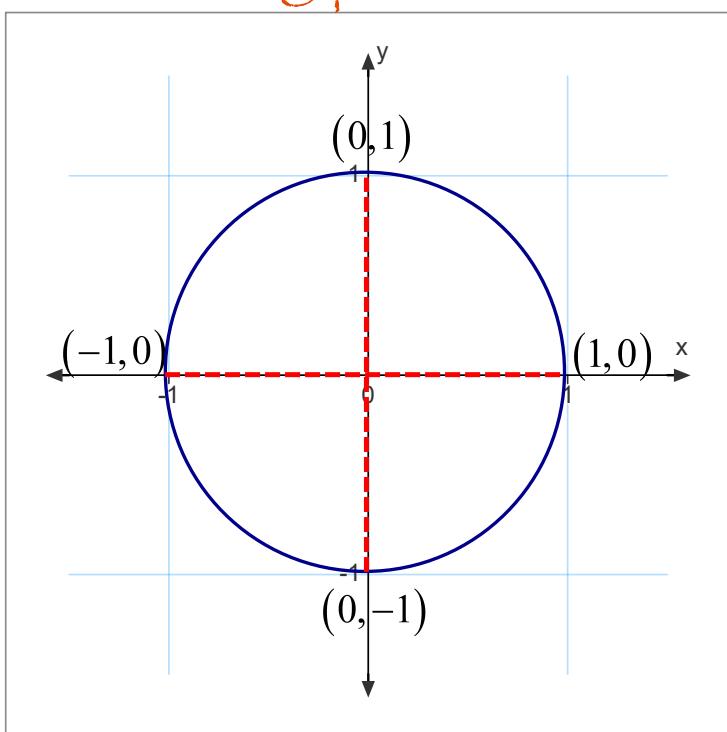
A trigonometric angle has its vertex at the origin, its fixed initial side along the positive x -axis, and its terminal side created by

a rotation from the initial side.

Every angle corresponds with a trigonometric point.



Known trig points:



