Optimisation

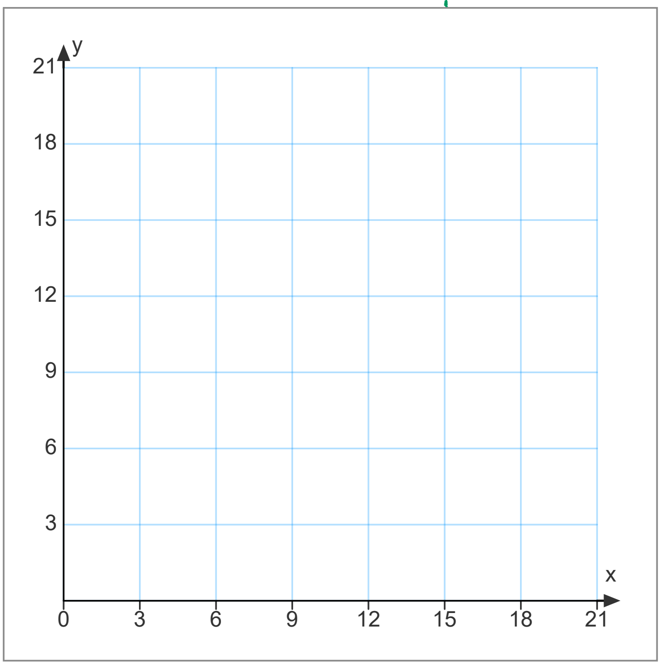
or Linear Programming

Example: A furniture manufacturer makes chairs and armchairs. The amount of time spent on making a chair is 3 hours and the amount of time spent on an armchair is 5 hours. In one week, the time spent on finishing these two pieces is equal to 45 hours.

a) Translate this situation into an equation.

b) Represent this equation on the Cartesian plane.

x				
y				



Determine the point of intersection of the following systems of linear equations.

a) $y = 20 - 7x$
$y = 3x + 5$

b) $y = 4x + 9$
$3x - 2y = 0$

c) $3x+5y=29$

$4x+2y=6$

Example: Chase, Sarah and Wendy went to the candy store. Chase bought five pieces of fudge and three pieces of bubble gum for a total of \$5.70. Sarah bought two pieces of fudge and ten pieces of bubble gum for a total of \$3.60. How much did Wendy pay for three pieces of fudge and 6 pieces of bubble gum?

Optimisation

Symbols:

$<$ less than, fewer than

$>$ greater than, more than, exceeds

$\leq$ less than or equal to, at most, maximum of, no more than

$\geq$ greater than or equal to, at least, minimum of, no less than

Examples with two variables:

1) At a school dance, students paid \$5 and guests paid \$8. The proceeds were more than \$1 200.

2) At a high school, at least twice as many girls as boys take chemistry.

3) The perimeter of a rectangle is less than $60cm$.

4) Rose and Eric went to New York and Boston. They spent at least twice as much time in New York as in Boston.

The inequalities presented in a situation are known as constraints - conditions that must be met.

Most situations have two extra constraints that are not mentioned. These are called the non-negative constraints and they exist when it is not possible for the variables to take on negative values.

$$x \geq 0$$
$$y \geq 0$$

Example: The maximum number of seats in a plane is 100. There must be at least 4 times as many seats in economy class as in business class.

The constraints are:

