Page 323

1. Find the equation of the circle centred at the origin with radius

a) $r = 2$ $x^2 + y^2 = 4$ **b)** $r = \sqrt{3}$ $x^2 + y^2 = 3$

2. Find the equation of the circle centred at the origin passing through A(–2, 3). $x^2 + y^2 = 13$

3. Consider the circle with equation: $x^2 + y^2 = 5$. Indicate if the following points belong to the circle.

a) A(–2, 1) *Yes* **b)** B(1, –2) *Yes* **c)** C(–2, 2) *No*

4. Consider the circle with equation: $x^2 + y^2 = 25$. Find the points M(x, y) of the circle that have

a) an x-coordinate equal to 4. $M_1(4, 3)$ and $M_2(4, -3)$

b) a y-coordinate equal to –2. $M_1\left(-\sqrt{21}, -2\right)$ and $M_2\left(\sqrt{21}, 2\right)$

Page 325

5. Consider the circle $\mathscr{C}$ of radius 3 units centred at O(0, 0) and the translation $t: (x, y) \rightarrow (x - 1, y + 2)$.

a) Find the equation of circle $\mathscr{C}$. $x^2 + y^2 = 9$

b) We draw circle $\mathscr{C}'$, image of circle $\mathscr{C}$, under translation t.

1. Determine the coordinates of the centre of circle $\mathscr{C}'$ and its radius.
   Centre (–1, 2); radius: 3
2. Find the equation of circle $\mathscr{C}'$. $(x + 1)^2 + (y - 2)^2 = 9$

6. Determine the centre ω and the radius r of the following circles.

a) $(x - 3)^2 + (y - 4)^2 = 16$ $\omega(3, 4); r = 4$

b) $(x + 2)^2 + (y - 1)^2 = 9$ $\omega(-2, 1); r = 3$

c) $(x + 3)^2 + (y + 1)^2 = 17$ $\omega(-3, -1); r = \sqrt{17}$

d) $\frac{(x+2)^2}{3} + \frac{(y-7)^2}{3} = 12$ $\omega(-2, 7); r = 6$

7. Find the equation of the circle, in the standard form, knowing the centre ω and a point M on the circle.

a) $\omega(1, 3)$ and M(1, 7) $(x - 1)^2 + (y - 3)^2 = 16$

b) $\omega(-4, 5)$ and M(2, –3) $(x + 4)^2 + (y - 5)^2 = 100$

8. The points A(–1, 2) and B(–3, –4) are the endpoints of a diameter AB of a circle. Find the equation, in the standard form, of this circle.
$(x + 2)^2 + (y + 1)^2 = 10$

9. The equation of a circle centred at the origin is $x^2 + y^2 = 16$.
Describe the translation which associates, with this circle, the circle of equation

a) $(x - 1)^2 + (y + 4)^2 = 16$: $(x, y) \rightarrow (x + 1, y - 4)$

b) $x^2 + (y - 3)^2 = 16$: $(x, y) \rightarrow (x, y + 3)$

c) $(x + 2)^2 + y^2 = 16$: $(x, y) \rightarrow (x - 2, y)$

d) $(x + 1)^2 + (y + 3)^2 = 16$: $(x, y) \rightarrow (x - 1, y - 3)$

Page 326

10. In each of the following cases, find the equation of circle $\mathscr{C}$ in the standard form and then in the general form.

a) Centre $(-1, 3)$; radius 2.

1. $(x + 1)^2 + (y - 3)^2 = 4$
2. $x^2 + y^2 + 2x - 6y + 6 = 0$

b) Centre $(0, -3)$; radius 2.

1. $x^2 + (y + 3)^2 = 4$
2. $x^2 + y^2 + 6y + 5 = 0$

c) Centre $(2, -1)$; $M(-1, 3) \in \mathscr{C}$.

1. $(x - 2)^2 + (y + 1)^2 = 25$
2. $x^2 + y^2 - 4x + 2y - 20 = 0$

d) $\overline{AB}$ is a diameter, $A(-2, 1)$ and $B(4, -3)$.

