Overview **Page 214**

1. a) $x^2 + y^2 = 1600$ **b)** $\frac{x^2}{2025} + \frac{y^2}{2809} = 1$ **c)** $(y + 3)^2 = 2(x + 4)$ **d)** $\frac{x^2}{64} - \frac{y^2}{36} = 1$

e) $x^2 + y^2 = 5625$ **f)** $(x + 10)^2 = -20(y - 20)$ **g)** $\frac{x^2}{841} + \frac{y^2}{441} = 1$ **h)** $\frac{x^2}{576} - \frac{y^2}{49} = -1$

Overview *(cont'd)* **Page 215**

2. a) 1) A parabola.
2) i) (20, 10) **ii)** (20, 10.25) **iii)** Does not apply.

b) 1) An ellipse.
2) i) (0, 12.5), (0, −12.5), (3.5, 0) and (−3.5, 0). **ii)** (0, 12) and (0, −12). **iii)** Does not apply.

c) 1) A hyperbola.
2) i) (0, 36) and (0, −36). **ii)** (0, 39) and (0, −39). **iii)** $y = \frac{12}{5}x$ and $y = -\frac{12}{5}x$.

d) 1) A parabola.
2) i) (−2, −9) **ii)** (−26, −9) **iii)** Does not apply.

e) 1) A hyperbola.
2) i) (22.5, 0) and (−22.5, 0). **ii)** (26.5, 0) and (−26.5, 0). **iii)** $y = \frac{28}{45}x$ and $y = -\frac{28}{45}x$.

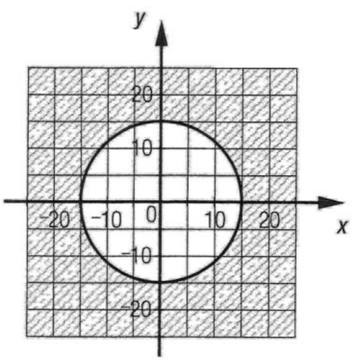
f) 1) An ellipse.
2) i) (0, 61), (0, −61), (11, 0) and (−11, 0). **ii)** (0, 60) and (0, −60). **iii)** Does not apply.

3. a) $x^2 + y^2 = 676$ **b)** $\frac{x^2}{576} + \frac{y^2}{1089} = 1$ **c)** $\frac{x^2}{400} + \frac{y^2}{1241} = 1$ **d)** $(y - 8)^2 = 8(x + 3)$

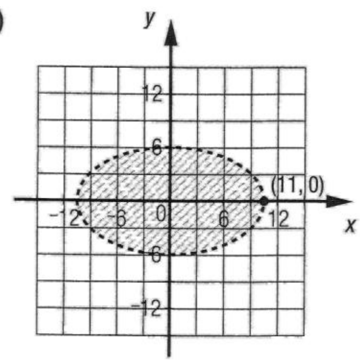
e) $(x - 12)^2 = 12(y + 4)$ **f)** $\frac{x^2}{576} - \frac{y^2}{4900} = -1$ **g)** $\frac{x^2}{196} - \frac{y^2}{506.25} = 1$

4. a) (35, 0) and (−35, 0). **b)** (0, 10) and (0, −10). **c)** (2, −4)
d) (−3, −0.5) **e)** (0, 4) and (0, −4). **f)** (13, 0) and (−13, 0).

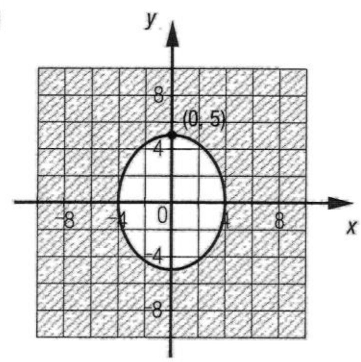
5. a)