Determining the Solution Set of a Linear Inequality

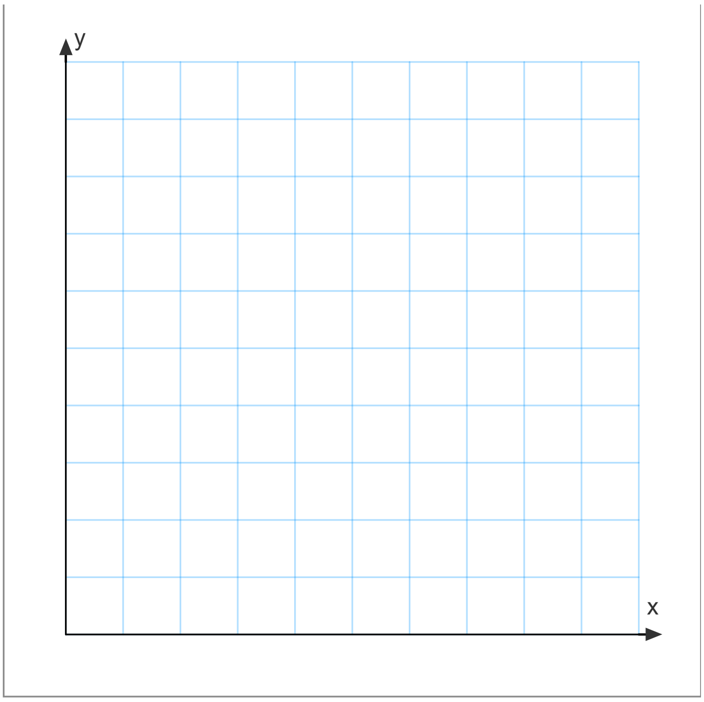
A local swimming pool uses a mixture of chlorine and bromine to purify the water. A litre of chlorine costs $10 and a litre of bromine costs $16. The pool manager buys a total of at least $240 worth of these products.

1. Variables:

2. Constraints:

1. Graph the line. Recall that if the inequality includes the equal sign, the line is drawn solid, but for a strict inequality, the line is broken.

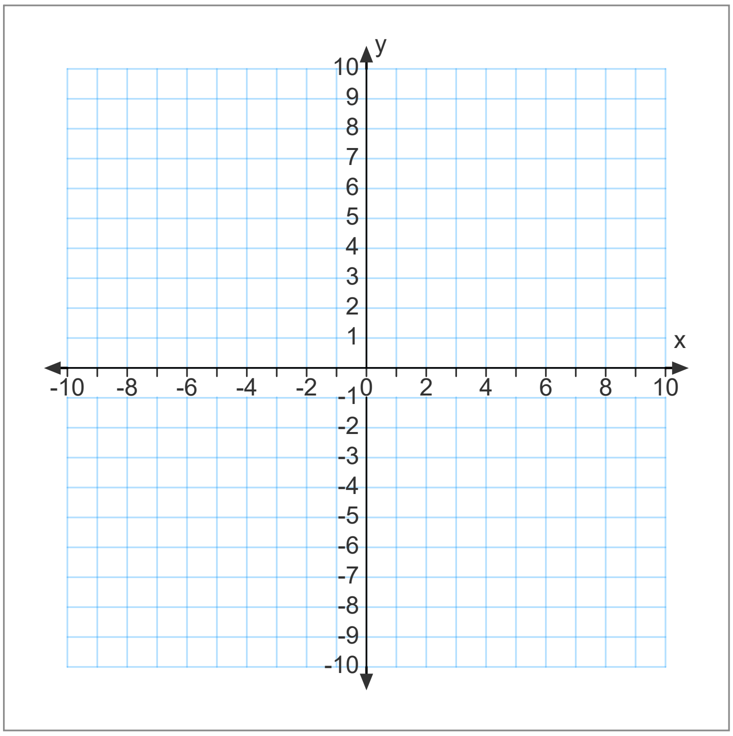
2. Choose a point. Test the point in the inequality.



