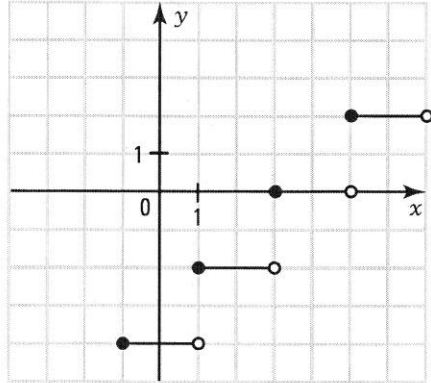


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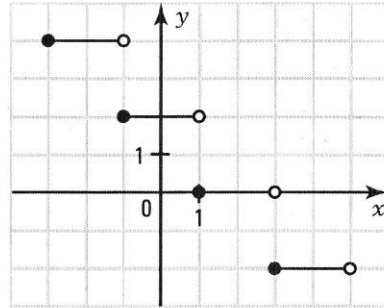
1. Represent the following functions and determine the set S of zeros.

a) $f(x) = 2\left[\frac{1}{2}(x - 1)\right] - 2$



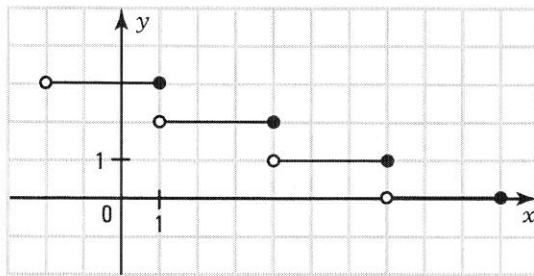
$S = [3, 5[$

b) $f(x) = -2\left[\frac{1}{2}(x + 1)\right] + 2$



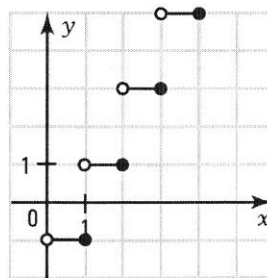
$S = [1, 3[$

c) $f(x) = \left[-\frac{1}{3}(x-1)\right] + 3$



$S =]7, 10]$

d) $f(x) = -2[-(x - 2)] + 1$



$S = \emptyset$

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2. The following rules define greatest integer functions. Write them in the form $y = a[b(x - h)] + k$.

a) $y = [2x - 1]$ $y = \left[2\left(x - \frac{1}{2}\right)\right]$ b) $y = -2[3x - 6]$ $y = -2[3(x - 2)]$

c) $y = \left[\frac{x-3}{2}\right]$ $y = \left[\frac{1}{2}(x-3)\right]$ d) $y = \left[\frac{x}{3} - 1\right]$ $y = \left[\frac{1}{3}(x-3)\right]$

3. Which geometric transformations apply the basic greatest integer function to the following functions?

- a) $f(x) = [x - 5]$ Horizontal translation of 5 units to the right.
- b) $f(x) = [x] + 2$ Vertical translation of 2 units upward.
- c) $f(x) = [x + 3] - 1$ Horizontal translation of 3 units to the left, followed by a vertical translation of one unit downward.
- d) $f(x) = -[x - 2]$ Horizontal translation of 2 units to the right, followed by a reflection about the x-axis.
- e) $f(x) = [-2x]$ Horizontal scale change followed by a reflection about the y-axis.

4. For each of the following greatest integer functions, determine

1. the length of one step and its type ($\bullet \rightarrow \circ$) or ($\circ \rightarrow \bullet$).
2. the counterstep height.
3. the set S of zeros.
4. the y -intercept.
5. the variation of the function.

a) $y = 2[-3(x - 1)] + 4$

1. length: $\frac{1}{3}$; $\circ \rightarrow \bullet$
2. height: 2
3. $S = \left[\frac{4}{3}, \frac{5}{3}\right]$
4. y -int.: 10
5. decreasing function

b) $y = \left\lfloor \frac{1}{2}(x + 1) \right\rfloor + 6$

1. length: 2; $\bullet \rightarrow \circ$
2. height: 2
3. $S = [-1, 1[$
4. y -int.: 0
5. increasing function

c) $y = 3[2(x + 1)] - 5$

1. length: $\frac{1}{2}$; $\bullet \rightarrow \circ$
2. height: 3
3. $S = \emptyset$
4. y -int.: 1
5. increasing function

d) $y = \frac{1}{2}[-4(x + 1)] + 2$

1. length: $\frac{1}{4}$; $\circ \rightarrow \bullet$
2. height: $\frac{1}{2}$
3. $S = \left[-\frac{1}{4}, 0\right]$
4. y -int.: 0
5. decreasing function

5. Determine the domain and range of the following functions.

a) $y = -3[5(x + 2)] - 7$

dom = \mathbb{R} ; ran = $\{y \mid y = -3m - 7, m \in \mathbb{Z}\}$

b) $y = \frac{1}{2}[-3(x - 1)] + 4$

dom = \mathbb{R} ; ran = $\{y \mid y = \frac{1}{2}m + 4, m \in \mathbb{Z}\}$

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6. Find the set of values of x for which

1. $f(x) \geq 0$.

