1. a) $30° = \frac{\pi}{6}$ rad **b)** $\frac{35\pi}{18}$ rad $= \mathbf{350°}$ **c)** $75° = \frac{5\pi}{12}$ rad

d) $27° = \frac{3\pi}{20}$ rad **e)** $-\frac{5\pi}{8}$ rad $= \mathbf{-112.5°}$ **f)** $270° = \frac{3\pi}{2}$ rad

g) $-36° = -\frac{\pi}{5}$ rad **h)** $\frac{8\pi}{5}$ rad $= \mathbf{288°}$ **i)** $-78° = -\frac{13\pi}{30}$ rad

2. a) 1st quadrant. **b)** 3rd quadrant. **c)** 2nd quadrant. **d)** 1st quadrant.

e) 4th quadrant. **f)** 4th quadrant. **g)** 1st quadrant. **h)** 2nd quadrant.

3. a) **1)** Domain: $\mathbb{R}$ **2)** Range: $[1, 7]$ **3)** Period: 10

b) **1)** Domain: $\mathbb{R}$, excluding $\frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$. **2)** Range: $\mathbb{R}$ **3)** Period: π

c) **1)** Domain: $\mathbb{R}$ **2)** Range: $[-7, -3]$ **3)** Period: $\frac{2\pi}{3}$

d) **1)** Domain: $\mathbb{R}$ **2)** Range: $[17, 19]$ **3)** Period: 4

e) **1)** Domain: $\mathbb{R}$ **2)** Range: $[-6, 8]$ **3)** Period: π

f) **1)** Domain: $\mathbb{R}$, excluding $\frac{11}{6} + \frac{n}{3}$, $n \in \mathbb{Z}$. **2)** Range: $\mathbb{R}$ **3)** Period: $\frac{1}{3}$

4. a)

$$\frac{(\sin x \cot x)^2}{1 + \sin x} = 1 - \sin x$$

$$\frac{\sin^2 x \frac{\cos^2 x}{\sin^2 x}}{1 + \sin x} = 1 - \sin x$$

$$\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$$

$$\frac{1 - \sin^2 x}{1 + \sin x} = 1 - \sin x$$

$$\frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} = 1 - \sin x$$

$$1 - \sin x = 1 - \sin x$$

b)

$$1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$1 - \sin^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$$

$$2\cos^2 x - 1 = 2\cos^2 x - 1$$

c)

$$\tan x + \cot x = \sec x \operatorname{cosec} x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \operatorname{cosec} x$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \sec x \operatorname{cosec} x$$

$$\frac{1}{\cos x \sin x} = \sec x \operatorname{cosec} x$$

$$\frac{1}{\cos x} \times \frac{1}{\sin x} = \sec x \operatorname{cosec} x$$

$$\sec x \operatorname{cosec} x = \sec x \operatorname{cosec} x$$

d)

$$(\tan x - \cot x)\sin x \cos x = \sin^2 x - \cos^2 x$$

$$\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right)\sin x \cos x = \sin^2 x - \cos^2 x$$

$$\left(\frac{\sin^2 x - \cos^2 x}{\cos x \sin x}\right)\sin x \cos x = \sin^2 x - \cos^2 x$$

$$\sin^2 x - \cos^2 x = \sin^2 x - \cos^2 x$$

e)
$$\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x}{\sin x + \cos x} \times \frac{\sec x}{\sec x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x \sec x}{(\sin x + \cos x)\sec x} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\sin x \frac{1}{\cos x}}{(\sin x + \cos x)\frac{1}{\cos x}} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\tan x}{1 + \tan x}$$
$$\frac{\tan x}{1 + \tan x} = \frac{\tan x}{1 + \tan x}$$

f)
$$\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2\sec x$$
$$\frac{\cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} + \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} = 2\sec x$$
$$\frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{1 - \sin^2 x} = 2\sec x$$
$$\frac{\cos x + \cos x}{\cos^2 x} = 2\sec x$$
$$\frac{2\cos x}{\cos^2 x} = 2\sec x$$
$$2\frac{1}{\cos x} = 2\sec x$$
$$2\sec x = 2\sec x$$

g)
$$(1 + \tan x)^2 + (1 - \tan x)^2 = 2\sec^2 x$$
$$1 + 2\tan x + \tan^2 x + 1 - 2\tan x + \tan^2 x = 2\sec^2 x$$
$$1 + \tan^2 x + 1 + \tan^2 x = 2\sec^2 x$$
$$\sec^2 x + \sec^2 x = 2\sec^2 x$$
$$2\sec^2 x = 2\sec^2 x$$

