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1. For each of the following parabolas with vertex V(0, 0), determine

1. the concavity. 2. the coordinates of the focus. 3. the equation of the directrix.

a) $x^2 = 6y$
1. *Upwards*
2. *F(0; 1.5)*
3. *y = -1.5*

b) $x^2 = -4y$
1. *Downwards*
2. *F(0, -1)*
3. *y = 1*

c) $y^2 = -12x$
1. *To the left*
2. *F(-3, 0)*
3. *x = 3*

d) $y^2 = 8x$
1. *To the right*
2. *F(2, 0)*
3. *x = -2*

2. A parabola with vertex V(0, 0), open to the left, passes through point A(–5, –5). What is its equation? $y^2 = -5x$

3. A parabola with vertex V(0, 0), passes through point A(4, 4). What is its equation if the parabola is

a) open upwards? $x^2 = 4y$ **b)** open to the right? $y^2 = 4x$

4. Complete the following table knowing that each parabola has vertex V(0, 0).

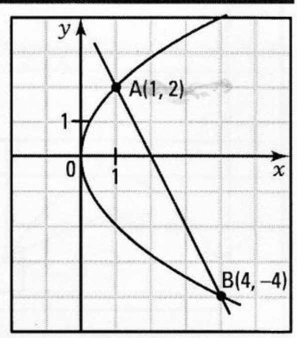
Equation of the parabola	Focus	Equation of the directrix	Equation of the axis of symmetry	A point on the parabola
$x^2 = -6y$	F(-1.5, 0)	$y = 1.5$	$x = 0$	A(6, -6)
$y^2 = 8x$	F(2, 0)	$x = -2$	$y = 0$	A(2, –4)
$x^2 = 2y$	$F\left(0, \frac{1}{2}\right)$	$y = -\frac{1}{2}$	$x = 0$	A(–4, 8)
$y^2 = -x$	$F\left(-\frac{1}{4}, 0\right)$	$x = \frac{1}{4}$	$y = 0$	A(-1, 1)

5. Consider the parabola $\mathscr{P}$ with equation: $y^2 = 4x$ and the line l with equation: $2x + y - 4 = 0$.

a) Determine algebraically the intersection points of parabola $\mathscr{P}$ and line l.

$\begin{cases} y^2 = 4x \\ y = -2x + 4 \end{cases} \Rightarrow (-2x + 4)^2 = 4x ; 4x^2 - 20x + 16 = 0$ A(1, 2); B(4, -4)

b) Represent the parabola and line l in the Cartesian plane and verify the answers found in a).



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c) Determine

1. the coordinates of the vertex V of parabola $\mathscr{P}'$. *V(2, 3)*
2. the coordinates of the focus F′ of parabola $\mathscr{P}'$. *F′(2, 4)*
3. the equation of the directrix l' of parabola $\mathscr{P}'$. *y = 2*
4. the equation of the axis of symmetry of $\mathscr{P}'$. *x = 2*

d) Deduce the equation (in the standard form) of parabola $\mathscr{P}'$ from the equation of parabola $\mathscr{P}$.

$(x + 2)^2 = 4(y + 3)$

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6. Complete the following table.

Equation of the parabola	Parameter c	Concavity	Coordinates of the vertex	Coordinates of the focus	Equation of the directrix
$(x+3)^2 = 4(y-1)$	*1*	*Upwards*	*V(–3, 1)*	*F(–3, 2)*	$y = 0$
$(y-2)^2 = 2(x+1)$	$\frac{1}{2}$	*To the right*	*V(–1, 2)*	$F\left(\frac{-1}{2}, 2\right)$	$x = \frac{-3}{2}$
$(x+1)^2 = -4(y+3)$	*1*	*Downwards*	*V(–1, –3)*	*F(–1, –4)*	$y = -2$
$(y+2)^2 = -2(x-1)$	$\frac{1}{2}$	*To the left*	*V(1, –2)*	$F\left(\frac{1}{2}, -2\right)$	$x = \frac{3}{2}$