3. Shade on the side of the line where the inequality is TRUE.

Example: Graph the solution set of the inequality

$$5x + 2y < 12$$



$5x + 2y = 12$

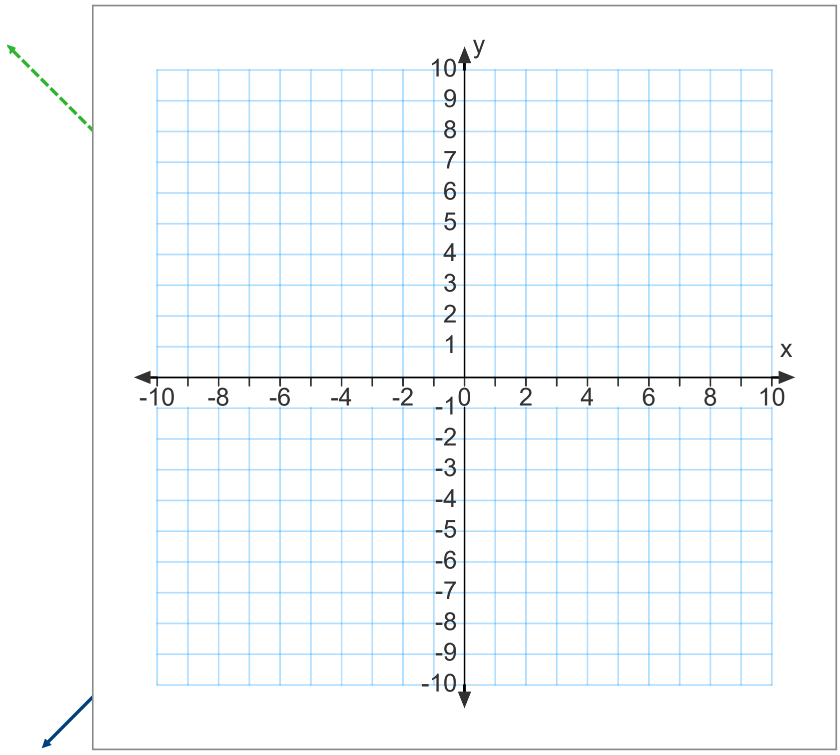
x	y

Systems of Linear Inequalities

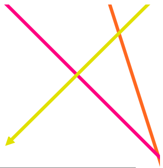
Solving a system of linear inequalities means finding all the points that satisfy all the inequalities.

Example: Determine the solution set of the following system.

