

6. The following functions have the rule $f(x) = a \sin b(x - h) + k$.

Find the zeros of each function over

- the interval $[h, h + p]$ where p is the period of the function.
- the set of all real numbers.

a) $f(x) = 2 \sin \frac{\pi}{6}(x - 2) + 1$ $p = \frac{2\pi}{\pi/6} = 12$

$$\sin \frac{\pi}{6}(x - 2) = -\frac{1}{2}$$

$$\frac{\pi}{6}(x - 2) = \frac{7\pi}{6} \text{ or } \frac{\pi}{6}(x - 2) = \frac{11\pi}{6}$$

$$x = 9 \text{ or } x = 13$$

$$1. S = \{9, 13\} \text{ over } [2, 14]$$

$$2. S = \{9 + 12n\} \cup \{13 + 12n\}$$

b) $f(x) = -2 \sin \frac{\pi}{3}(x + 1) + \frac{1}{2}$ $p = \frac{2\pi}{\pi/3} = 6$

$$\sin \frac{\pi}{3}(x + 1) = 0.25$$

$$\frac{\pi}{3}(x + 1) = 0.25 \text{ or } \frac{\pi}{3}(x + 1) = 2.89$$

$$x = -0.76 \text{ or } x = 1.76$$

$$1. S = \{-0.76; 1.76\} \text{ over } [-1, 5]$$

$$2. S = \{-0.76 + 6n\} \cup \{1.76 + 6n\}$$

c) $f(x) = \sin 2(x - \pi) + 1$ $p = \frac{2\pi}{2} = \pi$

$$\sin 2(x - \pi) = -1$$

$$2(x - \pi) = \frac{3\pi}{2}$$

$$x = \frac{7\pi}{4}$$

$$1. S = \left\{\frac{7\pi}{4}\right\} \text{ over } [\pi, 2\pi]$$

$$2. S = \left\{\frac{7\pi}{4} + \pi n\right\}$$

d) $f(x) = 6 \sin\left(x + \frac{\pi}{2}\right) - 3$ $p = \frac{2\pi}{1} = 2\pi$

$$\sin\left(x + \frac{\pi}{2}\right) = \frac{1}{2}$$

$$x + \frac{\pi}{2} = \frac{\pi}{6} \text{ or } x + \frac{\pi}{2} = \frac{5\pi}{6}$$

$$x = -\frac{\pi}{3} \text{ or } x = \frac{\pi}{3}$$

$$1. S = \left[-\frac{\pi}{3}, \frac{\pi}{3}\right] \text{ over } \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$2. S = \left[-\frac{\pi}{3} + 2\pi n\right] \cup \left[\frac{\pi}{3} + 2\pi n\right]$$

e) $f(x) = -3 \sin \frac{\pi}{4}x + 6$ $p = \frac{2\pi}{\pi/4} = 8$

$$\sin \frac{\pi}{4}x = 2$$

$$1. S = \emptyset$$

$$2. S = \emptyset$$

f) $f(x) = -2 \sin \frac{\pi}{8}(x + 2) + \sqrt{2}$ $p = \frac{2\pi}{\pi/8} = 16$

$$\sin \frac{\pi}{8}(x + 2) = \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{8}(x + 2) = \frac{\pi}{4} \text{ or } \frac{\pi}{8}(x + 2) = \frac{3\pi}{4}$$

$$1. S = \{0, 4\} \text{ over } [-2, 14]$$

$$2. S = \{0 + 16n\} \cup \{4 + 16n\}$$

7. Determine the zeros of the function $f(x) = -2 \sin \frac{\pi}{12}(x + 5) - 1$ over the interval $[90, 150]$.

$$\sin \frac{\pi}{12}(x + 5) = -\frac{1}{2}$$

$$\frac{\pi}{12}(x + 5) = \frac{7\pi}{6} \text{ or } \frac{\pi}{12}(x + 5) = \frac{11\pi}{6}$$

$$x = 9 \text{ or } x = 17$$

The zeros are: 105, 113, 129, 137.