1. $(x - 1)^2 + (y + 1)^2 = 13$
2. $x^2 + y^2 - 2x + 2y - 11 = 0$

11. For each of the following circles,

1. write the equation of the circle in the standard form.
2. find the centre and the radius of the circle.

a) $x^2 + y^2 - 2x + 4y - 4 = 0$

$(x - 1)^2 + (y + 2)^2 = 9$

$\omega(1, -2)$; $r = 3$

b) $x^2 + y^2 + 8x + 4y + 19 = 0$

$(x + 4)^2 + (y + 2)^2 = 1$

$\omega(-4, -2)$; $r = 1$

c) $x^2 + y^2 - 4x + 6y - 4 = 0$

$(x - 2)^2 + (y + 3)^2 = 17$

$\omega(2, -3)$; $r = \sqrt{17}$

d) $x^2 + y^2 - x + 3y - 1.5 = 0$

$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 4$

$\omega\left(\frac{1}{2}, \frac{-3}{2}\right)^2$; $r = 2$

12. Explain why each of the following equations is not that of a circle.

a) $x^2 + y^2 - 4x + 6y + 14 = 0$ $(x - 2)^2 + (y + 3)^2 = -1$. The sum of 2 squares cannot be negative.

b) $x^2 - y^2 - 2x - 4y - 19 = 0$ The coefficient of y^2 cannot be negative.

13. Determine, if they exist, the intersection points of circle $\mathscr{C}$ with line l.

a) $\mathscr{C}: (x - 1)^2 + (y + 2)^2 = 9$
$l: y = -x + 2$
(1, 1) and (4, −2)

b) $\mathscr{C}: (x + 1)^2 + (y - 2)^2 = 2$
$l: x - y + 1 = 0$
(0, 1)

c) $\mathscr{C}: (x + 2)^2 + (y + 1)^2 = 1$
$l: x - y - 1 = 0$
No intersection point.

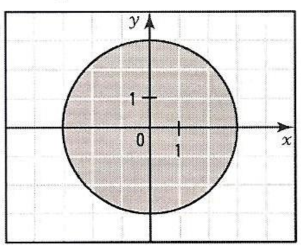
d) $\mathscr{C}: x^2 + y^2 - 2x + 4y + 1 = 0$
$l: x - y - 1 = 0$
(1, 0) and (−1, −2)

e) $\mathscr{C}: x^2 + y^2 = 9$
$l: x + y = 5$
No intersection point.

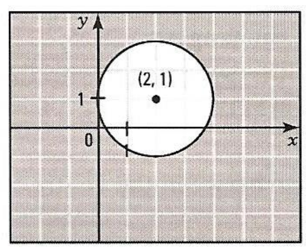
f) $\mathscr{C}: x^2 + y^2 - 2x + 4y - 20 = 0$
$l: 3x + 4y - 20 = 0$
(4, 2)

14. Represent the solution set of the following inequalities in the Cartesian plane.

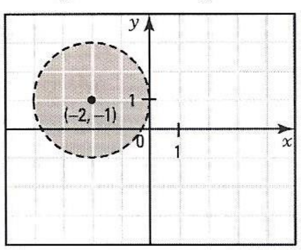
a) $x^2 + y^2 \leqslant 9$



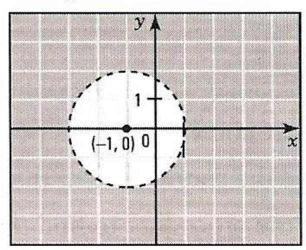
b) $(x - 2)^2 + (y - 1)^2 \geqslant 4$



c) $x^2 + y^2 + 4x + 2y + 1 < 0$

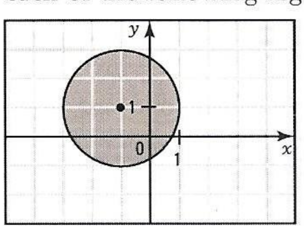


d) $x^2 + y^2 + 2x - 3 > 0$



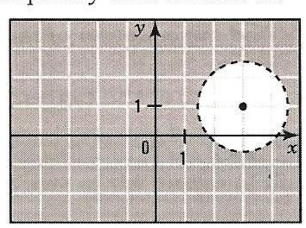
15. For each of the following regions, determine the inequality that defines it.

a)



$(x + 1)^2 + (y - 1)^2 \leqslant 4$

b)



$(x - 3)^2 + (y - 1)^2 > \frac{9}{4}$

16. Consider the circle with equation: $(x + 1)^2 + (y - 2)^2 = 25$.

a) Verify that point A(2, 6) is a point on this circle. $(2 + 1)^2 + (6 - 2)^2 = 25$

b) What are the coordinates of the centre ω of the circle? $\omega(-1, 2)$

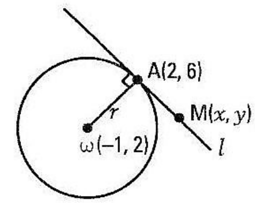
c) Explain the procedure to find the equation of the line l tangent to the circle at point A.