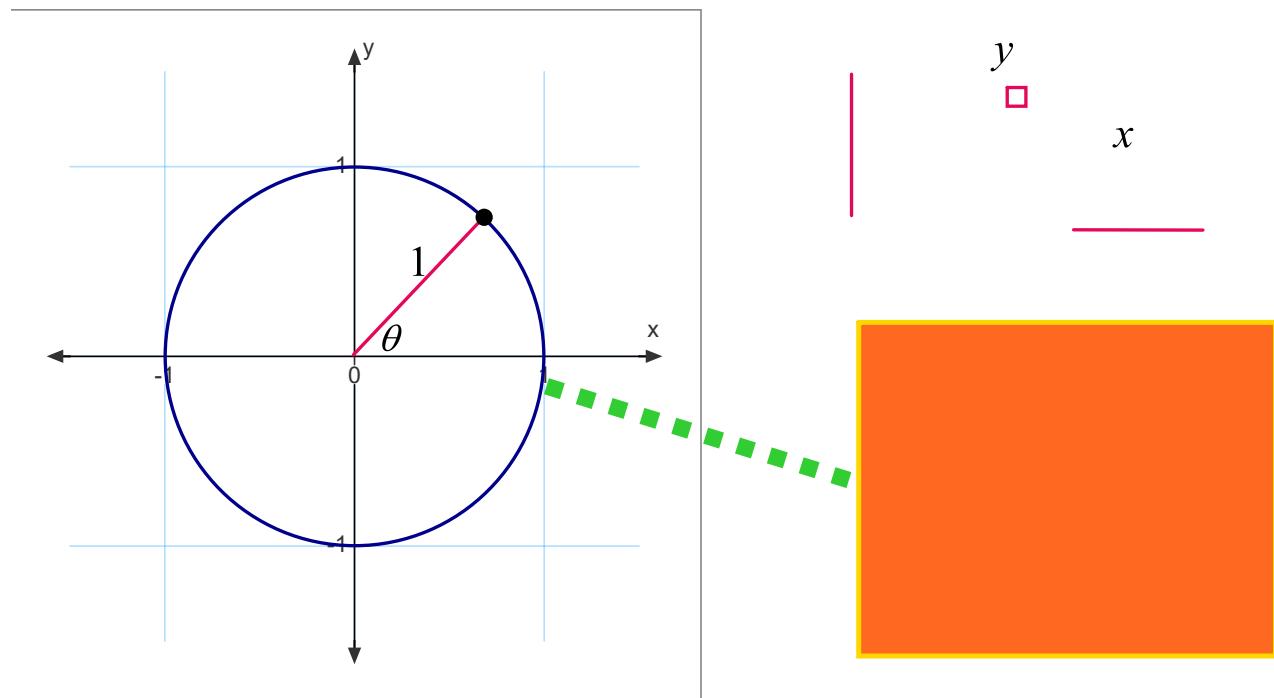
$$P(0^\circ) = (1,0)$$

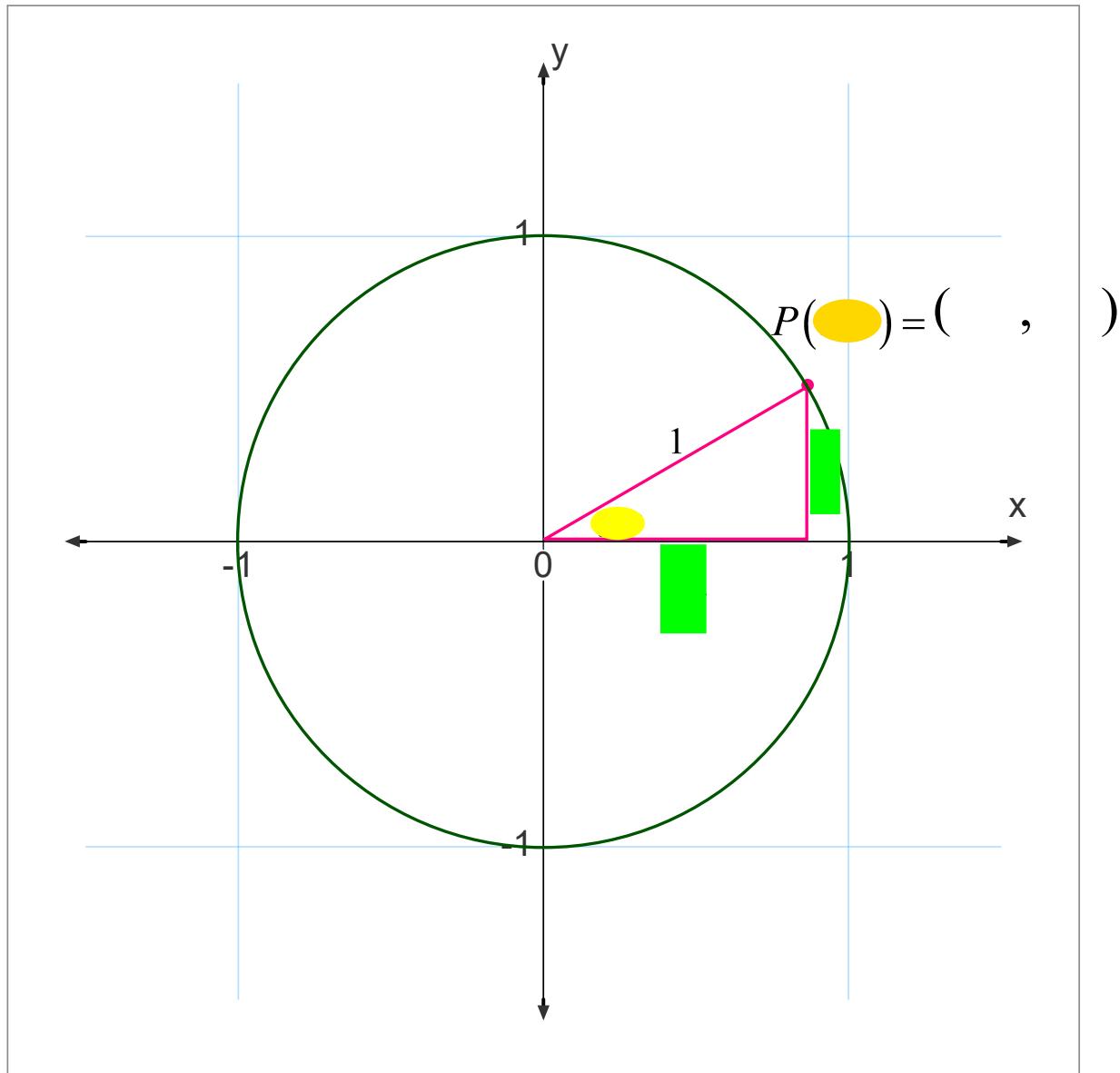
