1. Find the equation in the standard form of each of the following conics.

a) Ellipse with foci $F_1(-8, 0)$ and $F_2(8, 0)$ whose major axis measures 20 units. $\frac{x^2}{100} + \frac{y^2}{36} = 1$

b) Hyperbola with foci $F_1(-10, 0)$ and $F_2(10, 0)$ whose transverse axis has a length of 12 units.
$\frac{x^2}{36} - \frac{y^2}{64} = 1$

c) Circle centred at $O(0, 0)$ passing through $A(-2, 3)$. $x^2 + y^2 = 13$

d) Parabola with vertex $V(0, 0)$ and focus $F(0, -3)$. $x^2 = -12y$

2. For each of the conics defined by the following equations, describe the conic by giving:
- for a circle, the centre and the radius,
- for an ellipse, the coordinates of its foci and its vertices,
- for a hyperbola, the coordinates of its foci, its vertices and the equations of the asymptotes,
- for a parabola, the coordinates of its vertex, its focus and the equation of the directrix.

a) $x^2 + y^2 - 5 = 0$. *Circle of radius $\sqrt{5}$ with centre O(0,0)*

b) $x^2 + 4y^2 - 16 = 0$.

$\frac{x^2}{16} + \frac{y^2}{4} = 1$; *Ellipse centred at the origin; vertices: (–4, 0), (4, 0), (0, –2), (0, 2);*

foci: $(-\sqrt{12}, 0)$ *and* $(\sqrt{12}, 0)$

c) $4x^2 - 9y^2 - 36 = 0$.

$\frac{x^2}{9} - \frac{y^2}{4} = 1$; *Hyperbola centred at the origin; vertices: (–3, 0), (3, 0);*

foci: $(-\sqrt{13}, 0)$ *and* $(\sqrt{13}, 0)$ *and asymptotes:* $y = \frac{2}{3}x$ *and* $y = -\frac{2}{3}x$.

d) $y^2 + 2x - 2y + 7 = 0$.

$(y - 1)^2 = -2(x + 3)$; *Parabola with vertex (–3, 1) open to the left;*

focus: $\left(-\frac{7}{2}, 1\right)$ *and directrix:* $x = -\frac{5}{2}$.

3. In each of the following cases, name the geometric locus and give its equation.

a) Set of points $M(x, y)$ located at a distance of 3 units from the origin. *Circle:* $x^2 + y^2 = 9$.

b) Set of points $M(x, y)$ whose distances from the point $(-1, 2)$ and the line with equation $x = 3$ are equal. *Parabola:* $(y - 2)^2 = -8(x - 1)$.

c) Set of points $M(x, y)$ such that the absolute value of the difference of the distances from point M to points $(-5, 0)$ and $(5, 0)$ is equal to 8 units.

Hyperbola: $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

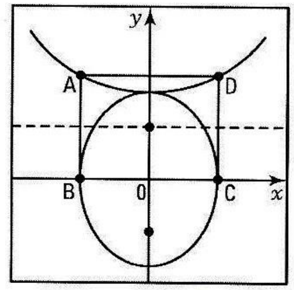
d) Set of points $M(x, y)$ such that the sum of the distances from point M to points $(0, -4)$ and $(0, 4)$ is equal to 10 units.

Ellipse: $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

4. The ellipse centred at the origin on the right has a major axis measuring 10 units and a minor axis measuring 8 units.

The vertex of the parabola on the right coincides with one of the vertices of the ellipse and its directrix passes through one of the foci of the ellipse.

Calculate the area of rectangle ABCD if line segment BC corresponds to the horizontal minor axis of the ellipse.

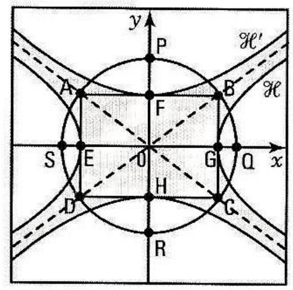


Ellipse: $\frac{x^2}{16} + \frac{y^2}{25} = 1$; *Parabola:* $x^2 = 8(y - 5)$

$B(-4, 0)$; $C(4, 0)$; $A(-4, 7)$; $D(4, 7)$.

Area of rectangle ABCD $= 56\ u^2$.

5. Consider the circle centred at 0 passing through point A(–4, 3) and the rectangle ABCD whose sides are parallel to the axes. Consider the hyperbola $\mathcal{H}$ whose transverse axis is on the x-axis, whose vertices are points E and G and whose foci are points S and Q and the hyperbola $\mathcal{H}'$ whose transverse axis is on the y-axis, whose vertices are points F and H and whose foci are points P and R.



a) Determine the system of inequalities representing the shaded region.

$\mathcal{H}: \frac{x^2}{16} - \frac{y^2}{9} = 1$; $\mathcal{H}': \frac{x^2}{16} - \frac{y^2}{9} = -1$; $\begin{cases} \frac{x^2}{16} - \frac{y^2}{9} \leq 1 \\ \frac{x^2}{16} - \frac{y^2}{9} \geq -1 \end{cases}$

b) What can be said about the supports of the diagonals of rectangle ABCD. Justify your answer.