h)
$$\frac{\tan^2 x}{1 + \tan^2 x} \times \frac{1 + \cot^2 x}{\cot^2 x} = \sin^2 x \sec^2 x$$
$$\frac{\tan^2 x}{\sec^2 x} \times \frac{\operatorname{cosec}^2 x}{\cot^2 x} = \sin^2 x \sec^2 x$$
$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} \times \frac{\frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \sin^2 x \sec^2 x$$
$$\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x \times \frac{1}{\sin^2 x} \times \frac{\sin^2 x}{\cos^2 x} = \sin^2 x \sec^2 x$$
$$\sin^2 x \frac{1}{\cos^2 x} = \sin^2 x \sec^2 x$$
$$\sin^2 x \sec^2 x = \sin^2 x \sec^2 x$$

Overview (*cont'd*)

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5. a) $x = \frac{10}{3} + 4n$ and $x = \frac{14}{3} + 4n$, $n \in \mathbb{Z}$.

b) $x = 2 + 16n$ and $x = 10 + 16n$, $n \in \mathbb{Z}$.

c) $x = 6n$, $n \in \mathbb{Z}$

d) $x = \frac{2}{9} + \frac{2}{3}n$ and $x = \frac{4}{9} + \frac{2}{3}n$, $n \in \mathbb{Z}$.

e) $x = \frac{\pi + 4}{4} + n\pi$, $n \in \mathbb{Z}$

f) $x = \frac{5\pi}{16} + n\pi$, $n \in \mathbb{Z}$

6. a) $f(x) = -\sin x$ or $f(x) = \cos\pi\left(x + \frac{\pi}{2}\right)$.

b) $f(x) = -3\cos x$ or $f(x) = 3\cos(x + \pi)$.

c) $f(x) = 2\sin 3x + 1$ or $f(x) = 2\cos 3\left(x - \frac{\pi}{6}\right) + 1$.

d) $f(x) = \sin\pi x - 1$ or $f(x) = \cos\pi\left(x - \frac{1}{2}\right) - 1$.

e) $f(x) = \sin x$ or $f(x) = \cos\left(x - \frac{\pi}{2}\right)$.

f) $f(x) = -10\sin x - 20$ or $f(x) = 10\cos\left(x + \frac{\pi}{2}\right) - 20$.

Overview (*cont'd*)

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7. a) No solution.

b) $\left\{-\frac{25\pi}{9}, -\frac{23\pi}{9}, -\frac{19\pi}{9}, -\frac{17\pi}{9}, -\frac{13\pi}{9}, -\frac{11\pi}{9}, -\frac{5\pi}{9}, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \frac{19\pi}{9}, \frac{23\pi}{9}, \frac{25\pi}{9}\right\}$

c) $-\frac{5\pi}{4}$

d) No solution.

e) $\left\{-\frac{71\pi}{24}, -\frac{67\pi}{24}, -\frac{47\pi}{24}, -\frac{43\pi}{24}, -\frac{23\pi}{24}, -\frac{19\pi}{24}, \frac{\pi}{24}, \frac{5\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{49\pi}{24}, \frac{53\pi}{24}\right\}$

f) $\left\{-\frac{11\pi}{4}, -\frac{9\pi}{4}, -\frac{7\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}\right\}$

8. a) $\ldots \cup \left]-\frac{35\pi}{4}, -\frac{25\pi}{4}\right[\cup \left]\frac{5\pi}{4}, \frac{15\pi}{4}\right[\cup \left]\frac{45\pi}{4}, \frac{55\pi}{4}\right[\cup \ldots$

b) No solution.

c) $\ldots \cup]-1, 0[\cup]3, 4[\cup]7, 8[\cup \ldots$

d) $\ldots \cup \left[-\frac{7\pi}{4}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{7\pi}{4}\right] \cup \left[\frac{9\pi}{4}, \frac{15\pi}{4}\right] \cup \ldots$

e) No solution.

f) $\ldots \cup \left[\frac{\pi}{3}, 0\right[\cup \left[\frac{5\pi}{6}, \frac{\pi}{2}\right[\cup \left[\frac{4\pi}{3}, \pi\right[\cup \ldots$

9. a) $\frac{5}{13}$ **b)** $\frac{13}{12}$ **c)** $\frac{5}{12}$ **d)** $\frac{13}{5}$ **e)** $\frac{12}{5}$