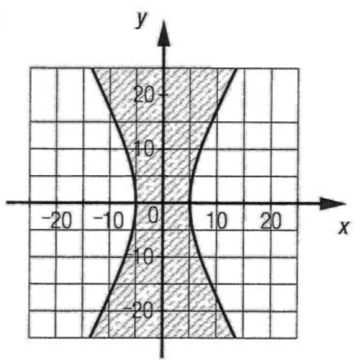
b)



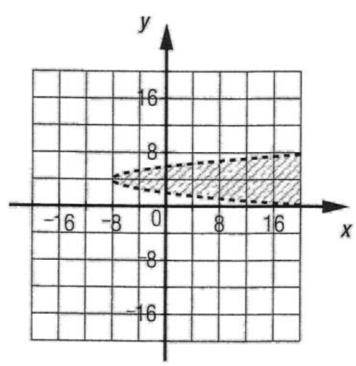
c)



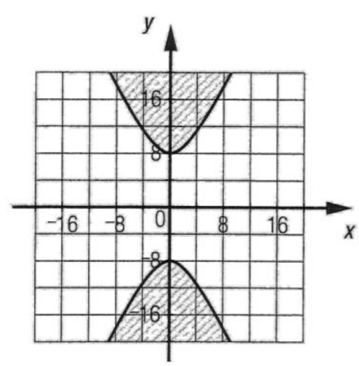
d)



e)



f)

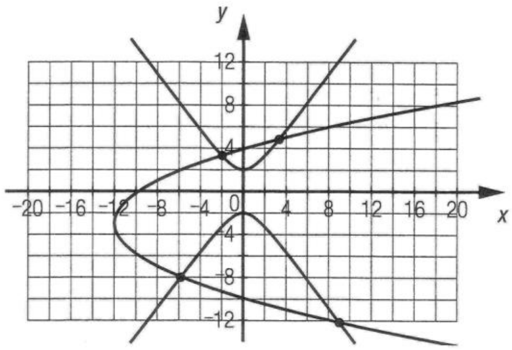


Overview *(cont'd)*

6. a) $\frac{x^2}{841} + \frac{y^2}{400} \geq 1$ **b)** $x^2 + y^2 \leq 289$ **c)** $\frac{x^2}{576} - \frac{y^2}{100} > 1$

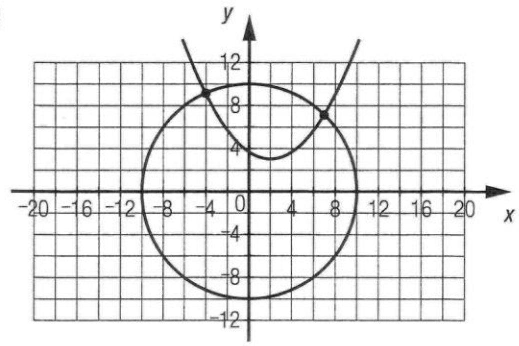
d) $(y + 5)^2 < -2(x - 7)$ **e)** $\frac{x^2}{12.25} - \frac{y^2}{144} \geq -1$ **f)** $(x + 10)^2 > 15(y - 5)$

7. a)



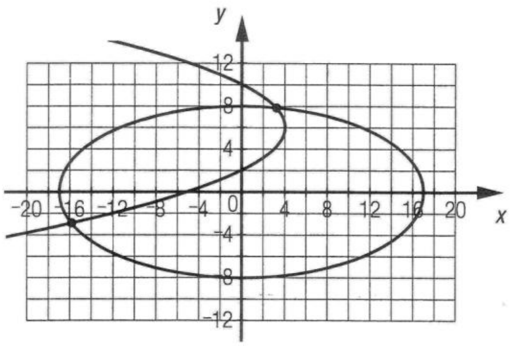
($\approx$ -5.8, $\approx$ -7.98), ($\approx$ -1.99, $\approx$ 3.33), ($\approx$ 3.29, $\approx$ 4.82) and ($\approx$ 9, $\approx$ -12.17).

b)



($\approx$ -4.07, $\approx$ 9.14) and ($\approx$ 6.99, $\approx$ 7.15).