$$P(90^\circ) = (0,1)$$

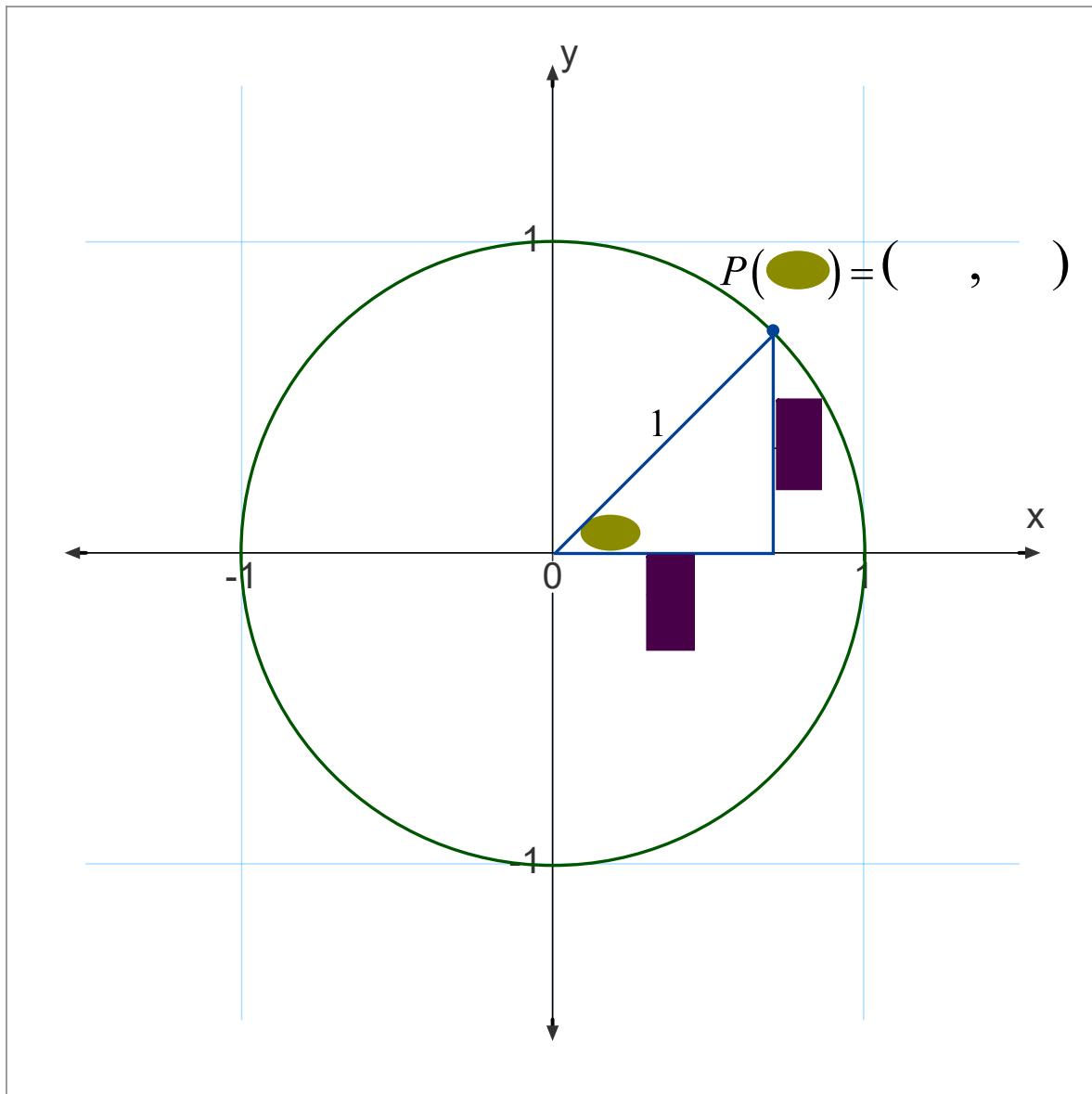
$$P(180^\circ) = (-1,0)$$

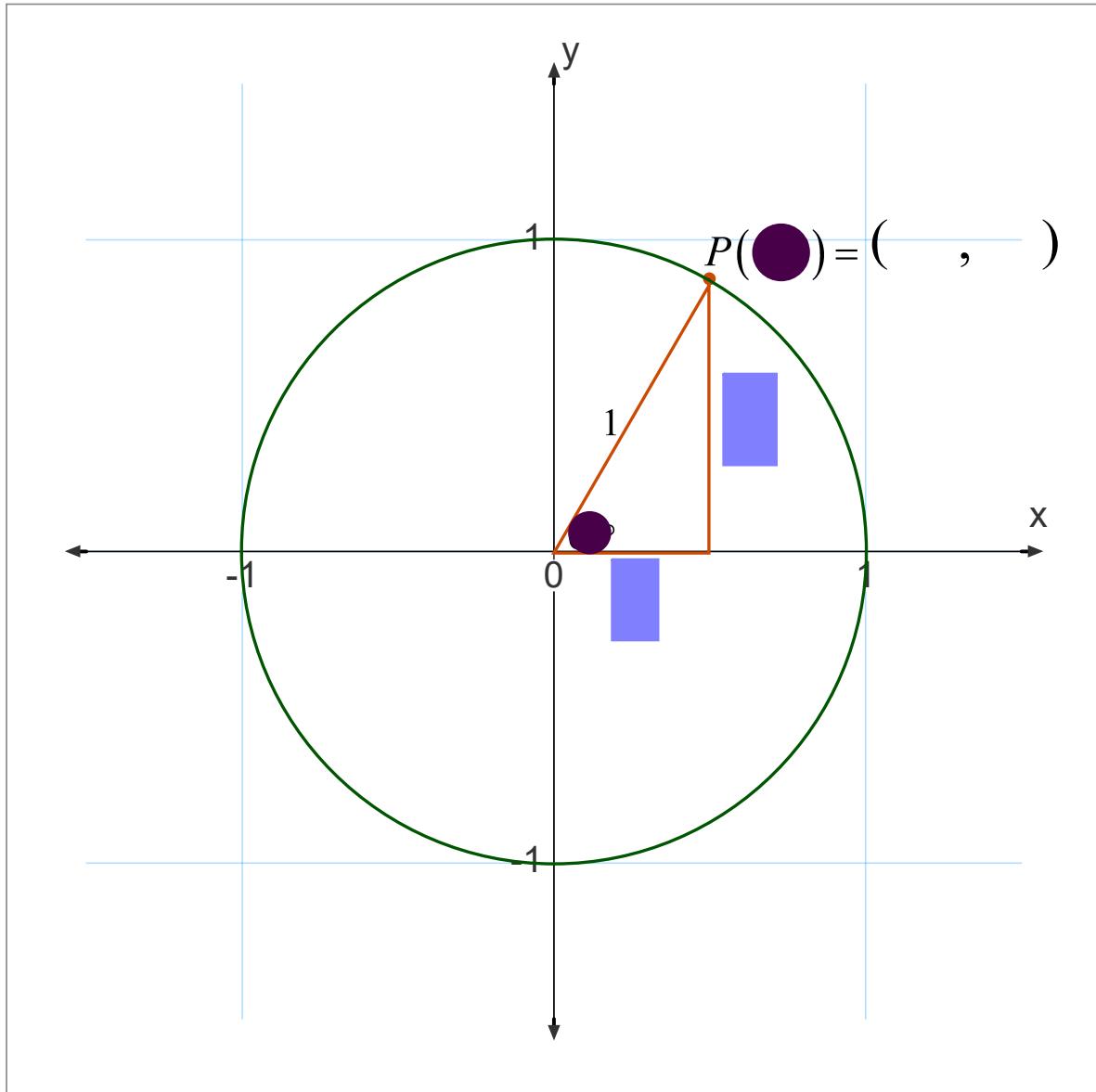
