Practice 6.2

1. a) 1) (−2, 0) 2) $x = 2$
   b) 1) (0, 0.5) 2) $y = -0.5$
   c) 1) $\left(\frac{33}{4}, -3\right)$ 2) $x = \frac{7}{4}$
   d) 1) $\left(12, \frac{-25}{4}\right)$ 2) $y = \frac{25}{4}$
   e) 1) (5, 14) 2) $x = -21$
   f) 1) (0, −2.1) 2) $y = -8.3$
2. a) $y^2 = -20x$
   b) $x^2 = -80y$
   c) $(y - 8)^2 = 0.4(x + 3)$
   d) *Several answers possible. Example:* $(x - 5)^2 = 32(y + 10)$ or $(x - 5)^2 = -32(y + 10)$.
   e) $(y - 12)^2 = -24(x - 9)$
3. a) 1) (9, 0) and (−9, 0). 2) (15, 0) and (−15, 0).
   b) 1) (24, 0) and (−24, 0). 2) (30, 0) and (−30, 0).
   c) 1) (0, 5) and (0, −5). 2) (0, 13) and (0, −13).
   d) 1) (4, 0) and (−4, 0). 2) (8.5, 0) and (−8.5, 0).
   e) 1) (0, 20) and (0, −20). 2) (0, 20.5) and (0, −20.5).
   f) 1) (0, 11) and (0, −11). 2) (0, 61) and (0, −61).
4. a) $\frac{x^2}{100} - \frac{y^2}{576} = 1$
   b) $\frac{x^2}{225} - \frac{y^2}{64} = -1$
   c) $\frac{x^2}{441} - \frac{y^2}{400.15} \approx 1$
   d) $\frac{x^2}{5.0625} - \frac{y^2}{100} = -1$
   e) Since the equation of the asymptote is $y = -\frac{7}{24}x$, you can present the following proportion:
   $\frac{7}{24} = \frac{b}{a}$
   $\frac{7}{24} = \frac{28}{a}$
   $a = 96$
   The equation of the hyperbola is therefore $\frac{x^2}{9216} - \frac{y^2}{784} = -1$.

Practice 6.2 *(cont'd)*

5. a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
   b) $\frac{x^2}{225} - \frac{y^2}{64} = -1$
   c) $\frac{x^2}{576} - \frac{y^2}{100} = -1$
   d) $\frac{x^2}{1089} - \frac{y^2}{3136} = 1$
   e) $\frac{x^2}{3024.8} - \frac{y^2}{2304} \approx -1$
   f) $\frac{x^2}{16} - \frac{y^2}{1} = 1$
6.

Equation of the parabola	Coordinates of the vertex	Coordinates of the focus	Equation of the directrix	Distance between the focus and the directrix
$(y - 8)^2 = -44(x + 2)$	**(−2, 8)**	**(−13, 8)**	**$x = 9$**	**22**
$(x + 13)^2 = 2(y - 10)$	**(−13, 10)**	**(−13, 10.5)**	**$y = 9.5$**	**1**
$(x - 10)^2 = -24(y + 12)$	(10, −12)	**(10, −18)**	$y = -6$	**12**
$(x - 2)^2 = -26(y + 2)$	(2, −2)	(2, −8.5)	**$y = 4.5$**	**13**
$(y - 8)^2 = -32(x - 14)$	(14, 8)	**(6, 8)**	$x = 22$	**16**
$(y + 2)^2 = 10.8(x - 2)$	(2, −2)	(4.7, −2)	**$x = -0.7$**	**5.4**

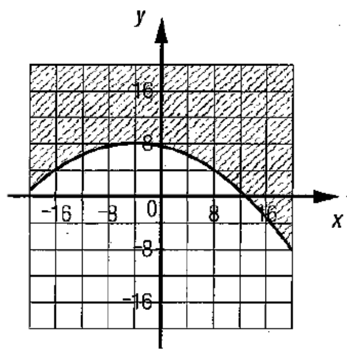
7. a) $x^2 = 24y$ **b)** $(x - 5)^2 = 28.8(y + 9)$ **c)** $(y + 2)^2 = 2(x + 4)$

d) $(x + 10)^2 = -80(y - 15)$ **e)** $y^2 = -8x$ **f)** $(y - 0.5)^2 = 0.4(x + 1.5)$

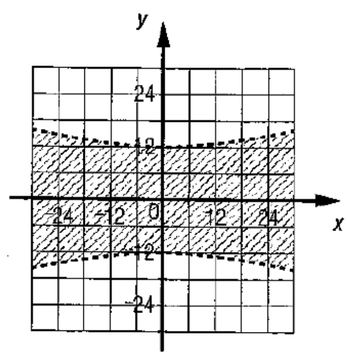
8.

Equation of the hyperbola	Coordinates of the vertices	Coordinates of the foci	Equation of the asymptotes
$\frac{x^2}{5929} - \frac{y^2}{1296} = 1$	**(77, 0)** **(-77, 0)**	**(85, 0)** **(-85, 0)**	$y = \frac{36}{77}x$ $y = -\frac{36}{77}x$
$\frac{x^2}{6400} - \frac{y^2}{324} = -1$	(0, 18) (0, -18)	**(0, 82)** **(0, -82)**	$y = -\frac{9}{40}x$ $y = \frac{9}{40}x$
$\frac{x^2}{4225} - \frac{y^2}{5184} = -1$	(0, 72) (0, -72)	(0, 97) (0, -97)	$y = \frac{72}{65}x$ $y = -\frac{72}{65}x$
$\frac{x^2}{64} - \frac{y^2}{992.25} = -1$	**(0, 31.5)** **(0, -31.5)**	**(0, 32.5)** **(0, -32.5)**	$y = \frac{63}{16}x$ $y = -\frac{63}{16}x$
$\frac{x^2}{2.25} - \frac{y^2}{4} = 1$	(1.5, 0) (-1.5, 0)	(2.5, 0) (-2.5, 0)	$y = \frac{4}{3}x$ $y = -\frac{4}{3}x$

