

Difference of squares

$$\begin{aligned}
 1. \quad \frac{(x-1)^2 - 9}{(x-4)} &= \frac{(x-1+3)(x-1-3)}{(x-4)} \\
 &= \frac{(x+2)(x-4)}{(x-4)} \\
 &= \boxed{x+2}
 \end{aligned}$$

2. Area of square = x^2 Area of rectangle = $2x^2 - 7x - 30$

$$\textcircled{1} \quad x^2 = 2x^2 - 7x - 30$$

$$x^2 - x^2 = 2x^2 - x^2 - 7x - 30$$

$$0 = x^2 - 7x - 30$$

(a) factor :

$$0 = (x-10)(x+3)$$

$$x-10=0 \quad \text{or} \quad x+3=0$$

$$x=10 \quad \text{or} \quad x=-3$$

$$x > 0$$

$$\therefore x=10$$

b) quadratic formula

$$x = \frac{-7 \pm \sqrt{49 - 4(1)(-30)}}{2}$$

$$x = \frac{-7 \pm \sqrt{169}}{2}$$

$$x = \frac{-7 \pm 13}{2}$$

$$\begin{aligned}
 x_1 &= \frac{-7 + 13}{2} \\
 &= \frac{20}{2} \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 x_2 &= \frac{-7 - 13}{2} \\
 &= -\frac{20}{2} \\
 &= -3
 \end{aligned}$$

$$x = 10$$

② Area = $2x^2 - 7x - 30$ P = ?

$$= L \times W$$

factor $2x^2 - 7x - 30$

$$a \cdot c = -60 \quad (\text{mn} \cdot n)$$

$$b = -7 \quad (\text{m+n})$$

$$-12, 5$$

$$2x^2 - 12x + 5x - 30$$

$$2x(x-6) + 5(x-6)$$

$$(x-6)(2x+5)$$

$$x=10$$

$$\boxed{\begin{array}{|c|c|} \hline 2x+5 & \\ \hline \end{array}}_{x=6}$$

$$\boxed{\begin{array}{|c|c|} \hline 25 & \\ \hline 4 & \\ \hline \end{array}}$$

$$\begin{aligned}
 \therefore P &= 2(25) + 2(4) \\
 &= \boxed{58 \text{ cm}}
 \end{aligned}$$

$$3. \quad \begin{array}{r} 2x+3 \\ \hline x-4) 2x^2 - 5x - 12 \\ \underline{- (2x^2 - 8x)} \\ 3x - 12 \\ \underline{- (3x - 12)} \\ 0 \end{array}$$

Answer : $2x+3$

$$\text{OR} \quad \begin{aligned} & \frac{2x^2 - 5x - 12}{x-4} \quad \begin{array}{l} ac = -24 \\ b = -5 \\ \hline -8, 3 \end{array} \\ &= \frac{2x^2 - 8x + 3x - 12}{x-4} \\ &= \frac{2x(x-4) + 3(x-4)}{x-4} \\ &= \frac{(x-4)(2x+3)}{(x-4)} \\ &= 2x+3 \end{aligned}$$

$$4. \quad \text{Square APQD : Area} = x^2 \quad \text{Rectangle PBQC : Area} = x(x+8) \\ = x^2 + 8x$$

$$\begin{aligned} \text{Rectangle ABCD : Area} &= 120 = x^2 + x^2 + 8x \\ 120 &= 2x^2 + 8x \\ 0 &= 2x^2 + 8x - 120 \\ \frac{0}{2} &= \frac{2x^2 + 8x - 120}{2} \\ 0 &= x^2 + 4x - 60 \end{aligned}$$