$$y > -x$$
$$y \leq x$$

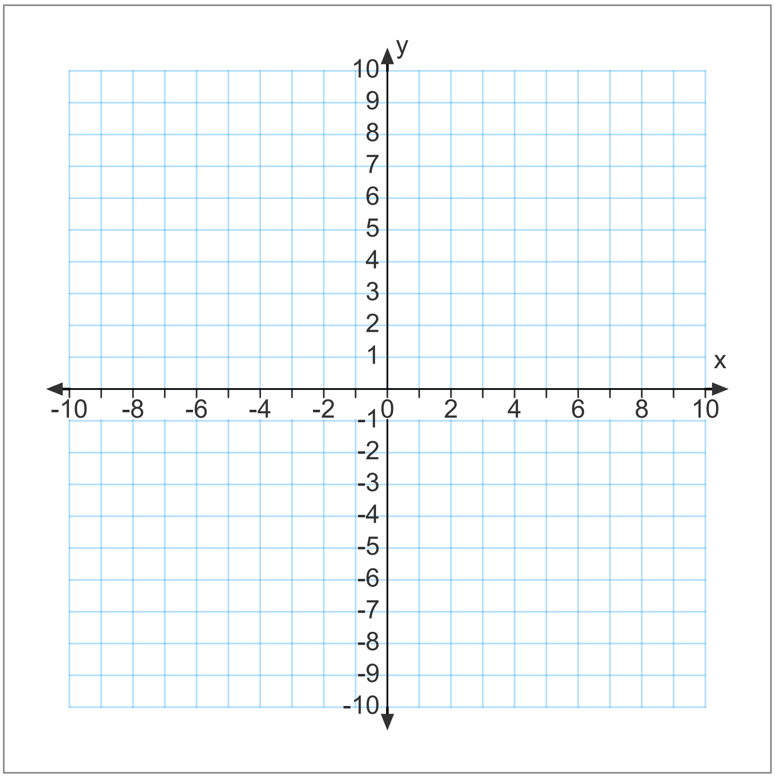


y
x
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10



Graph the following system of inequalities

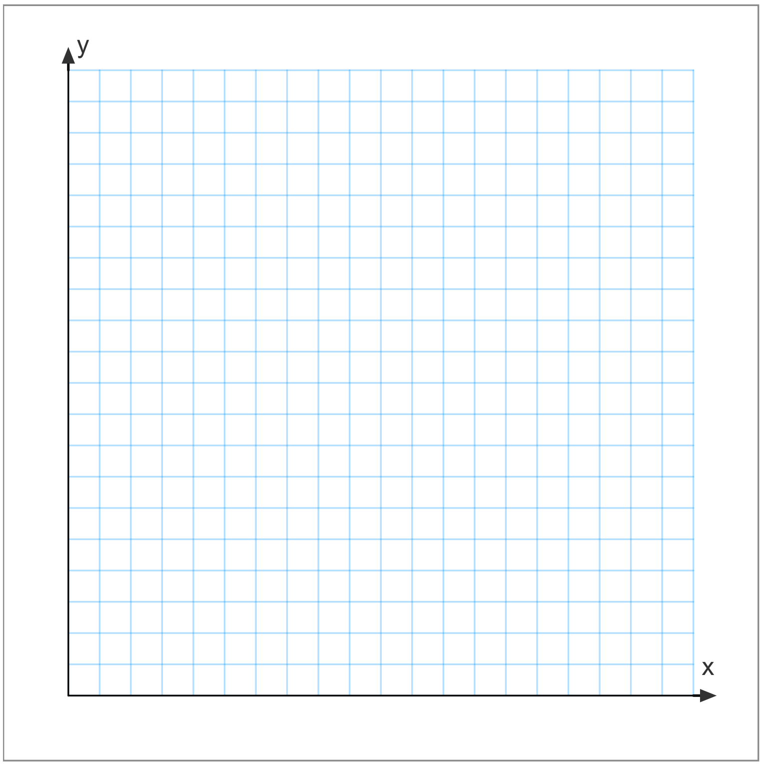
- $y \geq -x - 2$
- $y \leq x + 4$
- $3x + y \leq 8$



The figure created by the solution set of all the inequalities is called the polygon of constraints.

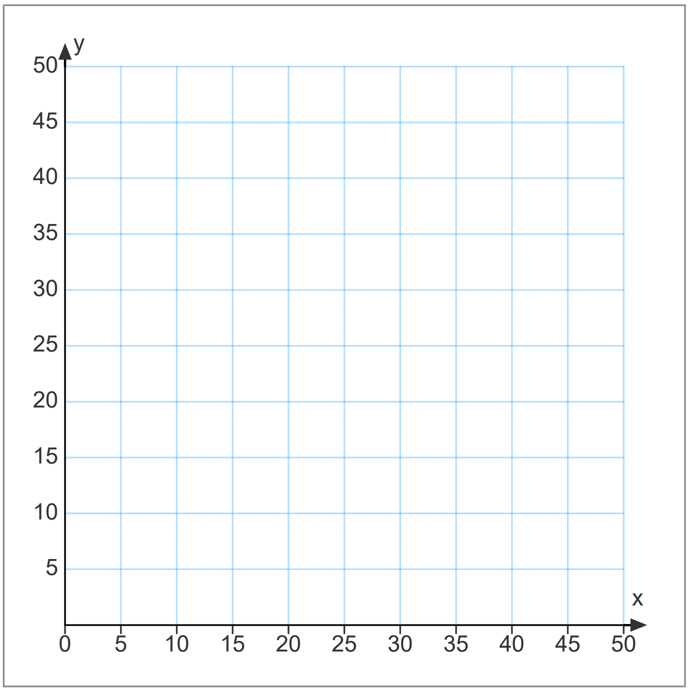
Example: A municipal garden grows red roses and white roses. There are at most 400 roses in total. The number of red roses increased by 80 is greater than twice the number of white roses.

Graph the solution set.



Not a polygon of constraints (Unbound).

Example: An orchestra has no more than 30 members.
There are at least twice as many musicians who play string instruments as musicians who play wind instruments.
Graph the solution set.


y
50
45
40
35
30
25
20
15
10
5
x
0
5
10
15
20
25
30
35
40
45
50

Objective Function

A system of linear inequalities has many solutions. Depending on the situation, some of these solutions (usually one) are better than the others and will be called the optimal solution . What determines this optimal solution is the objective function or objective rule .

Example:

A farmer grows cherries & raspberries on a piece of land that is at most $16ha$ in area. Each hectare of cherries requires 5 days of work and each hectare of raspberries, 3 days of work. The farmer has no more than 60 days available. He decides that the space for raspberries will be at most 3 times the amount of space for cherries. Each hectare of cherries and rasperries produces revenues of $\$3000$ and $\$5000$ respectively. What is the maximum revenue the farmer can earn?