$$P(270^\circ) = (0,-1)$$

$$P(360^\circ) = (1,0)$$









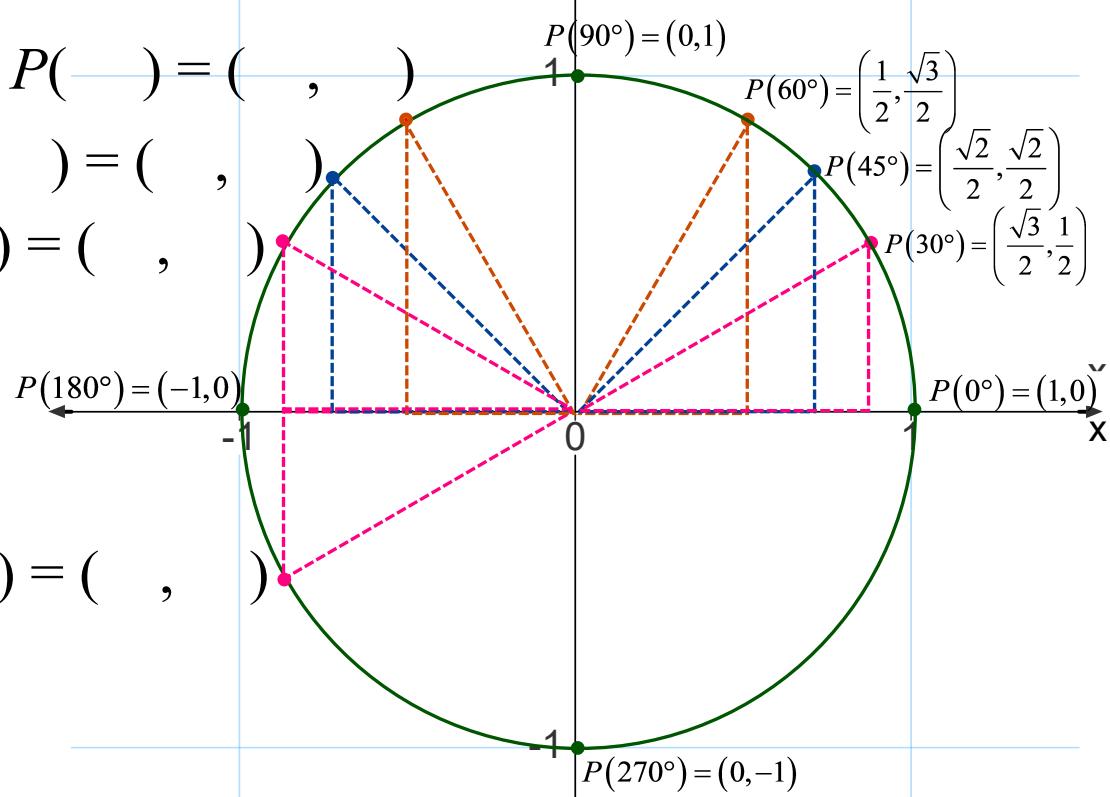
By reflecting these points around the axes, we can determine 12 more points with exact coordinates.