$$\text{factor: } x^2 + 4x - 60 = 0$$

$$(x+10)(x-6) = 0$$

$$x+10 = 0 \quad \text{or} \quad x-6 = 0$$

$$x = -10 \quad x = 6$$

$$x > 0 \quad \therefore x = 6$$

$$\begin{aligned} \text{Rectangl. PBQC} : A &= x(x+8) \\ &= 6(14) \\ &= 84 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{x+5}{\cancel{x^2-16}} + \frac{3}{x-4} = \frac{x+5}{(x+4)(x-4)} + \frac{3}{x-4} \\
 & \text{difference of squares} \\
 & = \frac{x+5}{(x+4)(x-4)} + \frac{3}{x-4} \left(\frac{x+4}{x+4} \right) \\
 & = \frac{x+5}{(x+4)(x-4)} + \frac{3x+12}{(x+4)(x-4)} \\
 & = \frac{4x+17}{(x+4)(x-4)} \stackrel{\text{or}}{=} \frac{4x+17}{x^2-16}
 \end{aligned}$$

Answer: B

$$\begin{array}{r}
 6. \quad \frac{2c^2 - 5c + 1}{c+3} \\
 \underline{- (2c^3 + 6c^2)} \\
 \underline{- 5c^2 - 14c} \\
 \underline{- (-5c^2 - 15c)} \\
 \underline{c+3} \\
 \underline{- (c+3)} \\
 \underline{0}
 \end{array}$$

Answer: $2c^2 - 5c + 1$

$$7. \quad \text{Triangle: Area} = \frac{(x)(3x-12)}{2} \quad \text{Rectangle: Area} = (15)(?)$$

$$\textcircled{1} \quad \text{Triangle: } x^2 + (3x-12)^2 = 52^2 \quad (\text{Pythagoras' Theorem})$$

$$x^2 + 9x^2 - 72x + 144 = 2704$$

$$10x^2 - 72x + 144 = 2704$$

$$10x^2 - 72x - 2560 = 0$$

$$x = \frac{72 \pm \sqrt{5184 - 4(10)(-2560)}}{20}$$

$$x = \frac{72 \pm \sqrt{107584}}{20}$$

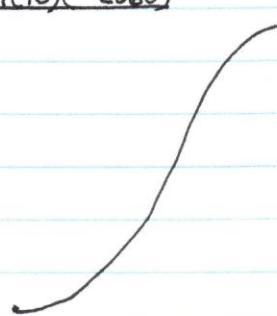
$$x = \frac{72 \pm 328}{20}$$

$$x_1 = \frac{72 + 328}{20} = \frac{400}{20}$$

$$x_2 = \frac{72 - 328}{20} = \frac{-256}{20}$$

$$x = \{20, -12.8\} \quad x > 0$$

$$\therefore x = 20$$



Hilary

$$7. \quad \textcircled{2} \quad \text{Area of Triangle} = \frac{x(3x-12)}{2} = \frac{20(60-12)}{2}$$

$$= \frac{20 \cdot 48}{2}$$

$$= 480 \text{ cm}^2$$

$$\textcircled{3} \quad \text{Area of Rectangle} = 15(\text{Length of Base})$$

$$15(L) = 480$$

$$\text{Length of Base} = \frac{480}{15}$$

$$= 32 \text{ cm}$$

grouping

$$8. \quad \frac{a^3b + 4a^2b - ab - 4b}{a^2 - 1} = \frac{b(a^3 + 4a^2 - a - 4)}{a^2 - 1}$$

*difference
of squares*

$$= b \left[\frac{a^3 - a + 4a^2 - 4}{(a+1)(a-1)} \right]$$

$$= b \left[\frac{a(a^2 - 1) + 4(a^2 - 1)}{(a+1)(a-1)} \right]$$

$$= b \left[\frac{\cancel{a^2-1}(a+4)}{\cancel{a^2-1}} \right]$$

$$= b(a+4)$$

$$9. \quad \frac{6ab - 15a + 12b - 30}{6b - 15} = \frac{3(2ab - 5a + 4b - 10)}{3(2b - 5)}$$