7. Complete the following table.

Equation of the parabola	Parameter c	Concavity	Coordinates of the vertex	Coordinates of the focus	Equation of the directrix
$(x-1)^2 = -6(y+2)$	$\frac{3}{2}$	Downwards	V(1, –2)	$F\left(1, \frac{-7}{2}\right)$	$y = \frac{-1}{2}$
$(x+1)^2 = 4(y-3)$	1	Upwards	V(–1, 3)	*F(–1, 4)*	$y = 2$
$(y-2)^2 = -2(x+1)$	$\frac{1}{2}$	*To the left*	V(–1, 2)	$F\left(-\frac{3}{2}, 2\right)$	$x = \frac{-1}{2}$
$(y+1)^2 = 6(x-2)$	$\frac{3}{2}$	*To the right*	*V(2, –1)*	$F\left(\frac{7}{2}, -1\right)$	$x = \frac{1}{2}$

8. Determine the equation of the parabola in each of the following cases.

a) The parabola has focus F(1, 4) and directrix $l: y = 2$. $(x-1)^2 = 4(y-3)$

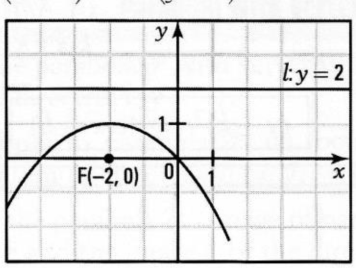
b) The parabola has vertex V(–1, 2), is open to the right and passes through point A(3, 6).
$(y-2)^2 = 4(x+1)$

c) The parabola has vertex V(1, 3), is open downwards and passes through point A(3, 1).
$(x-1)^2 = -2(y-3)$

9. In each of the following cases,

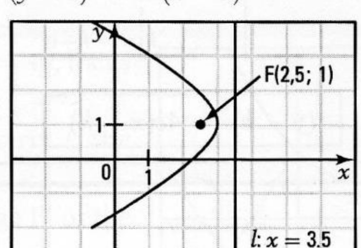
1. draw the parabola.
2. locate the focus F.
3. draw the directrix l.

a) $(x+2)^2 = -4(y-1)$



x	–5	–4	–2	0	1
y	–1.25	0	1	0	–1.25

b) $(y-1)^2 = -2(x-3)$



x	0	1	2.5	3	
y	–1.5 / 3.5	–1 / 3	0 / 2	1	

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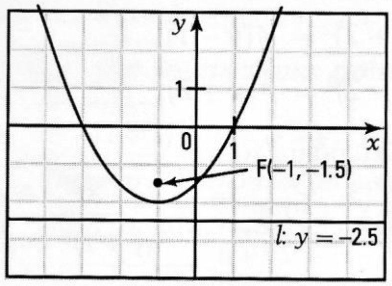
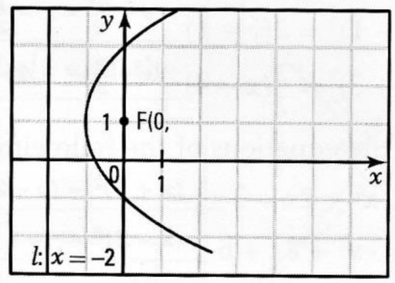
c) $(x + 1)^2 = 2(y + 2)$



x	−4	−3	−1	1	2
y	2.5	0	−2	0	2.5

d) $(y - 1)^2 = 4(x + 1)$



x	−1	0	1
y	1	−1 / 3	−1.8 / 3.8

10. Write the equations of the following parabolas in the general form.

a) $(x-1)^2 = 2(y+1)$ $y = \frac{x^2}{2} - x - \frac{1}{2}$ **b)** $(x+2)^2 = -4(y-1)$ $y = -\frac{x^2}{4} - x$

c) $(y+3)^2 = \frac{3}{2}(x-4)$ $x = \frac{2}{3}y^2 + 4y + 10$ **d)** $(y-2)^2 = \frac{-1}{2}(x+2)$ $x = -2y^2 + 8y - 10$

11. Write the equations of the following parabolas in the standard form.

a) $y = x^2 + 2x + 3$ $(x+1)^2 = (y-2)$ **b)** $x = y^2 - 6y + 10$ $(y-3)^2 = (x-1)$

c) $y = -x^2 + 4x + 6$ $(x-2)^2 = -(y-10)$ **d)** $x = -y^2 - 2y + 1$ $(y+1)^2 = -(x-2)$

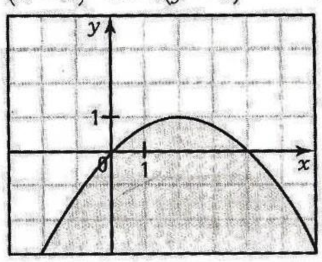
12. Determine the coordinates of the focus F and the equation of the directrix l of the following parabolas.