- a)** $f(x) = -2 \sin \frac{\pi}{8}(x - 5) + 3$

1 $p = 16$

2. $A = 2$

3. $\text{ran } f = [1, 5]$

c) $f(x) = 5 \sin \frac{4\pi}{3}(x+1) - 4$

1. $p = \frac{3}{2}$

2 **A = 5**

$$3. \quad \text{ran } f = [-9, 1]$$

b) $f(x) = 3 \sin 12\left(x + \frac{\pi}{2}\right) + 5$

$$1. \quad p = \frac{\pi}{6}$$

$$A = 3$$

$$3 \quad \text{ran } f = [2, 8]$$

d) $f(x) = 10 \sin \frac{6}{5} \left(x - \frac{\pi}{4} \right) + 4$

$$1. \quad p = \frac{5\pi}{3}$$

$$A = 10$$

$$3 \quad \text{ran } f = [-6, 14]$$

- a) $f(x) = 4 \sin \frac{\pi}{6}(x+1) - 3$ -1

b) $f(x) = -2 \sin \frac{\pi}{3}(x-2) + 2$ $\sqrt{3} + 2$

c) $f(x) = 2 \sin 2\left(x - \frac{\pi}{4}\right) + 4$ 2

d) $f(x) = 3 \sin \pi(x + 5) - 1$ -

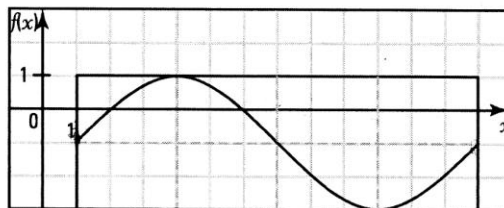
- a)** $f(x) = 2 \sin \frac{\pi}{6}(x-1) - 1$

Zeros: $\sin \frac{\pi}{6}(x - 1) = \frac{1}{2}$

$$\frac{\pi}{6}(x - 1) = \frac{\pi}{6} \text{ or } \frac{\pi}{6}(x - 1) = \frac{5\pi}{6}$$

$x = 2$ $x = 6$

$$f(x) \geq 0 \text{ over } [2 + 12n, 6 + 12n]$$



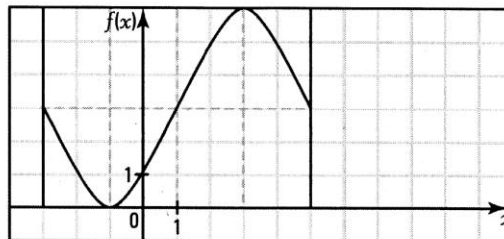
b) $f(x) = -3 \sin \frac{\pi}{4}(x + 3) + 3$

Zeros: $\sin \frac{\pi}{4}(x + 3) = 1$

$$\frac{\pi}{4}(x + 3) = \frac{\pi}{2}$$

$$x = -1$$

$$f(x) \geq 0 \text{ over } \mathbb{R}$$



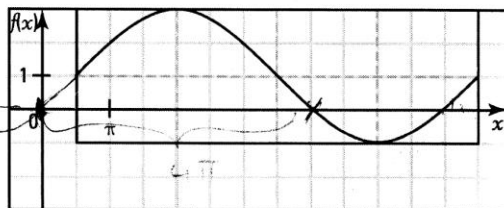
c) $f(x) = 2 \sin \frac{1}{3} \left(x - \frac{\pi}{2} \right) + 1$

Zeros: $\sin \frac{1}{3} \left(x - \frac{\pi}{2} \right) = -\frac{1}{2}$

$$\frac{1}{3}\left(x - \frac{\pi}{2}\right) = \frac{7\pi}{6} \text{ or } \frac{1}{3}\left(x - \frac{\pi}{2}\right) = \frac{11\pi}{6}$$

$$x = 4\pi \qquad x = 6\pi$$

$$f(x) \geq 0 \text{ over } \left[-\frac{1}{2} + 6\pi n, 4\pi + 6\pi n \right] \cup \left[6\pi + 6\pi n, \frac{3}{2} + 6\pi n \right]$$