10. a) $-\frac{1}{2}$ b) $\frac{\sqrt{3}}{3}$ c) $-\frac{2\sqrt{3}}{3}$ d) $-\frac{\sqrt{3}}{2}$ e) -2

11. a) $\frac{\sqrt{3}}{2}$ b) $\frac{\pi}{3}$ c) 1 d) $\frac{\sqrt{2}}{2}$ e) $\frac{\pi}{2} + n\pi$ where $n \in \mathbb{Z}$. f) $-\frac{\pi}{4}$

12. The time taken for the wheel to make one complete turn corresponds to the period of the function whose rule is $h = 14\sin 15(t - 15) + 18$; therefore, $\frac{2\pi}{15}$ s, which is approximately 0.42 s.

Overview (*cont'd*) **Page 143**

13. a) $f(x) = 2\tan\frac{1}{2}x - 2$ b) $f(x) = -\frac{1}{2}\tan\left(x - \frac{\pi}{2}\right) + \frac{1}{2}$ c) $f(x) = \frac{1}{2}\tan \pi x + 0.5$

d) $f(x) = -2\tan\frac{\pi}{2}x$ e) $f(x) = 4\tan\frac{\pi}{2}x$ f) $f(x) = \tan\frac{1}{2}(x + \pi) + 2$

14. a) $x \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ b) $x \in \left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$ c) $x \in \left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

d) $x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ e) $x = \frac{3\pi}{2}$ f) $x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

Overview (*cont'd*) **Page 144**

15. a) $\frac{2\sqrt{10}}{7}$ b) $\frac{2\sqrt{10}}{3}$ c) $\frac{7}{3}$ d) $\frac{7\sqrt{10}}{20}$ e) $\frac{3\sqrt{10}}{20}$ f) $\frac{3}{4}$

g) $\frac{\sqrt{7}}{3}$ h) $\frac{4}{3}$ i) $\frac{4\sqrt{7}}{7}$ j) $\frac{3\sqrt{7}}{7}$ k) $\frac{6\sqrt{10} + 3\sqrt{7}}{28}$ l) $\frac{9 + 2\sqrt{70}}{28}$

16. a) $f(x) \geq 0$ if $x \in \left[-2, -\frac{5}{3}\right] \cup \left[-\frac{1}{3}, \frac{1}{3}\right] \cup \left[\frac{5}{3}, 2\right]$;

$f(x) \leq 0$ if $x \in \left[-\frac{5}{3}, -\frac{1}{3}\right] \cup \left[\frac{1}{3}, \frac{5}{3}\right]$.

b) $f(x) \geq 0$ if $x \in \left[-\frac{23\pi}{6}, -\frac{7\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{17\pi}{6}\right]$;

$f(x) \leq 0$ if $x \in \left[-4\pi, -\frac{23\pi}{6}\right] \cup \left[-\frac{7\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{17\pi}{6}, 4\pi\right]$.

c) $f(x) \geq 0$ if $x \in \left[-4\pi, -\frac{29\pi}{12}\right] \cup \left[-\frac{13\pi}{12}, \frac{19\pi}{12}\right] \cup \left[\frac{35\pi}{12}, 4\pi\right]$;

$f(x) \leq 0$ if $x \in \left[-\frac{29\pi}{12}, -\frac{13\pi}{12}\right] \cup \left[\frac{19\pi}{12}, \frac{35\pi}{12}\right]$.

d) $f(x) \geq 0$ if $x \in \left[-\frac{7\pi}{4}, -\pi\right[\cup \left[-\frac{3\pi}{4}, 0\right[\cup \left[\frac{\pi}{4}, \pi\right[\cup \left[\frac{5\pi}{4}, 2\pi\right[$;

$f(x) \leq 0$ if $x \in \left]-2\pi, -\frac{7\pi}{4}\right] \cup \left]-\pi, -\frac{3\pi}{4}\right] \cup \left]0, \frac{\pi}{4}\right] \cup \left]\pi, \frac{5\pi}{4}\right]$.

e) $f(x) \geq 0$ if $x \in \left]-3\pi, -\frac{5\pi}{2}\right] \cup \left]-\pi, -\frac{\pi}{2}\right] \cup \left]\pi, \frac{3\pi}{2}\right] \cup \left]3\pi, \frac{7\pi}{2}\right]$;

$f(x) \leq 0$ if $x \in [-4\pi, -3\pi[\cup \left[-\frac{5\pi}{2}, -\pi\right[\cup \left[-\frac{\pi}{2}, \pi\right[\cup \left[\frac{3\pi}{2}, 3\pi\right[\cup \left[\frac{7\pi}{2}, 4\pi\right]$.