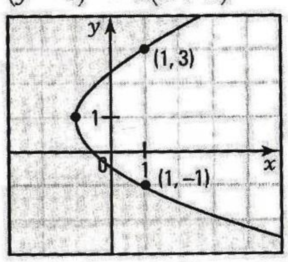
a) $y = x^2 - 2x - 1$; $(x-1)^2 = y + 2$; $F\left(1, -\frac{7}{4}\right)$; $l: y = -\frac{9}{4}$

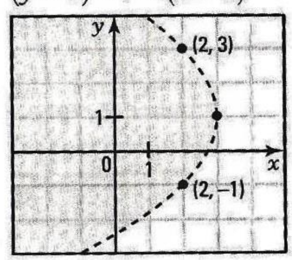
b) $x = \frac{1}{4}y^2 - y - 1$; $(y-2)^2 = 4(x+2)$; $F(-1, 2)$; $l: x = -3$

13. Represent the solution set of the following inequalities in the Cartesian plane.

a) $(x-2)^2 \leqslant -4(y-1)$ **b)** $(y-1)^2 \geqslant 2(x+1)$ **c)** $(y-1)^2 < -4(x-3)$

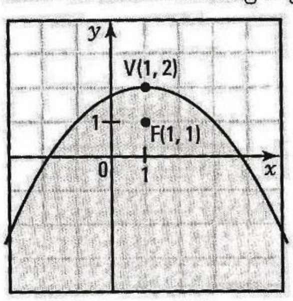






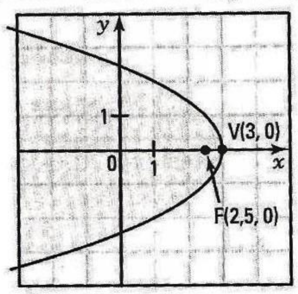
14. For each of the following regions, determine the inequality that defines it.

a)



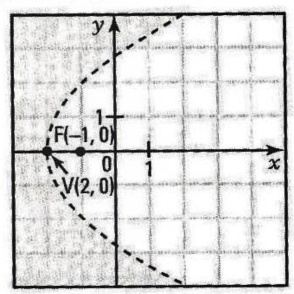
$(x-1)^2 \leqslant -4(y-2)$

b)



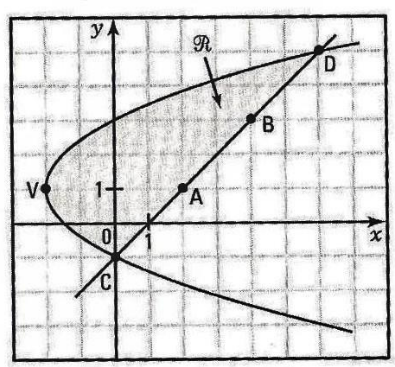
$y^2 \leqslant -2(x-3)$

c)



$y^2 > 4(x+2)$

15. Consider the parabola $\mathcal{P}$ with vertex V(−2, 1) and focus $F\left(\frac{-3}{2}, 1\right)$ and the line l passing through points A(2, 1) and B(4, 3).



a) Find the intersection points C and D of parabola $\mathcal{P}$ and line l. C(0, −1), D(6, 5)

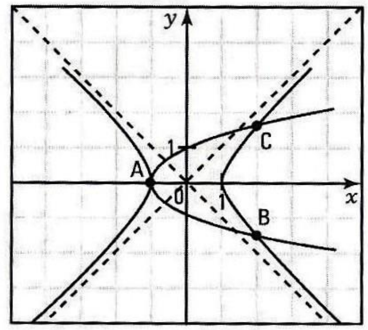
$\mathcal{P}: (y-1)^2 = 2(x+2)$; $l: y = x - 1$

b) Draw parabola $\mathcal{P}$ and line l and verify the results found in a).

c) Consider the closed region $\mathcal{R}$ whose boundary is parabola $\mathcal{P}$ and line l.