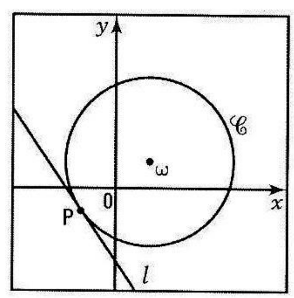
AC: $y = -\frac{3}{4}x$ *(shared asymptote for hyperbolas* $\mathcal{H}$ *and* $\mathcal{H}'$*)*

BD: $y = \frac{3}{4}x$ *(shared asymptote for hyperbolas* $\mathcal{H}$ *and* $\mathcal{H}'$*)*

6. A circle $\mathscr{C}$ centred at $\omega(-1, 2)$ passes through point A(2, 6). Find the equation of the line l tangent to the circle at point P(3, –1).

$\mathscr{C}: (x + 1)^2 + (y - 2)^2 = 25$; $l: y = \frac{4}{3}x - 5$

7. Circle $\mathscr{C}$ centred at $\omega(4, 3)$ on the right is tangent to line l with equation: $4x + 3y + 25 = 0$.



a) Find the equation of circle $\mathscr{C}$.

$r = d(\omega, l) = \frac{|4 \times 4 + 3 \times 3 + 25|}{\sqrt{4^2 + 3^2}} = \frac{50}{5} = 10$

$\mathscr{C}: (x - 4)^2 + (y - 3)^2 = 100$

b) Find the coordinates of the point of tangency P.

$P(-4, -3)$

8. Consider triangle ABC having vertices A(–5, 10), B(–7, –4) and C(9, 8).
Find the equation of the circumscribed circle of triangle ABC.
Recall that: The circumscribed circle's centre is the intersection point of the perpendicular bisectors of the sides of the triangle.

l_1 (perpendicular bisector of $\overline{AB}$): $y = -\frac{1}{7}x + \frac{15}{7}$; l_2 (perpendicular bisector of $\overline{AC}$): $y = 7x - 5$;

ω (centre of circumscribed circle) = (1, 2); radius of circumscribed circle = d(ω, A) = 10.

Circumscribed circle: $(x - 1)^2 + (y - 2)^2 = 100$.

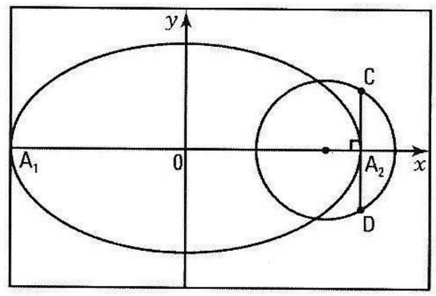
9. What is the intersection point of the directrices of parabolas
$\mathscr{P}_1: (y - 1)^2 = -8(x + 1)$ and $\mathscr{P}_2: (x + 3)^2 = 4(y + 2)$?

$l_1: x = 1$; $l_2: y = -3$ thus the intersection point is P(1, –3).

10. The difference between the radii of two concentric circles is 3 cm.
If the equation of the larger circle is: $x^2 + y^2 + 6x - 4y - 36 = 0$, determine the equation of the smaller circle.

Large circle: $(x + 3)^2 + (y - 2)^2 = 49$; small circle: $(x + 3)^2 + (y - 2)^2 = 16$

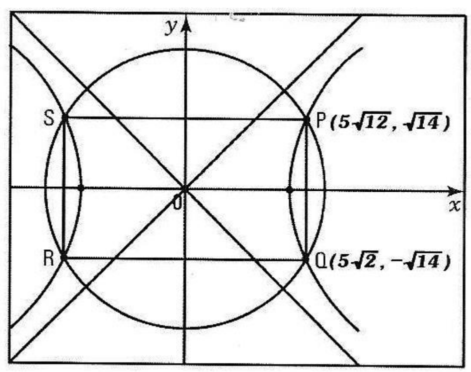
11. The major and the minor axes of the ellipse centred at the origin shown on the right measure 10 and 6 units respectively.
A circle of radius 2 units centred at one of the foci of the ellipse was drawn.
Determine the length of the chord CD knowing that $\overline{CD} \perp \overline{A_1A_2}$.



Ellipse: $\frac{x^2}{25} + \frac{y^2}{9} = 1$; Focus: F(4, 0); Circle: $(x - 4)^2 + y^2 = 4$

$C(5, \sqrt{3})$; $D(5, -\sqrt{3})$; $m\overline{CD} = 2\sqrt{3}$

12. The asymptotes of the hyperbola centred at the origin shown on the right are the lines with equations: $y = x$ and $y = -x$. Its transverse axis measures 12 units. The circle on the right, of radius 8 units and centred at the origin, intersects the hyperbola at four points P, Q, R and S.
Calculate the area of rectangle PQRS.