a) $f(x) = 3[3(x - 1)] + 2$

1. $x \in [1, +\infty[$
2. $x \in [-\infty, 1]$

2. $f(x) < 0$.

b) $f(x) = -3\left[\frac{1}{3}x + 2\right] + 6$

1. $x \in]-\infty, 3[$
2. $x \in [3, +\infty[$

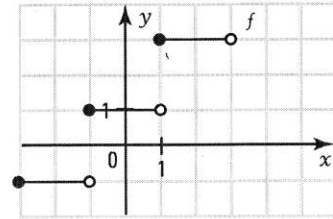
c) $f(x) = -2\left[\frac{1}{4}(x - 1)\right] - 4$

1. $x \in]-\infty, -3[$
2. $x \in [-3, +\infty[$

d) $f(x) = 3\left[\frac{-1}{2}(x + 2)\right] - 4$

1. $x \in]-\infty, -6[$
2. $x \in [-6, +\infty[$

7. Consider the function $f(x) = 2\left[\frac{1}{2}(x - 1)\right] + 3$ represented on the right.



- a) Represent the function $g(x) = 2\left[\frac{1}{2}(x + 1)\right] + 1$.

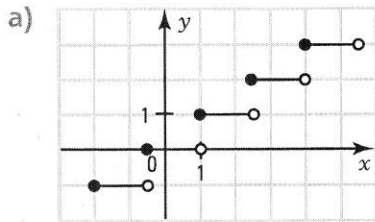
What do you notice?

The graph of g coincides with the graph of f .

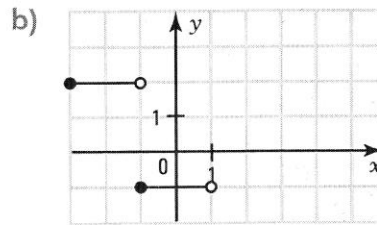
- b) Find the rule of a function h with the same graph as f .

For example, $h(x) = 2\left[\frac{1}{2}(x + 3)\right] - 1$

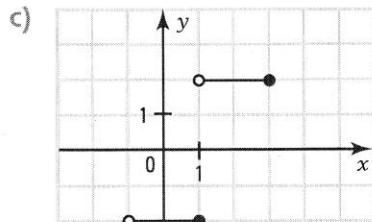
8. For each of the step functions represented below, find a rule corresponding to the function.



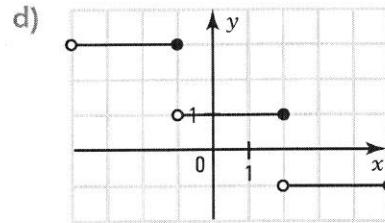
$$y = \left[\frac{2}{3}(x - 1)\right] + 1$$



$$y = -3\left[\frac{1}{2}(x + 1)\right] - 1$$



$$y = -4\left[-\frac{1}{2}(x - 1)\right] - 2$$

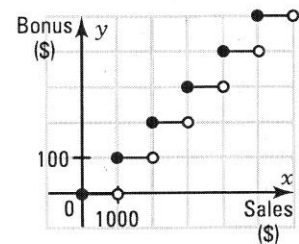


$$y = 2\left[-\frac{1}{3}(x - 2)\right] + 1$$

9. To motivate his salesmen, a sales manager awards a bonus of \$100 for every \$1000 in sales.

- a) Draw the Cartesian graph of the function which gives the awarded bonus y as a function of the amount of sales x .

- b) What is the rule of the function? $y = 100\left[\frac{x}{1000}\right] \quad (x \geq 0)$



10. A salesman receives a weekly base salary of \$150 and a \$50 bonus for every \$1000 in weekly sales.

a) Find the rule of the function which gives the weekly salary y as a function of the amount x of weekly sales.

$$y = 50 \left\lfloor \frac{x}{1000} \right\rfloor + 150$$

b) A salesman sells \$12 480 of merchandise in a week. What will his salary be?
\$750

c) For what amount of sales will the salesman receive a salary of \$1000?

An amount within the interval [17 000, 18 000[

d) Is it possible for a salesman to receive a salary equal to \$825?

$$50 \left\lfloor \frac{x}{1000} \right\rfloor + 150 = 825 \Leftrightarrow \left\lfloor \frac{x}{1000} \right\rfloor = 13.5$$

The last equation has no solution since $13.5 \notin \mathbb{N}$.

11. The cost y , in dollars, of mailing a package depends on its mass x , in grams. This cost is defined by the rule $y = -2.5 \left\lfloor -\frac{x}{100} \right\rfloor$.

a) What is the cost of mailing a package with a mass of 260 g? **\$7.50**

b) What is the mass of a package that costs \$12.50 to mail?

$$x \in]400, 500]$$

c) Explain in your words how to calculate the cost of mailing a package.

It costs \$2.50 for 100 g or less and \$2.50 more for every additional 100 g.

12. At a parking lot, the cost y of parking is calculated as follows: a minimum cost of \$2 for a parking time of less than 30 min. In addition, \$1.50 is charged for every 30 minute interval of parking time.

a) Draw the graph of the function which gives the total cost y , in dollars, of parking as a function of the parking time x , in hours.

b) Find the rule of the function.

$$y = 1.5[2x] + 2$$

c) What is the parking time corresponding to a cost of \$8?

$$x \in [2, 2.5[$$