Variables:

Constraints:

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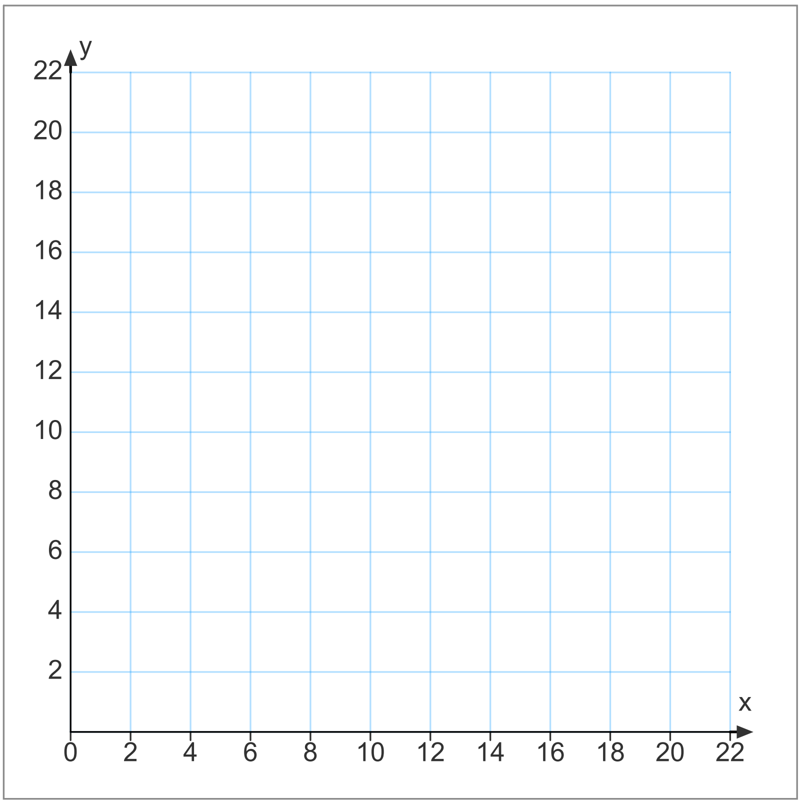
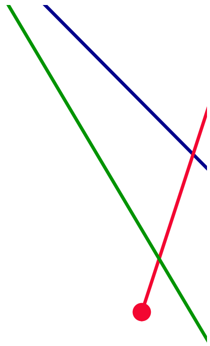
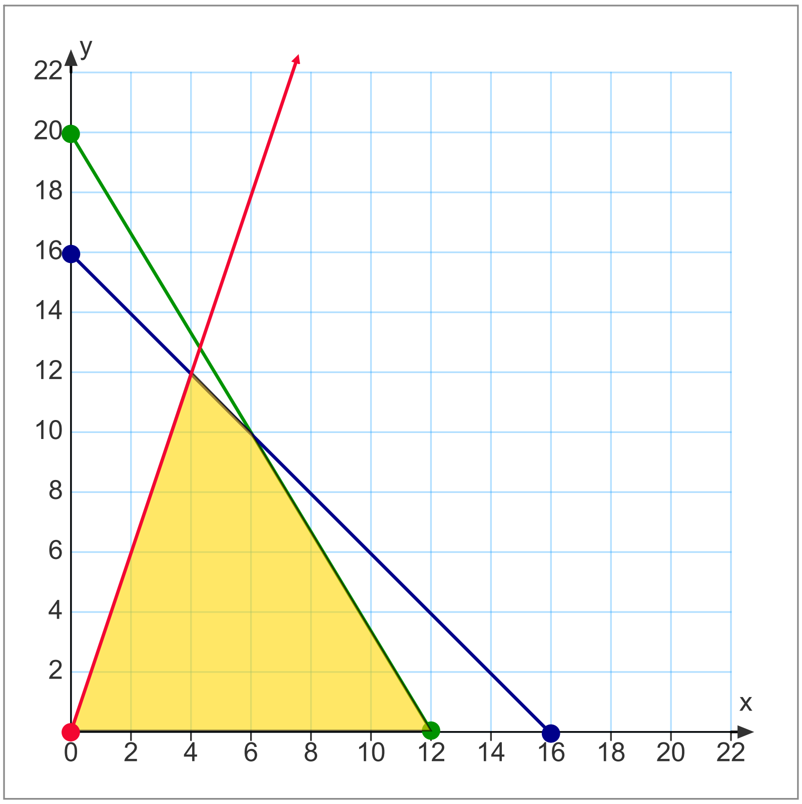
The farmer's objective is to make the maximum revenue possible given the constraints.