$$= \frac{a(2b - 5) + 2(2b - 5)}{2b - 5}$$

$$= \frac{(2b - 5)(a + 2)}{2b - 5}$$

$$= a + 2$$

$$10. \frac{6x^3y^3 - 11x^2y^2 + 18xy - 5}{3xy - 1}$$

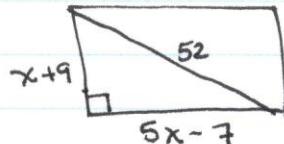
$$\begin{aligned}
 & \frac{2x^2y^2 - 3xy + 5}{3xy - 1} \\
 & 3xy - 1 \overline{)6x^3y^3 - 11x^2y^2 + 18xy - 5} \\
 & \quad - (6x^3y^3 - 2x^2y^2) \\
 & \quad - 9x^2y^2 + 18xy \\
 & \quad - (-9x^2y^2 + 3xy) \\
 & \quad 15xy - 5 \\
 & \quad - (15xy - 5) \\
 & \quad 0
 \end{aligned}$$

Answer: $2x^2y^2 - 3xy + 5$

$$11. \text{ Area} = 5x^2 + 38x - 63 \quad \left. \begin{array}{l} a \cdot c = -315 \\ b = 38 \end{array} \right\} 45, -7$$

$$5x^2 + 45x - 7x - 63$$

$$\begin{aligned}
 & 5x(x+9) - 7(x+9) \\
 & (x+9)(5x-7)
 \end{aligned}$$



$$(x+9)^2 + (5x-7)^2 = 52^2$$

$$x^2 + 18x + 81 + 25x^2 - 70x + 49 = 2704$$

$$26x^2 - 52x + 130 = 2704$$

$$26x^2 - 52x - 2574 = 0$$

$$\frac{26x^2 - 52x - 2574}{26} = 0$$

$$x^2 - 2x - 99 = 0$$

$$(x-11)(x+9) = 0$$

$$x-11=0 \text{ or } x+9=0$$

$$x=11 \quad x=-9$$

length, width > 0

$$\therefore x=11$$

$$\begin{array}{c}
 x=11 \\
 20 \boxed{ } \checkmark \\
 48
 \end{array}$$

$$\begin{array}{c}
 x=-9 \\
 0 \boxed{ } \\
 -52
 \end{array}$$

$$\begin{aligned}
 \therefore \text{Perimeter ABCD} &= 2(20) + 2(48) \\
 &= 40 + 96 \\
 &= 136 \text{ cm}
 \end{aligned}$$

grouping

$$12. \frac{a^2 - 4a + ab - 4b}{a^2 - 16} = \frac{a(a-4) + b(a-4)}{(a+4)(a-4)} = \frac{(a-4)(a+b)}{(a+4)(a-4)}$$

difference of squares

$$= \frac{a+b}{a+4}$$

Answer : C

13. a) $18x + 20$
 $2(9x + 10)$

b) $12x + 4y$
 $4(3x + y)$

c) $20xy + 16x$
 $4x(5y + 4)$

d) $m^8 + m^6$
 $m^6(m^2 + 1)$

e) $6x^2 - 2x + 4$
 $2(3x^2 - x + 2)$
 $\begin{matrix} a \cdot c = 6 \\ b = -1 \end{matrix}$
 $2(3x^2 - x + 2)$

f) $a^2 - ab - a$
 $a(a - b - 1)$

g) $7x^5y^2 + 21x^2y^3 + 14xy^4$
 $7xy^2(x^4 + 3xy + 2y^2)$

h) $x(m+n) + y(m+n)$
 $(m+n)(x+y)$

i) $c(a+b) - (a+b)$
 $(a+b)(c-1)$

j) $6x^2(3x-2) + 2x(3x-2) - 4(3x-2)$
 $(3x-2)(6x^2 + 2x - 4)$
 $(3x-2)(2)(3x^2 + x - 2)$
 $\underbrace{(3x-2)(2)(3x^2 + 3x - 2x - 2)}_{ac = -6, b = 1}$
 $(3x-2)(2)(3x(x+1) - 2(x+1))$
 $2(3x-2)(x+1)(3x-2)$
 $2(3x-2)^2(x+1)$

k) $ax + x^2 + ay + xy$
 $x(a+x) + y(a+x)$
 $(a+x)(x+y)$

l) $5ax + bx - 10ay - 14by$
 $x(5a+b) - 2y(5a + 7b)$
 $5ax - 10ay + bx - 14by$
 $5a(x-2y) + b(x-14y)$

m) $x^4 + x^3 + x^2 + x$
 $x(x^3 + x^2 + x + 1)$
 $x[x^2(x+1) + 1(x+1)]$
 $x(x+1)(x^2+1)$

