Practice 6.3

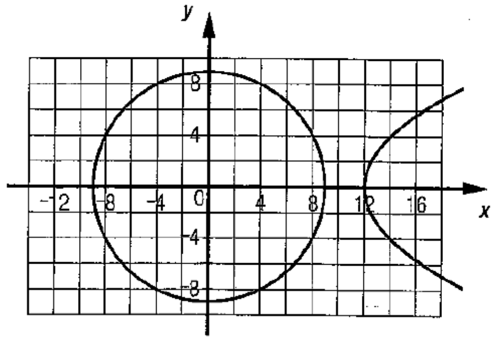
1. a) The equations of the curves are $x = -4$ and $(x - 8)^2 = -24(y - 12)$.
   The intersection points are (-4, 6).
   b) The equations of the curves are $y = 24$ and $x^2 + y^2 = 32^2$.
   The intersection points are ($\approx$ 21.17, 24) and ($\approx$ -21.17, 24).
   c) The equations of the curves are $y = 0.75x + 0.25$ and $\frac{x^2}{16} - \frac{y^2}{4} = -1$.
   The intersection points are (-4.2, -2.9) and (3, 2.5).
   d) The equations of the curves are $y = 2x + 12$ and $\frac{x^2}{144} + \frac{y^2}{256} = 1$.
   The intersection points are ($\approx$ 1.9, $\approx$ 15.8) and ($\approx$ -10.21, $\approx$ -8.41).
   e) The equations of the curves are $y = -0.6x$ and $(y - 8)^2 = 20(x + 35)$.
   The intersection points are (-30, 18) and ($\approx$ 58.89, $\approx$ -35.33).
   f) The equations of the curves are $y = x + 12$ and $\frac{x^2}{144} - \frac{y^2}{256} = 1$.
   The intersection points are (-12, 0) and ($\approx$ 42.86, $\approx$ 54.86).

Practice 6.3 (*cont'd*)

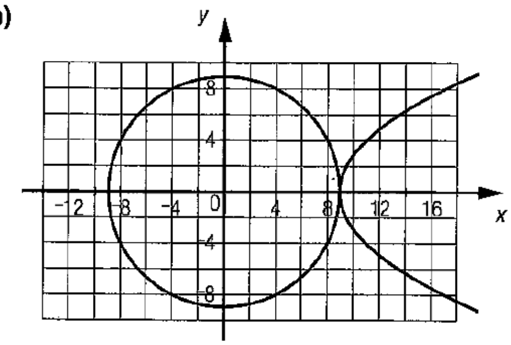
2. a) (15, 8) and (-15, 8).
   b) (-7.8, 4) and (-7.8, -4).
   c) (-7.5, 30), (0, -24) and (7.5, 30).
   d) (3.6, 4.8) and (3.6, -4.8).
   e) ($\approx$ 4.74, $\approx$ -4.9), ($\approx$ -4.74, $\approx$ -4.9), ($\approx$ 4.24, 4) and ($\approx$ -4.24, 4).
   f) (34, 0)
3. a) The equations of the curves are $y^2 = \frac{25}{12}x$ and $x^2 + y^2 = 169$.
   The intersection points are (12, 5) and (12, -5).
   b) The equations of the curves are $x^2 = 2(y + 9)$ and $\frac{x^2}{81} + \frac{y^2}{49} = 1$.
   The intersection points are ($\approx$ 2.09, $\approx$ -6.81), ($\approx$ -2.09, $\approx$ -6.81), ($\approx$ 5.4, $\approx$ 5.6) and ($\approx$ -5.4, $\approx$ 5.6).
   c) The equations of the curves are $y^2 = -40(x + 15)$ and $x^2 + y^2 = 625$.
   The intersection points are ($\approx$ -20.31, $\approx$ 14.58) and ($\approx$ -20.31, $\approx$ -14.58).
   d) The equations of the curves are $y^2 = -12x$ and $\frac{x^2}{4} - \frac{y^2}{16} = 1$.
   The intersection points are (-4, $\approx$ 6.93) and (-4, $\approx$ -6.93).
   e) The equations of the curves are $x^2 = -12(y - 29)$ and $\frac{x^2}{400} + \frac{y^2}{841} = 1$.
   The intersection points are ($\approx$ -19.83, -3.77), (0, 29) and ($\approx$ 19.83, -3.77).
   f) The equations of the curves are $x^2 = 8(y + 10)$ and $\frac{x^2}{225} - \frac{y^2}{64} = -1$.
   The intersection points are ($\approx$ 3.74, $\approx$ -8.25), ($\approx$ -3.74, $\approx$ -8.25), ($\approx$ 12.81, $\approx$ 10.52) and ($\approx$ -12.81, $\approx$ 10.52).