f) $f(x) \geq 0$ if $x \in \left[-2, -\frac{11}{6}\right[\cup \left[-\frac{3}{2}, -\frac{7}{6}\right[\cup \left[-\frac{5}{6}, -\frac{1}{2}\right[\cup \left[-\frac{1}{6}, \frac{1}{6}\right[\cup \left[\frac{1}{2}, \frac{5}{6}\right[\cup \left[\frac{7}{6}, \frac{3}{2}\right[\cup \left[\frac{11}{6}, 2\right]$;

$f(x) \leq 0$ if $x \in \left]-\frac{11}{6}, -\frac{3}{2}\right] \cup \left]-\frac{7}{6}, -\frac{5}{6}\right] \cup \left]-\frac{1}{2}, -\frac{1}{6}\right] \cup \left]\frac{1}{6}, \frac{1}{2}\right] \cup \left]\frac{5}{6}, \frac{7}{6}\right] \cup \left]\frac{3}{2}, \frac{11}{6}\right]$.

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17. a) $x = n\pi$ and $x = \frac{2\pi}{3} + 2n\pi$ and $x = \frac{4\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

b) $x = \frac{\pi}{4} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

c) $x \approx 0.6749 + 2n\pi$ and $x \approx -0.6749 + 2n\pi$, $n \in \mathbb{Z}$.

d) $x = \frac{\pi}{4} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

e) $x = \frac{\pi}{4} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

f) $x = \frac{\pi}{4} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

g) $x = \frac{\pi}{3} + n\pi$ and $x = \frac{2\pi}{3} + n\pi$, $n \in \mathbb{Z}$.

h) $x \approx 1.9979 + 2n\pi$ and $x \approx -1.9979 + 2n\pi$, $n \in \mathbb{Z}$.

i) $x = \frac{2\pi}{3} + 2n\pi$ and $x = \frac{4\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

18. Arc of the circle = 3450 km
Circumference of circle = $(2\pi \times 1520)$ km
The length of the arc of the circle is $\frac{345}{152}$ rad.

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19. a) *Several answers possible. Example:* $f(x) = 3\cos\pi(x - 1) + 1.5$ or $f(x) = 3\sin\pi\left(x - \frac{1}{2}\right) + 1.5$.

b) 1) {1, 3, 5} **2)** $\left\{\frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3}, \frac{13}{3}, \frac{17}{3}\right\}$

20. During the simulations, an explosion occurs at: 0.75 s, 2.75 s, 4.75 s, 6.75 s, 8.75 s, 10.75 s, 12.75 s and 14.75 s.

21. The device is saturated over the following intervals: [0, ≈ 0.01[s,]≈ 0.08, 0.11[s,]≈ 0.19, ≈ 0.21[s,]≈ 0.29, ≈0.31[s,]≈ 0.39, ≈ 0.41[s,]≈ 0.49, ≈ 0.51[s,]≈ 0.59, ≈ 0.61[s,]≈ 0.69, ≈ 0.71[s,]≈ 0.79, ≈ 0.81[s,]≈ 0.89, ≈ 0.91[s and]≈ 0.99, 1] s.

22. The length of this belt is approximately 43.62 cm.

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23. For $x \in [0, 2]$, find the zero of the cosine function. You obtain $x = \frac{\pi}{2}$, which is ≈ 1.57.
The length L of the blade is therefore approximately 1.57 mm.
Solve $\tan x = \cos x$ for $x \in [0, 2]$.
You obtain: $x \approx 0.67$.
$y \approx \tan 0.67$
$y \approx 0.79$
The height H is approximately 0.79 mm.

24. Considering the x-axis as the surface of the water, you must find two consecutive zeros for the function
$h = 250\cos\frac{\pi t}{15} + 125$ when the curve is found above the x-axis.
$t = 20$ s and $t = 10$ s. Therefore 20 s − 10 s = 10 s.
The water bomber takes 10 s to fill its tank.

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25. a) 1) ≈ 194.67 m **2)** ≈ 187.04 m

b) The general formula is $P = \frac{v\cos\theta}{g}(v\sin\theta + \sqrt{(v\sin\theta)^2 + 2gy_0})$.
By replacing y_0 by 0, you obtain: $P = \frac{v\cos\theta}{g}(v\sin\theta + \sqrt{(v\sin\theta)^2 + 2g \times 0})$