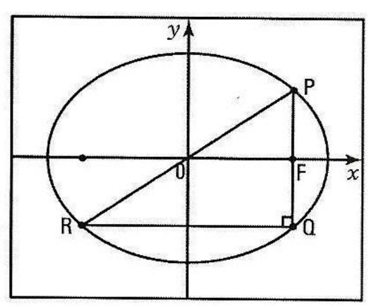
Hyperbola: $x^2 - y^2 = 36$. Circle: $x^2 + y^2 = 64$

$P(5\sqrt{2}, \sqrt{14})$; $Q(5\sqrt{2}, -\sqrt{14})$; $R(-5\sqrt{2}, -\sqrt{14})$;

$S(-5\sqrt{2}, \sqrt{14})$; $m\overline{PQ} = 2\sqrt{14}$; $m\overline{RQ} = 10\sqrt{2}$;

Area PQRS = $40\sqrt{7}$ u²

13. The major and the minor axes of the ellipse on the right measure 10 units and 6 units respectively. Calculate the area of the right triangle PQR inscribed inside the ellipse knowing that the side PQ passes through one of the foci of the ellipse.



Ellipse: $\frac{x^2}{25} + \frac{y^2}{9} = 1$; F(4, 0); $P\left(4, \frac{9}{5}\right)$

$Q\left(4, \frac{-9}{5}\right)$; $R\left(-4, \frac{-9}{5}\right)$; $m\overline{PQ} = \frac{18}{5}$; $m\overline{RQ} = 8$; Area $\triangle PQR = \frac{36}{5} = 7.2$ u²

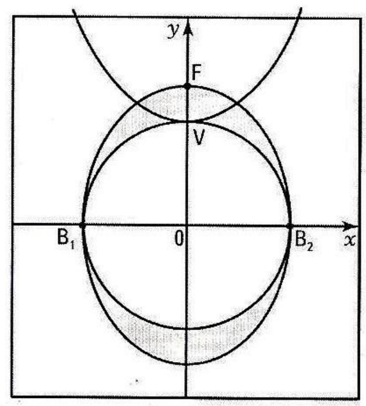
14.4

14. On the figure on the right, the circle centred at the origin passes through the vertex V of the parabola and the ellipse passes through its focus F. The minor axis B_1B_2 of the ellipse is a diameter for the circle.

If the equation of the parabola is $x^2 = 4(y - 3)$, describe the shaded region using a system of inequalities.

V(0, 3); F(0, 4); Circle: $x^2 + y^2 = 9$

Ellipse: $\frac{x^2}{9} + \frac{y^2}{16} = 1.$ $\begin{cases} x^2 + y^2 \geq 9 \\ \frac{x^2}{9} + \frac{y^2}{16} \leq 1. \end{cases}$

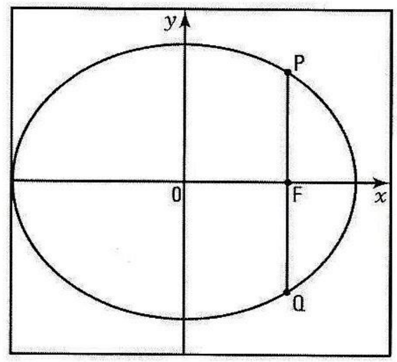


15. In an elliptical shaped pool, two points P and Q on the side of this pool are joined by a cable. This cable passes through a focus of the ellipse and is perpendicular to its major axis.

Knowing that the major axis and the minor axis measure 200 m and 160 m respectively, calculate the length of the cable when it is stretched.

Ellipse: $\frac{x^2}{100^2} + \frac{y^2}{80^2} = 1$ *; F(60, 0); P(60, 64); Q(60, –64);*

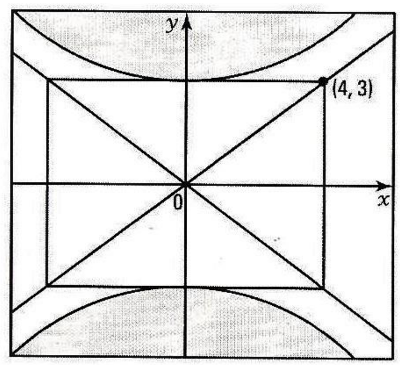
$m\overline{PQ} = 128$ *m.*



16. The interior region of a hyperbola centred at the origin has been shaded.

Knowing that one of the asymptotes of the hyperbola passes through point (4, 3) and that the distance between the vertices of the hyperbola is equal to 6 units, describe the shaded region using an inequality.

$\frac{x^2}{16} - \frac{y^2}{9} \leq -1$



17. Consider an ellipse centred at the origin with equation: $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Determine the equation of each of the parabolas whose vertex and focus are respectively the foci of the ellipse.

Parabola: vertex $F_1(-3, 0)$ ***and focus*** $F_2(3, 0)$***;*** $(x + 3) = 24y^2$

Parabola: vertex $F_2(3, 0)$ ***and focus*** $F_1(-3, 0)$***;*** $(x - 3) = -24y^2$