4. **a)** (0.4, -10.1) **b)** (-11, -22.5) and (17, -22.5). **c)** (-24, -10) and (24, -10).
   d) (19.2, 15) and (-19.2, -15). **e)** (40, 30) and (-30, -40). **f)** (-2, -11) and (-20, 7).
   g) ($\approx$ 12.71, $\approx$ 91.69) and ($\approx$ -8.04, $\approx$ -74.32). **h)** (4, -6) and ($\approx$ -4.94, $\approx$ -1.53). **i)** (-36, 0) and (60, 20).
5. **a)** No points. **b)** 2 points. **c)** 3 points. **d)** 1 point. **e)** 4 points. **f)** 2 points.
6. *Several answers possible. Examples:*

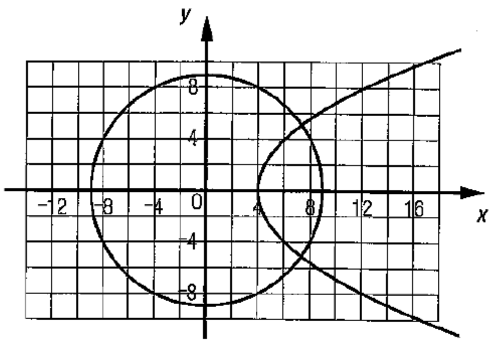
a)



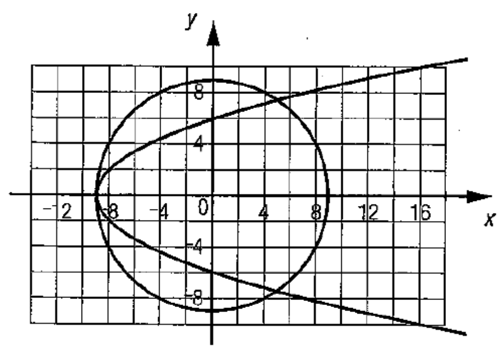
b)



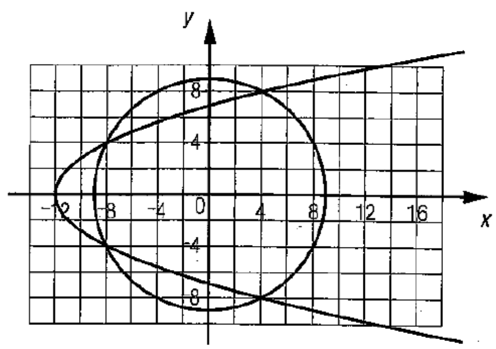
c)



d)



e)



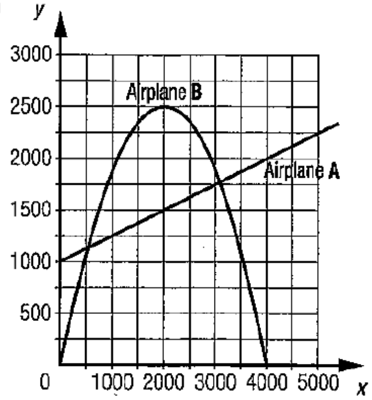
7. **a)** ($\approx$ 11.24, 6) and ($\approx$ -11.24, 6).
   b) ($\approx$ -12.96, $\approx$ 5.18) and (10.2, -6.4).
   c) (0, -8), ($\approx$ 14.12, $\approx$ 4.46) and ($\approx$ -14.12, $\approx$ 4.46).

8. **a) 1)** The line that passes through the centre of the bridge and points (0, 20) and (10, 0); therefore, the equation is $y = -2x + 20$.
   2) The equation of the ellipse associated with the outline of the lake is $\frac{x^2}{900} + \frac{y^2}{1600} = 1$ since the vertices are (30, 0) and (0, 40).

   b) 1) ($\approx$ -9.07, $\approx$ 38.13) **2)** ($\approx$ 22.91, $\approx$ -25.82)

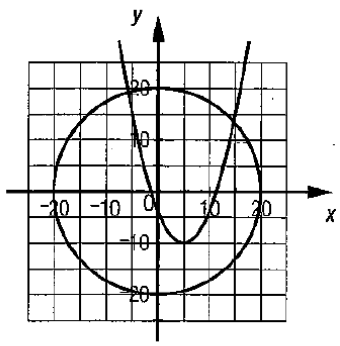
   c) The distance that separates the two piers is $\sqrt{(22.91 - -9.07)^2 + (-25.82 - 38.13)^2} \approx 71.5$ dam.