The objective rule for this farmer is


y
22
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
18
20
22
x





$$R = 3000x + 5000y$$

Points	Revenue

The optimal solutions (maximum or minimum) occur on the boundary of the solution set and usually occur only at the vertices.

Solving Optimisation Problems

1. Define the variables.
2. List the constraints.
3. Write the objective function.
4. Graph the polygon of constraints.
5. Determine the coordinates of the vertices of the polygon.
6. Identify the optimal solution - the maximum or minimum that solve the problem.

Example:

Joan wants to give at least 12 chocolates to her children at Easter. She intends to buy at least twice as many dark chocolates as milk chocolates, but no more than 20 dark chocolates.

One milk chocolate costs \$2.00 and one dark chocolate costs \$4.00.

How many of each type of chocolate should Joan buy in order to minimise her costs ?

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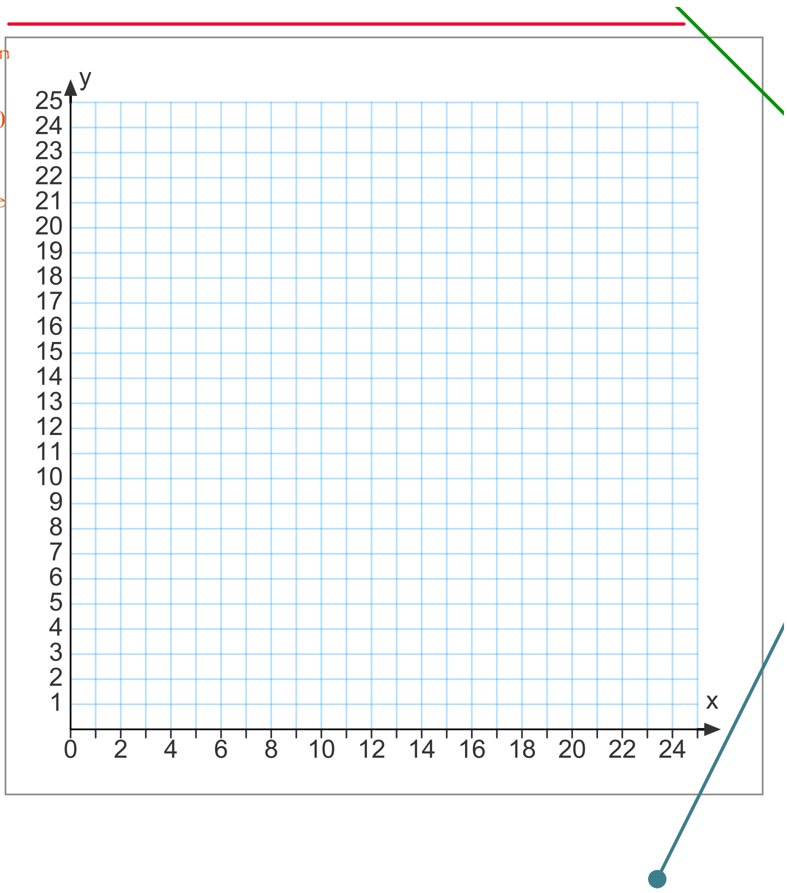
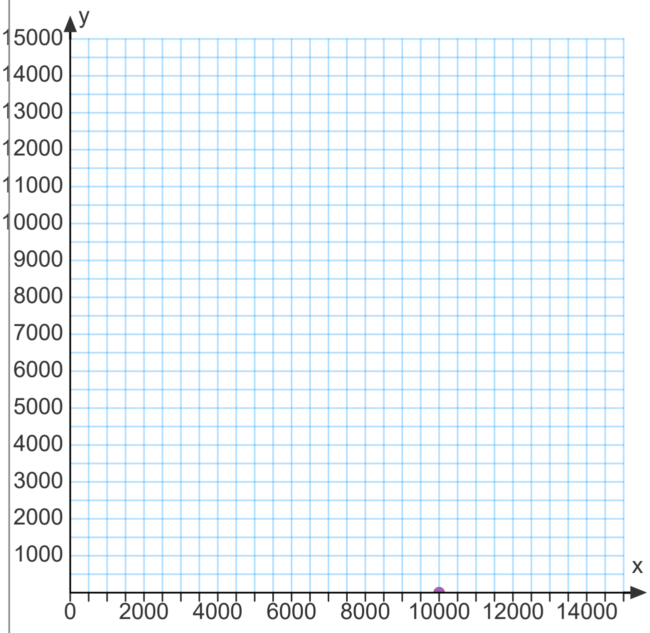
Vertices	Cost $C = 4x + 2y$