c)



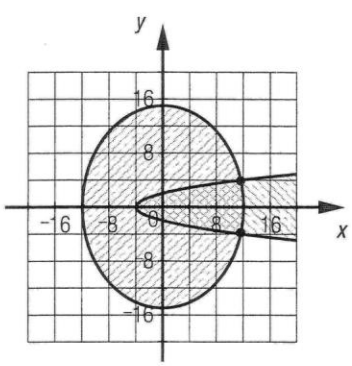
($\approx$ -15.84, $\approx$ -2.91) and ($\approx$ 3.13, $\approx$ 7.86).

Overview *(cont'd)*

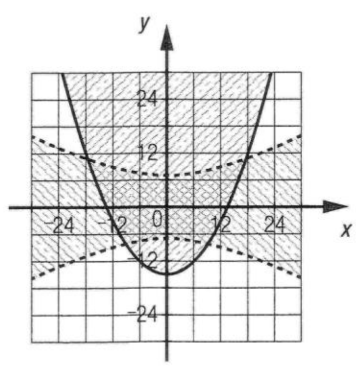
8.

Equation of the conic	Coordinates of the vertex or vertices	Coordinates of the focus or foci
$\frac{x^2}{6724} + \frac{y^2}{324} = 1$	**(82, 0) and (-82, 0)** **(0, 18) and (0, -18)**	**(80, 0)** **(-80, 0)**
$(y - 12)^2 = -9(x + 4)$	**(-4, 12)**	**(-6.25, 12)**
$\frac{x^2}{256} - \frac{y^2}{3969} = -1$	**(0, 63) and (0, -63)**	**(0, 65)** **(0, -65)**
$(x + 9)^2 = 0.5(y + 5)$	**(-9, -5)**	**(-9, -4.875)**
$\frac{x^2}{196} - \frac{y^2}{506.25} = 1$	**(14, 0) and (-14, 0)**	**(26.5, 0)** **(-26.5, 0)**

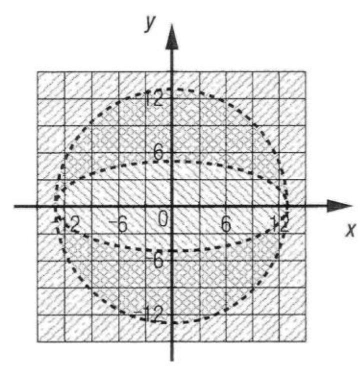
9. a)



b)



c)



10. The inequality associated with the secure swimming zone is in the form $(x - \text{h})^2 \leq 4\text{c}(y - \text{k})$. Using the graphical representation, it is possible to determine:

- the value of **c**: -5
- the coordinates of the vertex: $(x, 25)$
- the coordinates of a point through which the curve passes: (40, 20)

You therefore obtain $(40 - \text{h})^2 = 4 \times -5 \times (20 - 25)$ where h = 30 since based on the graphical representation, it is the only accepted value. The inequality associated with the secure swimming zone is therefore $(x - 30)^2 \leq -20(y - 25)$.

Overview *(cont'd)* **Page 218**

11. a) It is possible to deduce that the coordinates of the focus of the ellipse and of the vertex of the parabola are (20, 0) since a = 29 and b = 21 and that $\text{a}^2 = \text{b}^2 + \text{c}^2$.
The equation of the directrix of the parabola is $x = 29$ since it passes through the vertex of the ellipse. It is possible to deduce that parameter **c** = -9 since the vertex of the parabola is (20, 0).
The equation of the parabola associated with the trajectory of the swimmer is therefore $y^2 = -36(x - 20)$.

b) To determine the coordinates of the swimmer's entry point and exit point, find the intersection points between the parabola with equation $y^2 = -36(x - 20)$ and the ellipse with equation $\frac{x^2}{841} + \frac{y^2}{441} = 1$.
You obtain $\frac{x^2}{841} + \frac{-36(x - 20)}{441} = 1$ where $x \approx 8.9$ since the second value obtained does not correspond to the situation.
Since $y^2 \approx -36(8.9 - 20)$, $y \approx \pm 19.99$.