$5ax + bx - 10ay - 14by$

Hilary

$$n) 15xy + 20y^2 - 18x - 24y$$

$$o) \frac{4a^2 - 3a - 4ab + 3b}{12a - 9}$$

$$p) y^2 - 1$$

$$(y+1)(y-1)$$

$$5y(3x+4y) - 6(3x+4y)$$

$$(3x+4y)(5y-6)$$

$$\underline{a(4a-3) - b(4a-3)}$$

$$3(4a-3)$$

$$\frac{(4a-3)(a-b)}{3(4a-3)}, a \neq 3/4$$

$$\frac{a-b}{3}$$

$$q) x^8 - 9$$

$$(x^4 + 3)(x^4 - 3)$$

$$r) 16 - 25x^2$$

$$(4 + 5x)(4 - 5x)$$

$$s) xyz - x^3yz$$

$$xyz(1 - x^2)$$

$$xyz(1+x)(1-x)$$

$$t) x^3 - 9y + ax^2 - 9a$$

$$y(x^2 - a) + a(x^2 - a)$$

$$(x^2 - a)(y + a)$$

$$(x+3)(x-3)(y+a)$$

$$u) (a+b)^2 - 16$$

$$(a+b+4)(a+b-4)$$

$$v) (3x+y)^2 - (x-y)^2$$

$$(3x+y + (x-y))(3x+y - (x-y))$$

$$(4x)(2x+2y)$$

$$(4x)(2)(x+y)$$

$$w) x^2 + 2x + 1$$

$$(x+1)(x+1)$$

$$(x+1)^2$$

$$x) x^2 + 5x + 6$$

$$(x+2)(x+3)$$

$$y) x^2 + 4x - 12$$

$$(x+6)(x-2)$$

$$z) x^2 + 4x + 5$$

$$x^2 + 4x + 5$$

$$14) a) 12xy^4 - 64x^3y^3 + 40x^2y^2$$

$$4xy^2(3y^2 - 16x^2y + 10x)$$

$$b) 4x^2 + 28x - 120$$

$$4(x^2 + 7x - 30)$$

$$4(x+10)(x-3)$$

$$c) 25b^2 - 3ba^4$$

$$(5b + 6a^2)(5b - 6a^2)$$

$$d) 6x + yx - yz - bz$$

$$x(6 + y) - z(y + 6)$$

$$(y+6)(x-z)$$

14

e) $x^2 - x - 6$

$(x-3)(x+2)$

f) $4x^2 + 12x + 9$

$4x^2 = (2x)^2 \quad 9 = 3^2$

$12x = 2 \cdot 2x \cdot 3$

$(2x+3)^2$

g) $2x^2 + 15x + 7$

$$\begin{array}{l} a \cdot c = 14 \\ b = 15 \end{array} \quad \left. \begin{array}{l} 14, 1 \\ 1, 14 \end{array} \right\}$$

$2x^2 + 14x + 1x + 7$

$2x(x+7) + 1(x+7)$

$(x+7)(2x+1)$

h) $8x^2 - 26x + 15$

$$\begin{array}{l} a \cdot c = 120 \\ b = -26 \end{array} \quad \left. \begin{array}{l} 120, -10 \\ -10, 120 \end{array} \right\}$$

$8x^2 - 6x - 20x + 15$

$2x(4x-3) - 5(4x-3)$

$(4x-3)(2x-5)$

i) $18a^2 + 3ab - b^2$

$$\begin{array}{l} a \cdot c = -18 \\ b = 3 \end{array} \quad \left. \begin{array}{l} 6, -3 \\ -3, 6 \end{array} \right\}$$

$18a^2 + 6ab - 3ab - b^2$

$6a(3a+b) - b(3a+b)$

$(3a+b)(6a-b)$