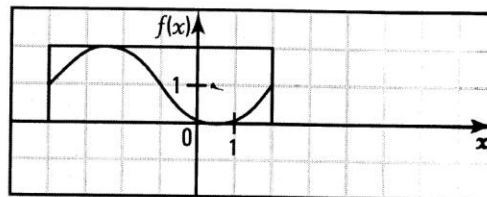


$$[0 + 6\pi n, 4\pi + 6\pi n] \\ [0, 4\pi] + 6\pi n, n \in \mathbb{Z} \quad \text{© Guén}$$

- 11.** For each of the following functions, determine the interval over which the function is decreasing.

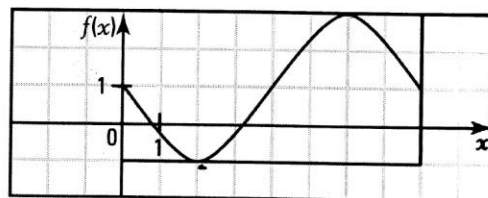
a) $f(x) = \sin \frac{\pi}{3}(x + 4) + 1$ dans $[-4, 2]$.

f is over $[-2.5, 0.5]$



b) $f(x) = -2 \sin \frac{\pi}{4}x + 1$ over $[0, 8]$

f is over $[0, 2] \cup [6, 8]$



- 12.** In an experiment, we define a function I which gives the electrical current across a cable, expressed in amperes, as a function of time x , expressed in seconds. The function I is defined by the rule: $I(x) = 4 \sin \frac{\pi}{6}(x + 8) + 4$.

A light bulb lights up when the intensity of the current is equal to 6 amperes.

Determine at what times the light bulb lights up during the first 30 seconds of the experiment.

$$4 \sin \frac{\pi}{6}(x + 8) + 4 = 6$$

The light bulb lights up at the moments

$$\sin \frac{\pi}{6}(x + 8) = \frac{1}{2}$$

5 s, 9 s, 17 s, 21 s and 29 s.

$$\frac{\pi}{6}(x + 8) = \frac{\pi}{6} \text{ or } \frac{\pi}{6}(x + 8) = \frac{5\pi}{6}$$

$$x = -7$$

$$x = -3$$

- 13.** In a company, the number of parts assembled varies according to a sinusoidal function defined by the rule: $f(x) = 200 \sin \left(\frac{\pi}{24}x \right) + 15$ where x represents the number of days elapsed since the beginning of the year.

Over the course of the first 50 days, for how many days was the number of parts assembled greater than or equal to 115?

$$200 \sin \left(\frac{\pi}{24}x \right) + 15 = 115$$

$$\sin \frac{\pi}{24}x = \frac{1}{2}$$

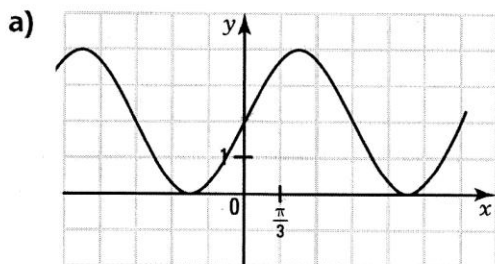
$$\frac{\pi}{24}x = \frac{\pi}{6} \text{ or } \frac{\pi}{24}x = \frac{5\pi}{6}$$

During 17 days, from the 4th day to the 20th day, inclusively.