$$P(\quad) = (\quad, \quad)$$

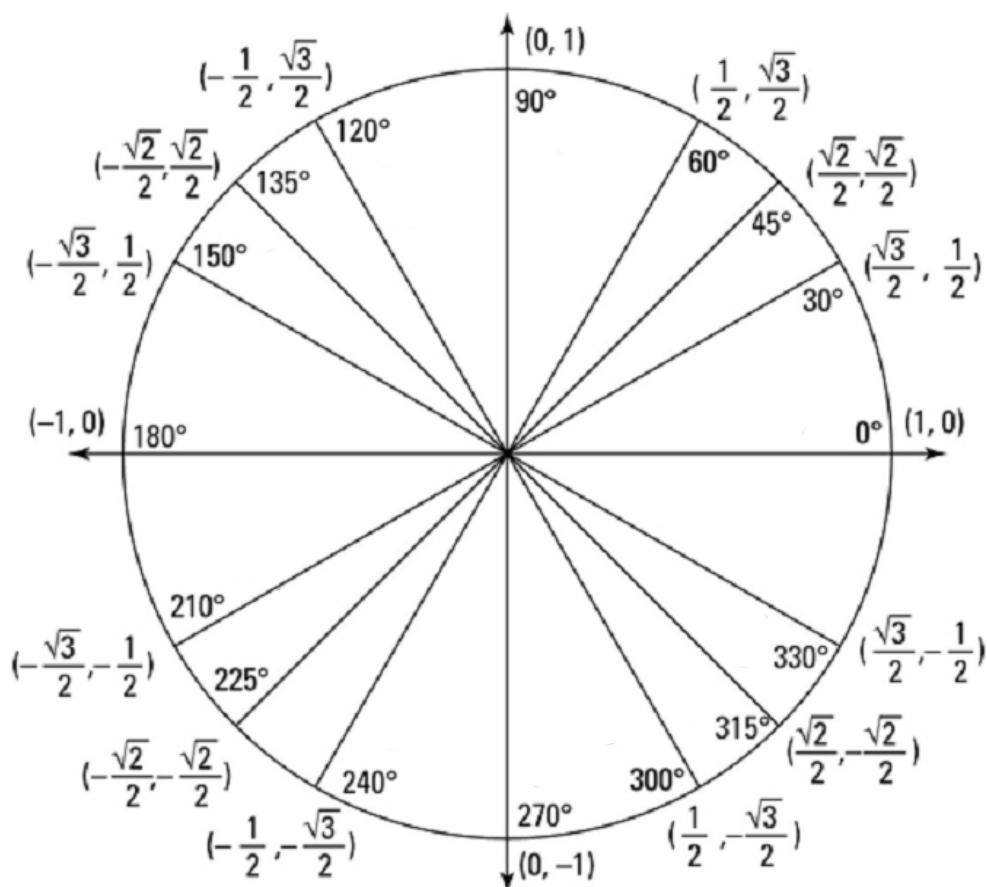
$$P(\quad) = (\quad, \quad)$$

$$P(\quad) = (\quad, \quad)$$

$$P(\quad) = (\quad, \quad)$$



This circle, with all of the exact trigonometric points, is known as the **trigonometric circle**.



Angles that have the same terminal arm , but different rotations, are called coterminal .

They are created by going around the circle more than once, or by going in a clockwise (−) direction.