Describe region $\mathcal{R}$ using a system of inequalities. $\begin{cases} (y-1)^2 \leqslant 2(x+2) \\ y \geqslant x - 1 \end{cases}$

16. Consider the hyperbola $\mathcal{H}$ with equation: $x^2 - y^2 = 1$ and the parabola $\mathcal{P}$ with equation: $x + 1 = y^2$.



a) Find the intersection points of the hyperbola and the parabola.

$x^2 - (x + 1) = 1 \Leftrightarrow x^2 - x - 2 = 0 \Leftrightarrow x = -1$ or $x = 2$

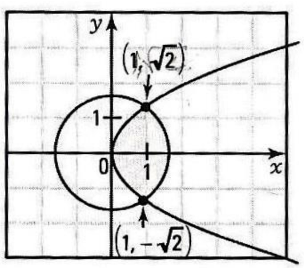
There are 3 intersection points.

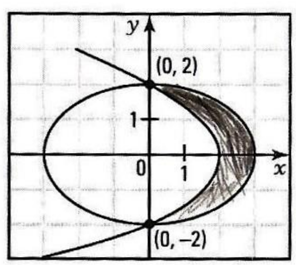
A(−1, 0), $B(2, -\sqrt{3})$, $C(2, \sqrt{3})$

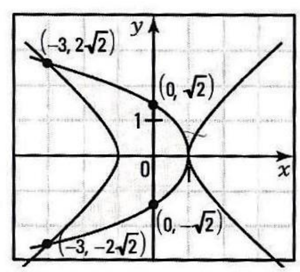
b) Represent, in the Cartesian plane, the region defined by the system $\begin{cases} x^2 - y^2 \leqslant 1 \\ x + 1 \geqslant y^2 \end{cases}$.

17. In each of the following cases, represent the region defined by the system.

a) $\begin{cases} x^2 + y^2 \leqslant 3 \\ y^2 \leqslant 2x \end{cases}$ **b)** $\begin{cases} y^2 \geqslant -2(x-2) \\ 4x^2 + 9y^2 - 36 \leqslant 0 \end{cases}$ **c)** $\begin{cases} x - 1 \leqslant \frac{-1}{2}y^2 \\ x^2 - y^2 \leqslant 1 \end{cases}$

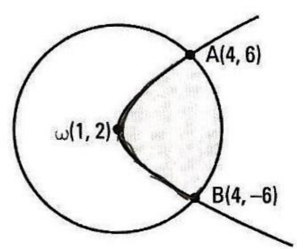






18. Consider the circle $\mathcal{C}$ and the parabola $\mathcal{P}$ on the right. The vertex of the parabola is the centre ω(1, 2) of the circle. The points A(4, 6) and B(4, −6) are the intersection points of the circle and the parabola.

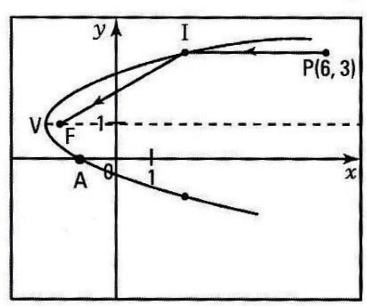
Describe, using a system, the shaded region.



$$\begin{cases} (y - 1)^2 + (y - 2)^2 \leqslant 25 \\ (y - 2)^2 \leqslant \frac{16}{3}(x - 1) \end{cases}$$

19. The parabola on the right, open to the right with vertex V(−2, 1) crosses the x-axis at point A(−1, 0).

Calculate the distance traveled by a light ray going from point P(6, 3) in a direction parallel to the x-axis, hitting the parabola at point I and reflected at the focus F.



Parabola: $(y - 1)^2 = x + 2$; $c = \frac{1}{4}$; $F\left(-\frac{7}{4}, 1\right)$.

I(2, 3): $m\overline{PI} = 4$ u; $m\overline{IF} = \frac{17}{4}$ u; distance traveled = 8.25 u.