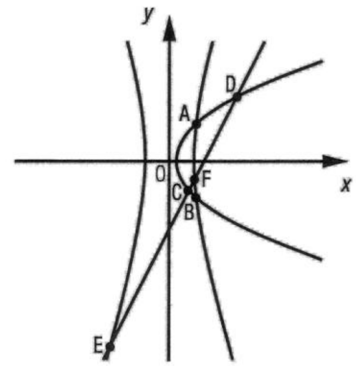
1) The coordinates of the swimmer's entry point are ($\approx$ 8.9, $\approx$ 19.99).
2) The coordinates of the swimmer's exit point are ($\approx$ 8.9, $\approx$ -19.99).

12. a) 1) Since the parabola has an axis of symmetry based on the coordinates of points (2, 6) and (6, 6), you can deduce that the coordinates of the vertex are (6 − 2, -2) = (4, -2).
The equation of this parabola is in the form $(x - \text{h})^2 = 4\text{c}(y - \text{k})$. By substituting the known data by these variables, you can determine the value of parameter **c**:
$(2 - 4)^2 = 4\text{c}(6 + 2)$
$\text{c} = 0.125$
The equation of the parabola is $(x - 4)^2 = 0.5(y + 2)$.

2) The line passes through the focus whose coordinates are (4, -2 + 0.125) = (4, -1.875).
Since the line passes through points (4, -1.875) and (2, -1), its equation is $y = -0.4375x - 0.125$.

b) Solve the system of equations:
$(x - 4)^2 = 0.5(y + 2)$
$y = -0.4375x - 0.125$
From this system, you obtain:
$$(x - 4)^2 = 0.5(-0.4375x - 0.125 + 2)$$
$x^2 - 7.78125x + 15.0625 = 0$, from where $x_1 \approx 3.62$ and $x_2 \approx 4.16$.
Therefore, $y_1 \approx -0.4375 \times 3.62 - 0.125$, or $y_1 \approx -1.71$, and $y_2 \approx -0.4375 \times 4.16 - 0.125$, or $y_2 \approx -1.95$.
The coordinates of the points where the bird could catch the fish are ($\approx$ 3.62, $\approx$ -1.71) and ($\approx$ 4.16, $\approx$ -1.95).

13. Determine the coordinates of 6 intersection points.



You can obtain the coordinates of points A and B by solving the system of equations $\frac{x^2}{49} - \frac{y^2}{576} = 1$ and $y^2 = 20(x - 2)$.
The coordinates of points A and B are respectively (≈ 7.66, ≈ 10.64) and (≈ 7.66, -10.64).
You can obtain the coordinates of points C and D by solving the system of equations $y = 2x - 20$ and $y^2 = 20(x - 2)$.
The coordinates of points C and D are respectively (≈ 5.7, ≈ -8.6) and (≈ 19.3, 18.6).
You can obtain the coordinates of points E and F by solving the system of equations $\frac{x^2}{49} - \frac{y^2}{576} = 1$ and $y = 2x - 20$.
The coordinates of points E and F are respectively (≈ -17.51, ≈ -55.01) and (≈ 7.19, -5.62).

14. a) Based on the definition of an ellipse, you know that the sum of the distances from point B to the point with coordinates (7.2, 4.2) and from this point to point C is 2a.
Therefore, $2a = \sqrt{(7.2 + 5.66)^2 + (4.2 - 0)^2} + \sqrt{(7.2 - 5.66)^2 + (4.2 + 0)^2}$, or ≈ 18.
The value of parameter **a** $= 18 \div 2 = 9$.
You can determine the value of parameter **b** using the relation $b^2 + c^2 = a^2$:
$b^2 + 5.66^2 \approx 9^2$
$b^2 \approx 48.96$
$b \approx 7$
The inequality that corresponds to the region associated with this archipelago is $\frac{x^2}{81} + \frac{y^2}{48.96} \leq 1$.