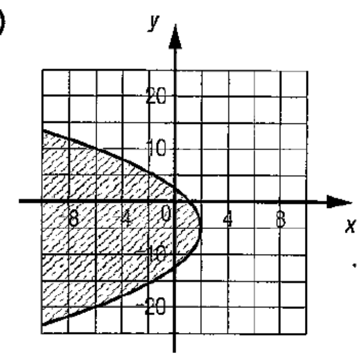
9. a)



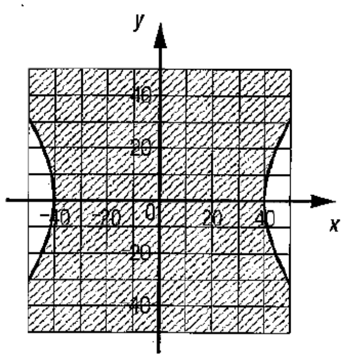
b)



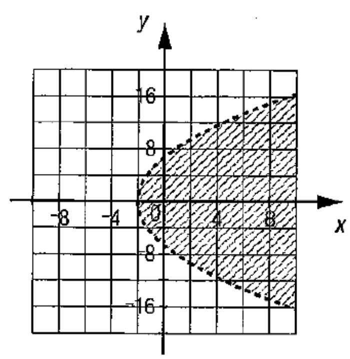
c)



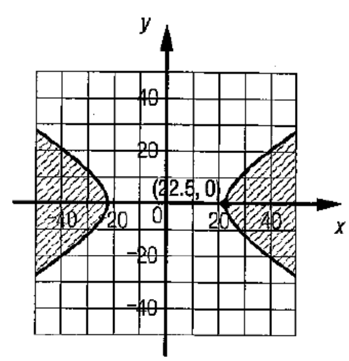
d)



e)



f)



10. $\frac{40}{9}$ is a simplified fraction of the expression $\frac{b}{a}$. You can therefore have b = 40 and a = 9 or b = 20 and a = 4.5 or b = 80 and a = 18, etc.

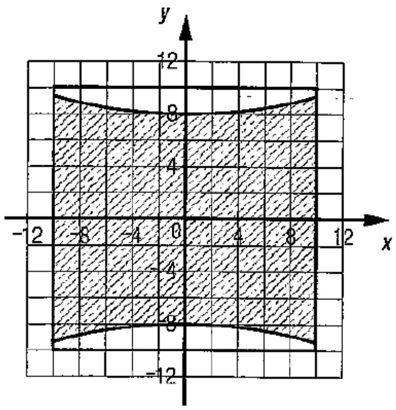
11. a) $\frac{x^2}{81} - \frac{y^2}{144} < 1$ **b)** $(y - 15)^2 \geq 60(x + 30)$ **c)** $(x - 8.5)^2 < -40(y - 4.5)$

d) $\frac{x^2}{256} - \frac{y^2}{900} \leq -1$ **e)** $\frac{x^2}{8100} - \frac{y^2}{3136} \geq 1$ **f)** $(x + 4)^2 \leq 6y$

12. a) 1) $\frac{x^2}{1} - \frac{y^2}{4} = 1$ 2) $y = 2x$ and $y = -2x$.

b) $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.

13. a) This flower garden corresponds to the shaded region of the inside of the square.

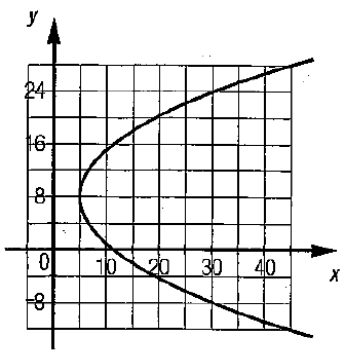


b) The minimum width of the flower garden corresponds to the distance between the two vertices and the hyperbola, which is 16 m.

c) No. This is not possible because the coordinates of the foci of the hyperbola are (0, 17) and (0, −17). The foci are therefore located outside of the lot.

14. a) Since the equation of the trajectory is $(y - 8)^2 = 10(x - 5)$, it can be deduced that the value of parameter **c** is $10 \div 4 = 2.5$. Using the coordinates of the vertex, determine the coordinates of the focus: $(5 + 2.5, 8) = (7.5, 8)$. The coordinates of the Sun are therefore (7.5, 8).

b)



c. The value of parameter **c** is 2.5, and it refers to the minimum distance between the vertex of the parabola and the directrix. The minimum distance that separates the trajectory of the comet and that of the satellite is 2.5 billion kilometres.

15. a) Since the coordinates of the vertex are (50, 0) and the trajectory passes through point (80, −11.25), the equation of the parabola associated with the trajectory of the submarine is $(x - 50)^2 = -80y$.

b) Find the value of y when $x = 0$:

$(0 - 50)^2 = -80y$

$2500 = -80y$

$-31.25 = y$

The maximum depth reached by the submarine is 31.25 m.

16. a) The coordinates of the vertex of the parabola are (0, 0), and the parabola passes through the point (20, 40). The equation of the parabola associated with the concave parabolic mirror is therefore $y^2 = 80x$.

b) The equation of the hyperbola centred at the origin where one of the branches is associated with the convex hyperbolic mirror is $\frac{x^2}{100} - \frac{y^2}{576} = 1$, since the coordinates of one of the vertices are (0, −10) and the coordinates of one of the foci are (−26, 0).