Examples: 135° and 495°

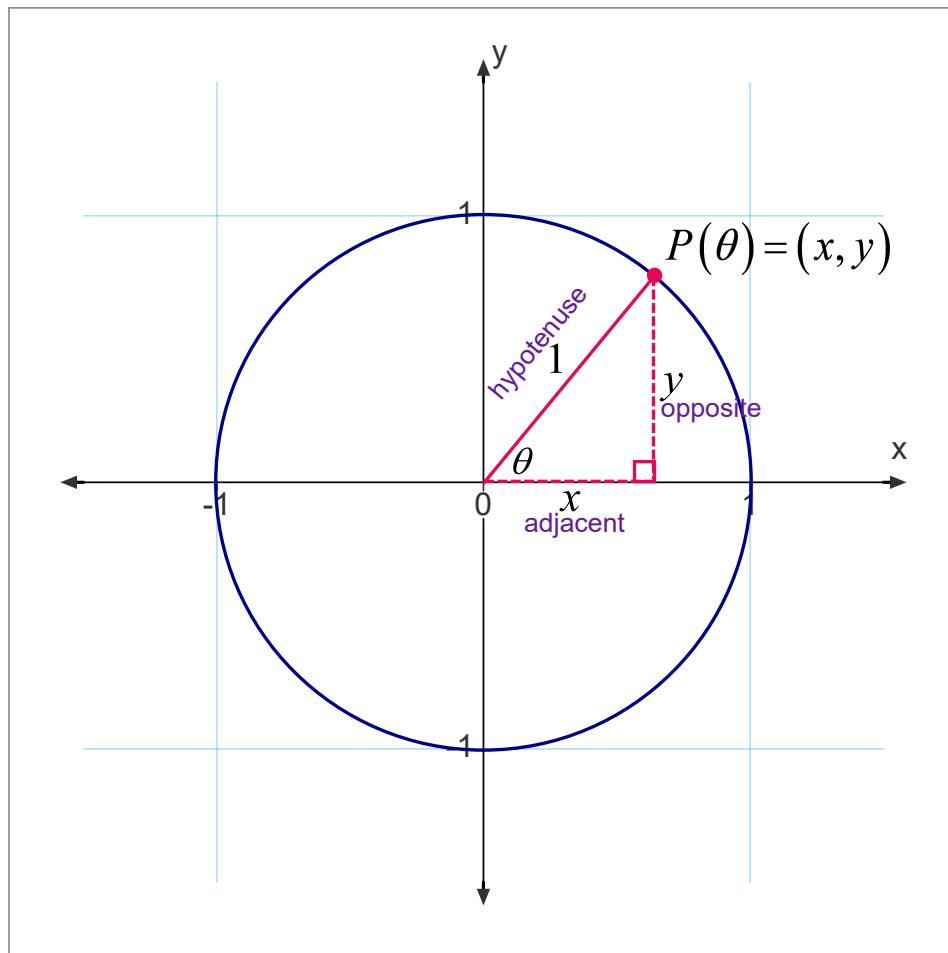
135° and -585°

Example: Determine the exact values of the coordinates of following points.