b) The length of the major axis of the ellipse is approximately 18 km, and the length of the minor axis is approximately 14 km.

c) You need to determine the intersection points between the ellipse with equation $\frac{x^2}{81} + \frac{y^2}{48.96} = 1$ and the line with equation $y = -0.4x - 1$:
$$\frac{x^2}{81} + \frac{(-0.4x - 1)^2}{48.96} = 1$$
$x_1 \approx -8.46$ and $x_2 \approx 7.42$, and $y_1 \approx 2.39$ and $y_2 \approx -3.97$.
The distance separating these two points is $\sqrt{(-8.46 - 7.42)^2 + (2.39 + 3.97)^2}$ km ≈ 17.11 km. The distance covered by the plane is approximately 17.11 km.

15. Since:
- the height of the viaduct is 8 m, then $b = 8$.
- the width of the viaduct is 34 m, then $a = 34 \div 2 = 17$.

The equation of the ellipse is $\frac{x^2}{289} + \frac{y^2}{64} = 1$.
Since:
- the x-axis is superimposed onto the directrix of the parabola, $x = 0$.
- the coordinates of the focus of the parabola and those of the vertices of the ellipse are the same, which is (0, 8).

Therefore, $c = 8 \div 2 = 4$; the vertex is $(0, 8 - 4) = (0, 4)$, and the equation of the parabola is $x^2 = 16(y - 4)$.
To find the intersection points, solve the equation:
$$\frac{16(y-4)}{289} + \frac{y^2}{64} = 1$$
$y_1 \approx 7.25$ and $y_2 \approx -10.79$.
The value of y_2 rejected based on the context.
$x^2 \approx 16(7.25 - 4)$
$x^2 \approx 52$
$x \approx \pm 7.21$
The coordinates of the two anchor points are therefore (≈ -7.21, ≈ 7.25) and (≈ 7.21, 7.25).

16. a) 1) The eccentricity of this ellipse is $\frac{m\,\overline{FA}}{m\,\overline{AB}} = \frac{2.6}{3.25} = 0.8$.

2) The coordinates of the vertex of the ellipse, which are $(x, 0)$, are presented. Since the eccentricity is 0.8, you have $\frac{-4 - x}{x + 6.25} = 0.8$.
Determine the value of the x-variable:
$-4 - x = 0.8x + 5$
$-1.8x = 9$
$x = -5$
The coordinates of the vertex V of the ellipse is (-5, 0).

b) The value of the ratio is 0.8. It corresponds to the eccentricity of this ellipse.

c) The coordinates of one of the foci are (0, 7) since $a^2 + c^2 = b^2$. In addition, the coordinates of one of the vertices are (0, 25).
Therefore, the eccentricity of the ellipse is $\frac{\text{distance of the origin to the focus}}{\text{distance of the origin to the vertex V}} = \frac{7}{25} = 0.28$.

17. Using the graphical representation and the equation of the asymptote, it is possible to deduce that the equation of the hyperbola is $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$.
The equation of the line associated with the top of the building is $y = 0.5x + 4$ since the segment passes through the points (0, 4) and (2, 5).
The equation of the parabola associated with the top of the door is $x^2 = -12(y + 3)$ since the equation of its directrix is $x = 0$ and the coordinates of its vertex is (0, -3).
The equation that corresponds to the ground is $y = -6$.

a) The coordinates of the upper right corner of the front (≈ 2.29, ≈ 5.14) are determined by calculating the intersection point between the hyperbola with the equation $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and the line with the equation $y = 0.5x + 4$.
The maximum height of the building is approximately $6 + 5.14 \approx 11.14$ m.

b) The coordinates of the doors' anchor points are determined by solving the following two systems of equations:
- $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and $y = -6$.
- $\frac{x^2}{1} - \frac{y^2}{6.25} = 1$ and $x^2 = -12(y + 3)$.