1. *We calculate the slope of the radius $\overline{\omega A}$.*
2. *We deduce the slope of the tangent l knowing that $\overline{\omega A} \perp l$.*
3. *We find the equation of the line l passing through point A(2, 6) knowing its slope.*

d) Find the equation of the tangent l at point A.

1. $a_{\overline{\omega a}} = \frac{6-2}{2+1} = \frac{4}{3}$ 2. $a_d = \frac{-3}{4}$ 3. $d: y = -\frac{3}{4}x + \frac{15}{2}$

17. Use the data from exercise 16. Justify the steps allowing us to find the equation of the tangent l at point A using the scalar product.



	Steps	Justifications
1.	$M(x, y) \in d \Leftrightarrow \overrightarrow{AM} \perp \overrightarrow{A\omega}$	*Property of the tangent.*
2.	$\Leftrightarrow \overrightarrow{AM} \cdot \overrightarrow{A\omega} = 0$	*Property of the scalar product.*
3.	$\Leftrightarrow (x - 2, y - 6) \cdot (-3, -4) = 0$	*Calculating the vector components.*
4.	$\Leftrightarrow -3(x - 2) - 4(y - 6) = 0$	*Calculating the scalar product.*
5.	$\Leftrightarrow -3x - 4y + 30 = 0$	*Equation of line l (general form).*
6.	$\Leftrightarrow y = -\frac{3}{4}x + \frac{15}{2}$	*Equation of line l (functional form).*

18. Find the equation of the tangent to the circle with equation: $x^2 + y^2 - 2x + 4y - 35 = 0$ at point A(3, 4).

$\mathscr{C}: (x - 1)^2 + (y + 2)^2 = 40$; $\omega(1, -2)$; $a_{\omega A} = 3$; $l: y = -\frac{1}{3}x + 5$

19. Find the equation of the circle with centre $\omega(-2, 3)$ if the circle is tangent to

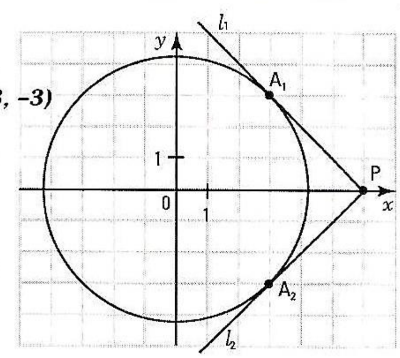
a) the x-axis. ***$(x + 2)^2 + (y - 3)^2 = 9$*** **b)** the y-axis. ***$(x + 2)^2 + (y - 3)^2 = 4$***

20. Consider the line l: $2x + y - 4 = 0$ and a point $\omega(1, -3)$.

a) Find the equation of the circle $\mathscr{C}$ centred at ω and tangent to line l.
Radius = $d(\omega, l) = \sqrt{5}$; $(x - 1)^2 + (y + 3)^2 = 5$

b) Find the coordinates of the point of tangency A. ***A(3, −2)***

21. Consider the circle $\mathscr{C}$ with equation: $x^2 + y^2 = 18$.



a) Find the coordinates of points A_1 and A_2 on the circle which have an x-coordinate equal to 3. ***$A_1(3, 3)$ and $A_2(3, -3)$***

b) Find the equation of the lines l_1 and l_2 tangent to circle $\mathscr{C}$ at points A_1 and A_2 respectively.
$l_1: y = -x + 6$ and $l_2: y = x - 6$

c) Show that lines l_1 and l_2 meet at a point P located on the x-axis.
The system $\begin{cases} y = -x + 6 \\ y = x - 6 \end{cases}$ has solution P(6, 0)

d) Show that the quadrilateral $0A_1PA_2$ is a square.
$m0A_1 = mA_1P = mPA_2 = m0A_2 = \sqrt{18} \Rightarrow 0A_1PA_2$ is a rhombus. $\angle 0A_1P$ is a right angle (Property of the tangent) $\Rightarrow 0A_1PA_2$ is a square, since one of the angles of the rhombus is right.

e) Calculate the area of the region bounded by the line segment PA_1, the line segment PA_2 and the arc of circle A_1A_2. ***Area of square $0A_1PA_2 = 18\ u^2$***
Area of the circular sector $A_10A_2 = \frac{18\pi}{4} u^2$ (one quarter of the disk). Requested area = $\left(18 - \frac{9\pi}{2}\right) u^2$.