a) $P(1860^\circ)$

b) $P(1035^\circ)$

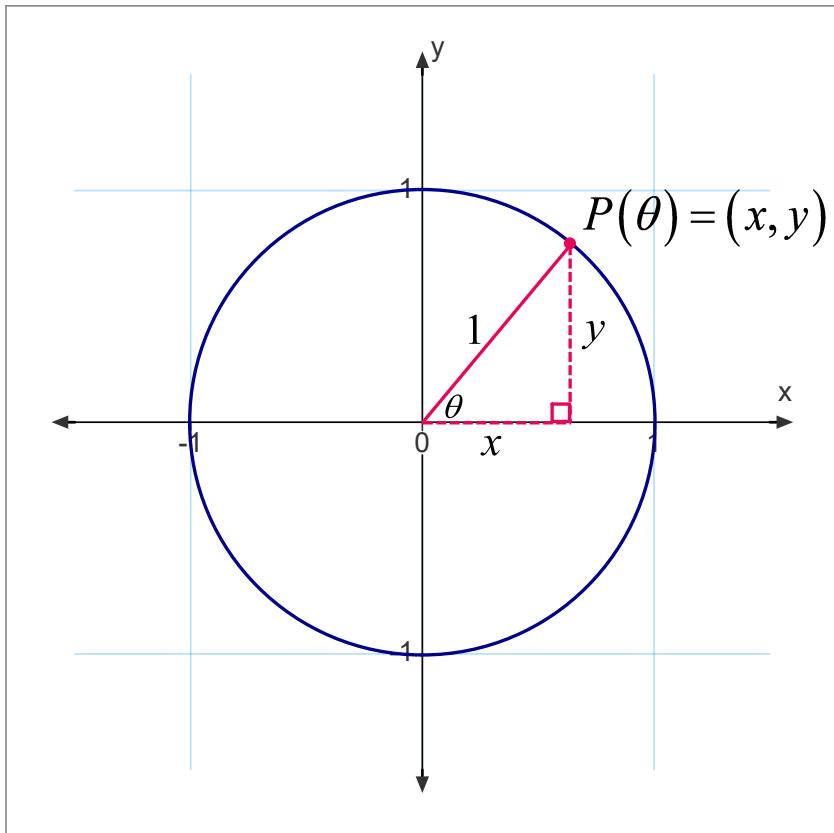
c) $P(-840^\circ)$



$$\cos \theta = ?$$

$$\sin \theta = ?$$

$$\tan \theta = ?$$



$$\cos \theta = \frac{x}{1}$$

$$\therefore x = \cos \theta$$

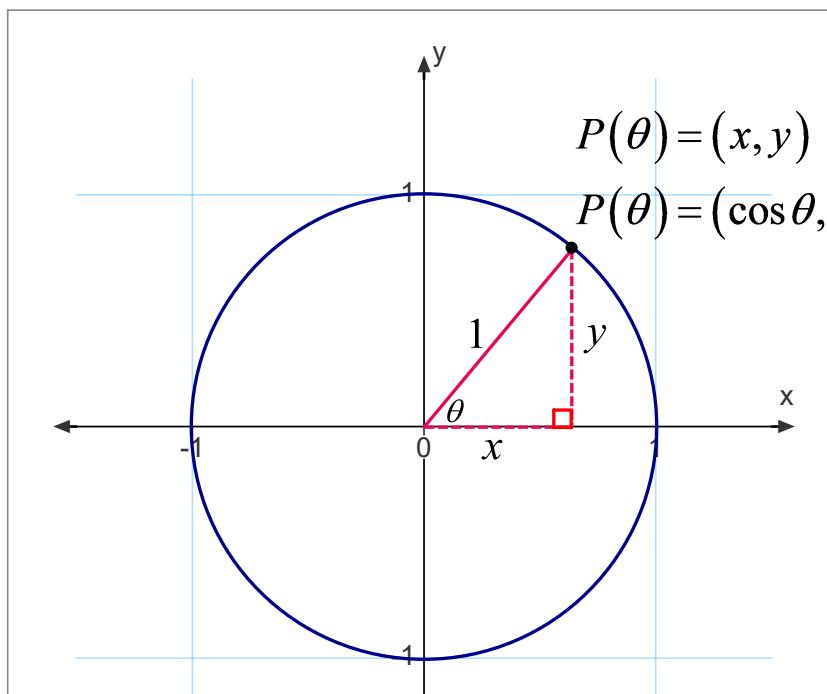
and

$$\sin \theta = \frac{y}{1}$$

$$\therefore y = \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$



$$P(\theta) = (x, y)$$
$$P(\theta) = (\cos \theta, \sin \theta)$$

The trig point for
any angle θ is
 $(\cos \theta, \sin \theta)$.

If ...

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

then ...

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Determine the exact values of the following.

a) $\tan 120^\circ$

b) $\sec 300^\circ$

c) $\csc 225^\circ$

d) $\cot 60^\circ$

If ... $\tan \theta = \frac{\sin \theta}{\cos \theta}$, then ... $\cot \theta = \frac{\cos \theta}{\sin \theta}$

The trig circle provides exact values of the sine & cosine (and, by calculation, the tangent) of the remarkable angles.

We can also determine the coordinates of a point corresponding to any angle.

What are the coordinates of the trig point that corresponds to 70° ?

$$P(70^\circ) = (\cos 70^\circ, \sin 70^\circ)$$

Determine the coordinates of the following trig points.

a) $P(220^\circ)$

b) $P(105^\circ)$

c) $P(330^\circ)$