The coordinates of the doors' four anchor points are (-2.6, -6), (2.6, -6), (≈ 1.62, ≈ -3.22) and (≈ -1.62, ≈ -3.22).

18. *Several answers possible. Example:*
The equation of a circle is $x^2 + y^2 = r^2$. It is possible to transform this equation into the equation of the ellipse $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$ in which parameters **a** and **b** are equal.
Since in an ellipse, you have the relation $a^2 = b^2 + c^2$, you therefore obtain $r^2 = r^2 + c^2$; therefore, $c = 0$.
Since the value of parameter **c** is 0, the foci are therefore located at the origin of the Cartesian plane.
Joseph is therefore correct.

19. The equation of the parabola is $(y - 30)^2 = 20(x - 10)$ since the coordinates of its vertex are (10, 30) and the equation of the directrix is $x = 5$.
Find the value of y when $x = 20$.
$(y - 30)^2 = 20(20 - 10)$
$(y - 30)^2 = 200$
$y_1 \approx 15.86$ and $y_2 \approx 44.14$.
The diameter of this car's headlight is $44.14 - 15.86 \approx 28.28$ cm.

20. a) The tulips are planted in the region defined by $x^2 + y^2 \leq 36$. This can be deduced using the point (3.6, 4.8):
$3.6^2 + 4.8^2 = 36$.

b) The roses are planted in the region defined by $\frac{x^2}{20.25} - \frac{y^2}{36} \leq -1$. This can be deduced using the coordinates of one of the vertices of the hyperbola (0, 6), the coordinates of one of the foci (0, 7.5) and the relation $a^2 + b^2 = c^2$.

c) The lilies are planted in the region defined by $y^2 \leq 12(x - 6)$. This can be deduced using the coordinates of the vertex (6, 0) and the coordinates of the focus (9, 0).

d) The daisies are planted in the region defined by $y^2 \leq -12(x + 6)$. This can be deduced using the coordinates of the vertex (-6, 0) and the coordinates of the focus (-9, 0).

21. The equation of the ellipse associated with this situation is $\frac{x^2}{36} + \frac{y^2}{100} = 1$ since the coordinates of one of the vertices are (6, 0) and those of the foci are (0, -8).

The equation of the parabola associated with this situation is $x^2 = -8(y - 8)$ since the coordinates of its vertex are (0, 8) and the equation of its directrix is $y = 10$.
The equation of the line associated with this situation is $y = 0.25x - 8$ since the line passes through the points whose coordinates are (0, -8) and (2, -7.5).

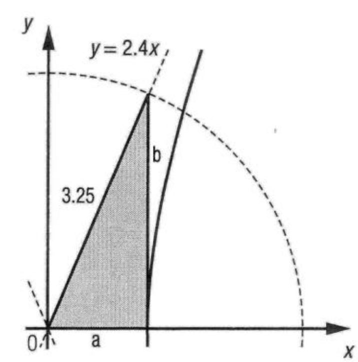
To determine the coordinates of points A and B, solve the system of equations $\frac{x^2}{36} + \frac{y^2}{100} = 1$ and $x^2 = -8(y - 8)$.

To determine the coordinates of points C and D, solve the system of equations $\frac{x^2}{36} + \frac{y^2}{100} = 1$ and $y = 0.25x - 8$.

The coordinates of point A are (≈ -5.4, ≈ 4.35). The coordinates of point B are (≈ 5.4, ≈ 4.35). The coordinates of point C are (≈ 4.33, ≈ -6.92). The coordinates of point D are (≈ -2.93, ≈ -8.73).