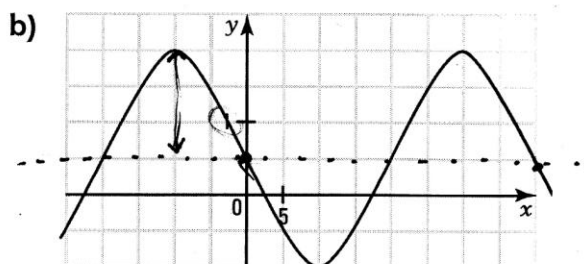
$$x = 4$$

$$x = 20$$

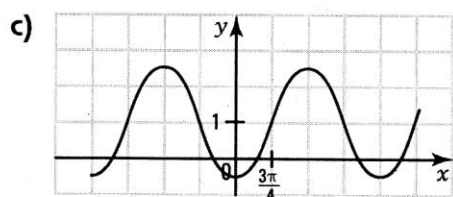
- 14.** Find a rule of the form $y = a \sin b(x - h) + k$ for each of the following functions.



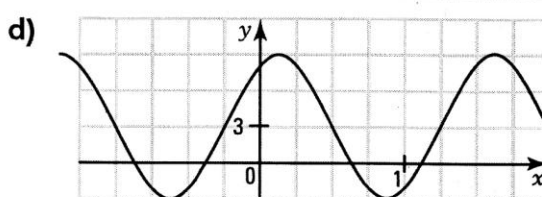
For example, $y = 2 \sin x + 2$



For example, $y = -1.5 \sin \frac{\pi}{20}x + 0.5$

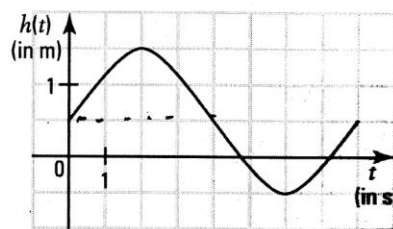


For example, $y = -1.5 \sin \frac{2}{3}\left(x + \frac{3\pi}{4}\right) + 1$



For example, $y = 6 \sin \frac{4\pi}{3}\left(x + \frac{1}{4}\right) + 3$

- 15.** The waves of an artificial lake are observed in a laboratory setting as indicated in the figure on the right. The graph represents the movement of one wave where t is time, expressed in seconds, and $h(t)$ is the height of the wave, in metres.



What is the height of the wave after 3 seconds?

$(h, k) = (0, 0.5) \quad h(t) = \sin \frac{\pi}{4}t + 0.5; \quad h(3) \approx 1.21 \text{ m}$

- 16.** Raphael is playing with a yo-yo for 30 seconds. The height of the yo-yo, in metres, varies as a function of time t , in seconds, according to the rule of a sinusoidal function. Initially, the yo-yo is at its maximum height of 2 m. After 4 seconds, the yo-yo reaches its minimum height of 1 m for the first time.

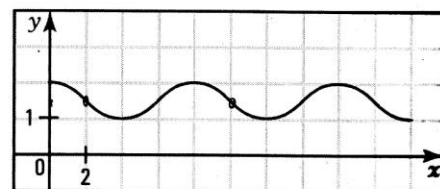
After how many seconds does Raphael's yo-yo reach a height of 1.8 m for the first time?

Rule of the function:

for example, $y = -0.5 \sin \frac{\pi}{4}(x - 2) + 1.5$.

$-0.5 \sin \frac{\pi}{4}(x - 2) + 1.5 = 1.8$

$x = 1.18$. After 1.18 seconds, the yo-yo reaches a height of 1.8 m for the first time.



- 17.** The flashing light of an electronic game follows the trajectory of a sinusoidal function as indicated in the graph on the right. If t represents the time (in seconds) and $h(t)$ represents the height (in dm) of the light, determine at what height the light is at the moment $t = 6$ s?

$(h, k) = (1.5) \quad y = -2.6 \sin \frac{\pi}{3}(x - 1) + 5$

$A = 2.6 \quad a = -2.6$

$b = \frac{2\pi}{p} = \frac{2\pi}{6} = \frac{\pi}{3} \quad \text{At the moment } t = 6 \text{ s, the light will be at a height of } 7.25 \text{ dm.}$