Example:

A chicken and turkey breeder raises fewer than 10 000 birds each year. She produces at least 4 times as many chickens as turkeys. Last year's sales encourage her to raise at least 6000 chickens and 1000 turkeys. Her profits are \$2.00 per chicken and \$7.00 per turkey.

How many of each type should she raise this year to maximise profits?



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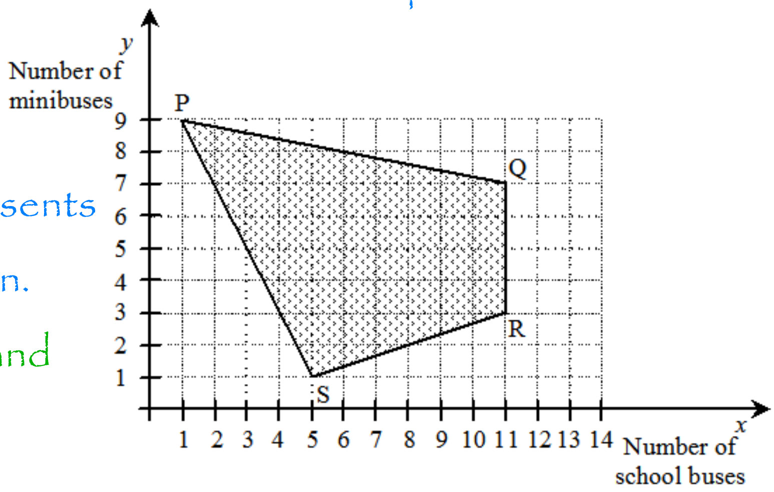
Vertices	$P = 2x + 7y$

When your optimal solution can't be used (vertex falls on a dotted line), choose a point inside the polygon that is close to that vertex.

Example: A school wants to minimize the transportation costs involved in taking students on a field trip. The following polygon of constraints represents the solutions for this situation.



Each school bus costs $80 and each minibus costs $40. The school needs to determine the minimum transportation cost.

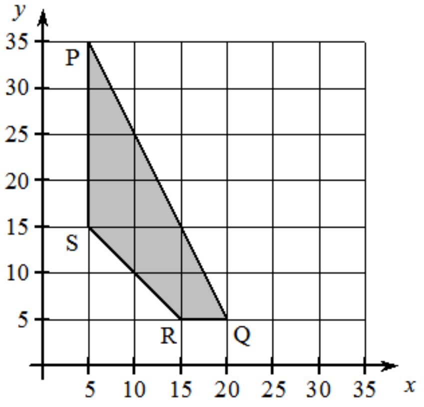
In this situation, how many solutions minimize the transportation cost involved in taking students on this field trip?

Vertices of the polygon of constraints	$C = 80x + 40y$

If the optimal solution (maximum or minimum) is achieved at two consecutive vertices (say P and S), then each point along the edge PS of the polygon of constraints is also a possible optimal solution.

$\therefore$ Number of solutions: ______

Example: Vincent works for a company that makes shelving units. Each week, he divides his time between assembly work and finishing work. The polygon of constraints represents the different constraints that Vincent faces.



Vertices
P(5, 35)
Q(20, 5)
R(15, 5)
S(5, 15)

x : number of hours spent on assembly work each week

y: number of hours spent on finishing work each week

Vincent is told that from now on he faces the following additional constraint: the number of hours spent on finishing work must be less than or equal to the number of hours spent on assembly work. He makes $10 an hour for assembly work and $8 an hour for finishing work.

By how many dollars does this constraint decrease Vincent's maximum possible weekly income?