9. a)



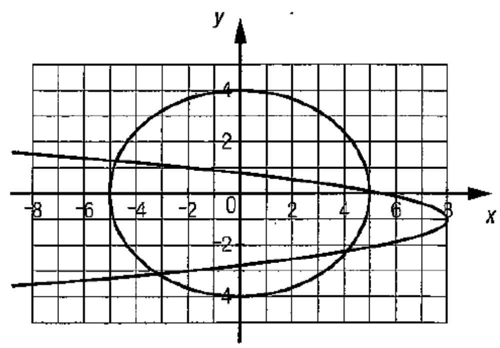
b) Solve the equation: $(x - 2000)^2 = -1600((0.25x + 1000) - 2500)$
$\Rightarrow x_1 \approx 3080.62$ and $x_2 \approx 519.38$.
The corresponding y-values are $y_1 \approx 1770.16$ and $y_2 \approx 1129.84$.
The coordinates of the points when Airplane **B** passes directly over Airplane **A** are ($\approx$ 519.38, $\approx$ 1129.84) and ($\approx$ 3080.62, $\approx$ 1770.16).

10. a)



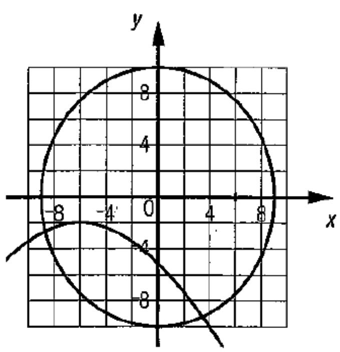
($\approx$ -5.8, $\approx$ 19.1) and ($\approx$ 14.7, $\approx$ 13.6).

b)



($\approx$ -4.7, $\approx$ 1.3), ($\approx$ -3.1, $\approx$ -3.1), ($\approx$ 4.1, $\approx$ -2.2) and ($\approx$ 4.99, $\approx$ 0.1).

c)



($\approx$ -8.69, $\approx$ -2.6) and ($\approx$ 3.35, $\approx$ -9.28).

11. a) Solve the equation: $x^2 + (0.5x - 40)^2 = 14\,400 \Rightarrow x_1 \approx 118.45$ and $x_2 \approx -86.45$.
The corresponding y-values are $y_1 \approx 19.22$ and $y_2 \approx -83.22$.
The intersection points are ($\approx$ -86.45, $\approx$ -83.22) and ($\approx$ 118.45, $\approx$ 19.22).

b) Solve the equation: $x^2 = -24((0.5x - 40) - 120) \Rightarrow x_1 \approx 56.26$ and $x_2 \approx -68.26$.
The corresponding y-values are $y_1 \approx -11.87$ and $y_2 \approx -74.13$.
The intersection points are ($\approx$ 56.26, $\approx$ -11.87) and ($\approx$ -68.26, $\approx$ -74.13).

c) Solve the equation: $-24(y - 120) + y^2 = 14\,400 \Rightarrow y_1 \approx 120$ and $y_2 \approx -96$.
The corresponding x-values are $x_1 = 0$ and $x_2 = \pm 72$.
The intersection points are (0, 120), (72, -96) and (-72, -96).

12. **a)** 1) The equation of the line that corresponds to the high wire is $y = \frac{x}{3} - 10$ since the line passes through the points (30, 0) and (60, 10).

2) The equation of the curve that corresponds to the canyon is $(x - 60)^2 = 20(y + 45)$ since the vertex of the parabola is (60, -45) and passes through the point (30, 0).

b) 1) (30, 0) 2) ($\approx$ 96.67, $\approx$ 22.22)

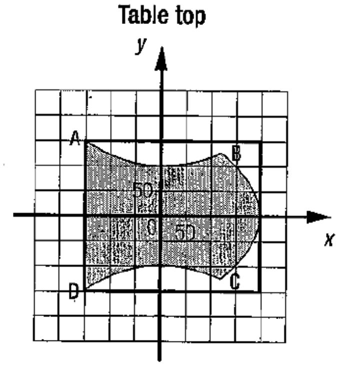
c) The distance covered by the tightrope walker is $\sqrt{(96.67 - 30)^2 + (22.22 - 0)^2}$ m $\approx$ 70.28 m.

d) The equation of the parabola is $(x - 60)^2 = 20(y + 45)$; therefore, $c = 20 \div 4 = 5$.
The coordinates of the vertex are (60, -45); the coordinates of the focus and the location of the camera are therefore (60, -45 + 5) = (60, -40).

13. **a)** For points A and D, find the intersection points between the hyperbola and the line. For points B and C, find the intersection points between the hyperbola and the parabola.
1) (-150, $\approx$ 141.42) 2) ($\approx$ 118.69, $\approx$ 127.52) 3) ($\approx$ 118.69, $\approx$ -127.52) 4) (-150, $\approx$ -141.42)

b) The minimum dimensions of the piece of wood are 350 cm by approximately 282.84 cm. The adjacent diagram represents this piece of wood.

