Rationalising the Denominator

We do not write expressions with a radical in the denominator.

Example: $\dfrac{6}{\sqrt{3}}$

We get rid of the radical in the denominator by a process called rationalising.

Multiply by a $\underbrace{\text{unit fraction}}$ $=1$ that will square the denominator.

$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Rationalise the denominators:

1. $\dfrac{4}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{4\sqrt{13}}{13}$

2. $\dfrac{5\sqrt{3}}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{5\sqrt{6}}{3 \cdot 2} = \dfrac{5\sqrt{6}}{6}$

Example: $\dfrac{5}{\sqrt{3}+\sqrt{6}} \cdot \dfrac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$

When more than 1 term is in the denominator, we must use conjugates to create a difference of squares.

$$\frac{5\cdot(\sqrt{3}-\sqrt{6})}{(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})} = \frac{5\sqrt{3}-5\sqrt{6}}{(\sqrt{3})^2-(\sqrt{6})^2} = \frac{5\sqrt{3}-5\sqrt{6}}{3-6}$$

$$= \frac{5\sqrt{3}-5\sqrt{6}}{-3}$$

$\underline{\underline{\text{OR}}}$ $\dfrac{5\sqrt{6}-5\sqrt{3}}{3}$

Example: $$\frac{\left(2\sqrt{5}+3\sqrt{2}\right)}{2\sqrt{2}-\sqrt{3}} \cdot \frac{\left(2\sqrt{2}+\sqrt{3}\right)}{2\sqrt{2}+\sqrt{3}}$$

$$= \frac{4\sqrt{10} + 2\sqrt{15} + \overset{6 \cdot 2}{6\sqrt{4}} + 3\sqrt{6}}{\underset{4 \cdot 2}{4\sqrt{4}} - \sqrt{9}}$$

$$= \frac{4\sqrt{10} + 2\sqrt{15} + 3\sqrt{6} + 12}{8 - 3}$$

$$= \frac{4\sqrt{10} + 2\sqrt{15} + 3\sqrt{6} + 12}{5}$$

Rationalise the denominators:

1. $\dfrac{3}{2\sqrt{5}+\sqrt{6}} \cdot \dfrac{2\sqrt{5}-\sqrt{6}}{2\sqrt{5}-\sqrt{6}} = \dfrac{6\sqrt{5}-3\sqrt{6}}{20-6} = \dfrac{6\sqrt{5}-3\sqrt{6}}{14}$

2. $\dfrac{4\sqrt{10}+2\sqrt{3}}{3\sqrt{2}-\sqrt{15}} \cdot \dfrac{3\sqrt{2}+\sqrt{15}}{3\sqrt{2}+\sqrt{15}} = \dfrac{12\sqrt{\overset{4\cdot 5}{20}} + 4\sqrt{\overset{25\cdot 6}{150}} + 6\sqrt{6} + 2\sqrt{\overset{9\cdot 5}{45}}}{18-15}$

$= \dfrac{12(2\sqrt{5}) + 4(5\sqrt{6}) + 6\sqrt{6} + 2(3\sqrt{5})}{3}$

$= \dfrac{30\sqrt{5} + 26\sqrt{6}}{3}$