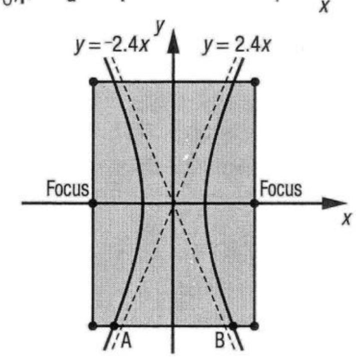
Overview (*cont'd*)

22. a) Since the piece of wood measures 6.5 m by 10 m, you can deduce that the coordinates of the foci are (-3.25, 0) and (3.25, 0).
Using this information and the equation of one of the asymptotes, determine the values of parameters **a** and **b**.
You know that $\frac{b}{a} = 2.4$; therefore, $b = 2.4a$ which allows you to obtain $b^2 = 5.76a^2$.
You also know that, in the hyperbola, $a^2 + b^2 = c^2$;
therefore $a^2 + 5.76a^2 = 3.25^2$.
You can determine that $a^2 = 1.5625$ and $b^2 = 9$.
The equation of the hyperbola is therefore $\frac{x^2}{1.5625} - \frac{y^2}{9} = 1$.



b) 1) The minimum width of the tabletop corresponds to the distance between the two vertices (-1.25, 0) and (1.25, 0) of the hyperbola, which is 2.5 m.

2) In the adjacent graphical representation, it consists of the distance between points A and B.
To determine these coordinates, solve the system of equations
$\frac{x^2}{1.5625} - \frac{y^2}{9} = 1$ and $y = -5$.
The coordinates of point A are (≈ -2.43, -5), and those of point B are (≈ 2.43, -5).
The maximum width of the tabletop is therefore 2.43 + 2.43 ≈ 4.86 m.



23. a) The relation $b^2 + c^2 = a^2$ allows you to deduce the value of parameter **c**: $56^2 + c^2 = 106^2$, $c = \pm 90$.
The coordinates of the foci are (-90, 0) and (90, 0).
Since the equation of the elliptical fence is $\frac{x^2}{106^2} + \frac{y^2}{56^2} = 1$ and you must find the coordinates of a point whose coordinates are (90, *y*), you can substitute *x* by 90 and isolate the *y*-variable:

$$\frac{90^2}{106^2} + \frac{y^2}{56^2} = 1$$
$$\frac{y^2}{3136} = \frac{784}{2809}$$
$$y^2 \approx 875.27$$
$$y \approx \pm 29.58$$

The coordinates of the four corners of the stage are (90, ≈ 29.58), (90, ≈ -29.58), (-90, ≈ 29.58) and (-90, ≈ -29.58).

b) Since you must look for the coordinates of a point whose coordinates are $(x, 41)$, you can substitute y by 41 in the equation of the ellipse and isolate the x-variable:

$$\frac{x^2}{106^2} + \frac{41^2}{56^2} = 1$$

$$\frac{x^2}{11\,236} = \frac{1455}{3136}$$

$$x^2 \approx 5213.13$$

$$x \approx \pm 72.2$$

The coordinates of the spectator are therefore $(\approx -72.2, 41)$.
The distance between this point and the origin of the Cartesian plane is $\sqrt{(-72.2)^2 + 41^2} \approx 83.03$ m.
The distance separating the spectator from the singer is approximately 83.03 m.

24. a) 1) The radius of the exterior circle associated with the red crown is $450 - 60 \div 2 = 195$ mm. The equation of this circle is $x^2 + y^2 = 38\,025$.

2) The radius of the interior circle associated with the red crown is $195 - 41 = 154$ mm. The equation of this circle is $x^2 + y^2 = 23\,716$.

b) You have the equation:

$$x^2 + (-x + 30)^2 = 23\,716$$

$2x^2 + -60x - 22\,816 = 0$, from where $x_1 \approx 122.86$ and $x_2 \approx -92.86$.
Therefore $y_1 \approx -122.86 + 30$ or ≈ -92.86 and $y_2 \approx 92.86 + 30$ or ≈ 122.86.
The coordinates of the intersection points are $(\approx 122.86, \approx -92.86)$ and $(\approx -92.86, \approx 122.86)$.