Simplifying Radicals

Many radicals can be simplified.

Example: $\sqrt{27}$

$\sqrt{m} \cdot \sqrt{n} = \sqrt{m \cdot n}$

1. Break down the radicand into two factors - one of which is a perfect square.

$\sqrt{27} = \sqrt{9 \cdot 3}$

2. Apply the multiplication property of radicals to create a coefficient times a radical.

$\sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

Example: $3\sqrt{72}$ $= 3\sqrt{9 \cdot 8}$

$= 3\sqrt{9} \cdot \sqrt{8}$

$= 3 \cdot 3 \cdot \sqrt{8}$

$= 9\sqrt{8}$

$= 9\sqrt{4 \cdot 2}$

$= 9\sqrt{4} \cdot \sqrt{2}$

$= 9 \cdot 2 \cdot \sqrt{2}$

$= 18\sqrt{2}$

OR/ $3\sqrt{36 \cdot 2}$

$= 3\sqrt{36} \cdot \sqrt{2}$

$= 3 \cdot 6 \cdot \sqrt{2}$

$= \underline{\underline{18\sqrt{2}}}$

Example: $4\sqrt{48}+2\sqrt{12}$

$4\sqrt{16\cdot 3}+2\sqrt{4\cdot 3}$

$4\cdot\sqrt{16}\cdot\sqrt{3}+2\sqrt{4}\cdot\sqrt{3}$

$4\cdot 4\cdot\sqrt{3}+2\cdot 2\cdot\sqrt{3}$

$16\sqrt{3}+4\sqrt{3}$

$20\sqrt{3}$

$4\sqrt{4\cdot 12}+2\sqrt{12}$

$8\sqrt{12}+2\sqrt{12}$

$10\sqrt{12}$

$10\sqrt{4\cdot 3}$

$10\cdot 2\sqrt{3}$

$20\sqrt{3}$

Try these!

1. $3\sqrt{5} \times \left(-2\sqrt{3}\right)$ $\sqrt{5 \cdot 3}$ $= -6\sqrt{15}$

2. $2\sqrt{3}\left(5\sqrt{3} + \sqrt{5}\right)$ $10 \cdot 3$

$10 \cdot \sqrt{9} + 2\sqrt{15} = 30 + 2\sqrt{15}$

3. $\left(4\sqrt{5} + 3\right)\left(4\sqrt{5} - 3\right)$ $= 16\sqrt{25} - 9$

$= 80 - 9 = 71$

4. $12\sqrt{30} \div 4\sqrt{5}$

$3\sqrt{30 \div 5} = 3\sqrt{6}$

5. $2\sqrt{75} - 2\sqrt{108} + 5\sqrt{75} - \sqrt{108} + 3\sqrt{12}$

$7\sqrt{75} - 3\sqrt{108} + 3\sqrt{12}$

$7\sqrt{25 \cdot 3} - 3\sqrt{36 \cdot 3} + 3\sqrt{4 \cdot 3}$

$7 \cdot 5 \cdot \sqrt{3} - 3 \cdot 6\sqrt{3} + 3 \cdot 2\sqrt{3}$

$35\sqrt{3} - 18\sqrt{3} + 6\sqrt{3} = 23\sqrt{3}$

Simplify

1. $3\sqrt{5} \times -2\sqrt{3} = -6\sqrt{15}$

2. $2\sqrt{3}\left(5\sqrt{3} + \sqrt{5}\right) = 2\sqrt{3}\cdot 5\sqrt{3} + 2\sqrt{3}\cdot\sqrt{5}$
$= 10\sqrt{9} + 2\sqrt{15}$
$= 10\cdot 3 + 2\sqrt{15}$
$= 30 + 2\sqrt{15}$

3. $\left(4\sqrt{5} + 3\right)\left(4\sqrt{5} - 3\right) = 4\sqrt{5}\cdot 4\sqrt{5} + 4\sqrt{5}\cdot(-3) + 3\cdot 4\sqrt{5} + 3\cdot(-3)$
$= 16\sqrt{25} - 12\sqrt{5} + 12\sqrt{5} - 9$
$= 16(5) - 9$
$= 80 - 9$
$= 71$

4. $2\sqrt{75} - 2\sqrt{108} + 5\sqrt{75} - \sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{75} - 3\sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{25\cdot 3} - 3\sqrt{36\cdot 3} + 3\sqrt{4\cdot 3} = 35\sqrt{3} - 18\sqrt{3} + 6\sqrt{3}$
$= 23\sqrt{3}$

5. $12\sqrt{30} \div 4\sqrt{5} = \dfrac{12\sqrt{30}}{4\sqrt{5}}$
$= 3\sqrt{\dfrac{30}{5}}$
$= 3\sqrt{6}$

Rationalising the Denominator

We do not write expressions with a radical in the denominator.

Example: $\dfrac{6}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

We get rid of the radical in the denominator by a process called rationalising.

Multiply by a $\overset{?=1}{\text{unit fraction}}$ that will square the denominator.

$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$$

Rationalise the denominators:

1. $\frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$

2. $\frac{5\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}}{3 \cdot 2} = \frac{5\sqrt{6}}{6}$

Example: $\dfrac{5}{\sqrt{3}+\sqrt{6}}$

When more than 1 term is in the denominator, we must use conjugates to create a difference of squares.

$$\frac{5}{\sqrt{3}+\sqrt{6}} \cdot \left(\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}\right) = \frac{5\sqrt{3}-5\sqrt{6}}{3-6} = \frac{5\sqrt{3}-5\sqrt{6}}{-3}$$

$$\text{OR } \frac{5\sqrt{6}-5\sqrt{3}}{3}$$

Example: $$\left(\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{2}-\sqrt{3}}\right)\cdot\left(\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}\right)$$

$$=\frac{4\sqrt{10}+2\sqrt{15}+6(2)+3\sqrt{6}}{4(2)-3}$$

$$=\frac{4\sqrt{10}+2\sqrt{15}+3\sqrt{6}+12}{5}$$