Table top

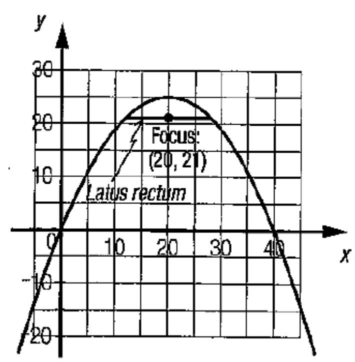


14. Note that the *latus rectum* is perpendicular to an axis of symmetry of the conic.

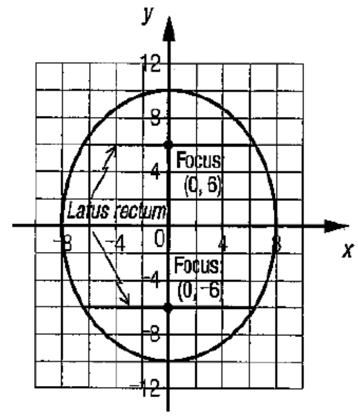
a) The equation of the parabola is $(x - 20)^2 = -16(y - 25)$. You can deduce that $c = -16 \div 4 = -4$ and that the vertex of the parabola is located at point (20, 25). The focus is therefore located at coordinates (20, 21).

You can determine that the intersection point between the parabola and the line with equation $y = 21$:
$(x - 20)^2 = -16(21 - 25)$
$(x - 20)^2 = 64$
$x_1 \approx 28$ and $x_2 \approx 12$.

The *latus rectum* is the segment that joins points (12, 21) and (28, 21). It therefore measures 16 u.



b) The equation of the ellipse is $\frac{x^2}{64} + \frac{y^2}{100} = 1$. You can deduce that $c = \pm 6$ since $a^2 + c^2 = b^2$ and that the foci are therefore located at coordinates (0, 6) and (0, −6).



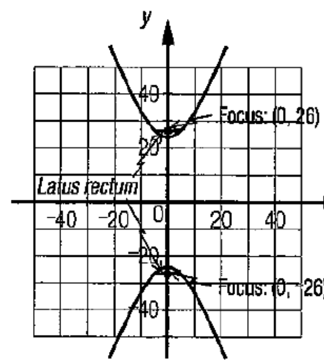
You can determine that the intersection point between the ellipse and the line with equation $y = 6$ or $y = -6$:

$$\frac{x^2}{64} + \frac{6^2}{100} = 1$$
$$\frac{x^2}{64} = \frac{16}{25}$$
$$x^2 = 40.96$$

$x_1 \approx 6.4$ and $x_2 \approx -6.4$.

The *latus rectum* is the segment that joins points (6.4, 6) and (−6.4, 6) as well as points (6.4, −6) and (−6.4, −6). It therefore measures 12.8 u.

c) The equation of the hyperbola is $\frac{x^2}{100} - \frac{y^2}{576} = -1$. You can deduce that $c = \pm 26$, since $a^2 + b^2 = c^2$, and that the foci are therefore located at coordinates (0, 26) and (0, −26)



You can determine that the intersection point between the hyperbola and the line with equation $y = 26$ or $y = -26$:

$$\frac{x^2}{100} - \frac{26^2}{576} = -1$$
$$\frac{x^2}{100} = \frac{25}{144}$$
$$x^2 = \frac{625}{36}$$

$x_1 \approx 4.17$ and $x_2 \approx -4.17$.

The *latus rectum* is the segment that joins points (≈ 4.17, 26) and (≈ −4.17, 26) as well as the points (≈ 4.17, −26) and (≈ −4.17, −26). It therefore measures approximately 8.33 u.

15. Solve the system of equations:

$$\frac{x^2}{40\,000} + \frac{y^2}{400} = 1$$
$$y = 0.05x + 5$$

You obtain the equation:

$$\frac{x^2}{40\,000} + \frac{(0.05x + 5)^2}{400} = 1$$

$$400x^2 + 100x^2 + 20\,000x + 1\,000\,000 = 16\,000\,000$$
$$500x^2 + 20\,000x - 15\,000\,000 = 0$$

$x_1 \approx 154.4$ and $x_2 \approx -194.36$,

of where: $y_1 \approx 0.05 \times 154.4 + 5 \approx 12.72$

$y_2 \approx 0.05 \times -194.36 + 5 \approx -4.72$

The coordinates of the probe's potential landing points are (≈ 154.36, ≈ 12.72) and (≈ −194.36, ≈ −4.72).