**1.** a) Convert $\frac{5\pi}{36}$ into degrees. ***25°***

b) Convert 75° into radians. $\boldsymbol{\frac{5\pi}{12}}$ ***rad***

**2.** A central angle measuring 54° sub-tends an arc on the circle.
If the circle has a radius of 12 cm, calculate the length of the sub-tended arc (round to the nearest tenth). ***11.3 cm***

**3.** What are the coordinates of the trigonometric point P($t$) located in the 3rd quadrant if $\cos t = -\frac{15}{17}$? $P(t) = \left(\frac{-15}{17}, \frac{-8}{17}\right)$

**4.** Evaluate $\sin\left(t - \frac{\pi}{4}\right)$ if $\frac{\pi}{2} < t < \pi$ and $\sin t = \frac{\sqrt{3}}{2}$.

$\cos t = -\frac{1}{2}$

$\sin\left(t - \frac{\pi}{4}\right) = \sin t \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos t = \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

**5.** Evaluate $\cos 2t$ if $\cos t = \frac{4}{5}$.

$\cos 2t = 2\cos^2 t - 1 = \frac{7}{25}$

**6.** Find the coordinates of the point P($t$), located in the 3rd quadrant, if $\tan t = \frac{8}{15}$?

$\sec^2 t = 1 + \tan^2 t = \frac{289}{225}$; $\sec t = \frac{-17}{15}$; $\cos t = \frac{-15}{17}$; $\sin t = \frac{-8}{17} \Rightarrow P(t) = \left(\frac{-15}{17}, \frac{-8}{17}\right)$

**7.** If $P(t) = \left(-\frac{3}{5}, \frac{4}{5}\right)$, find the coordinates of the point P($2t$).

$P(t) = (\cos 2t, \sin 2t) = (\cos^2 t - \sin^2 t, 2\sin t \cos t) = \left(\frac{-7}{25}, \frac{-24}{25}\right)$

**8.** Find the exact Cartesian coordinates of the trigonometric point $P\left(-\frac{19\pi}{6}\right)$.

$P\left(\frac{-19\pi}{6}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

**9.** Knowing that $\sin a = \frac{3}{5}$, $\cos a = \frac{4}{5}$, $\sin b = \frac{5}{13}$, $\cos b = \frac{12}{13}$, find the value of

a) $\sin(a + b)$ — $\sin a \cos b + \sin b \cos a = \frac{56}{65}$

b) $\cos(a - b)$ — $\cos a \cos b + \sin a \sin b = \frac{63}{65}$

c) $\tan(a + b)$ — $\frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$

**10.** Simplify $\frac{\sin 2\theta}{1 + \cos 2\theta}$. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$

**11.** Simplify $\frac{\sin 4a \cos 2a - \sin 2a \cos 4a}{\cos 2a \cos a + \sin 2a \sin a}$. $\frac{\sin(4a - 2a)}{\cos(2a - a)} = \frac{\sin 2a}{\cos a} = \frac{2\sin a \cos a}{\cos a} = 2\sin a.$

**12.** Given the trigonometric point $P(t) = \left(\frac{3}{5}, \frac{4}{5}\right)$, find the coordinates of the following trigonometric points.

**a)** $P\left(t + \frac{\pi}{2}\right)$ $\left(-\frac{4}{5}, \frac{3}{5}\right)$ **b)** $P\left(t - \frac{\pi}{2}\right)$ $\left(\frac{4}{5}, -\frac{3}{5}\right)$

**c)** $P(t + \pi)$ $\left(-\frac{3}{5}, \frac{-4}{5}\right)$ **d)** $P(\pi - t)$ $\left(-\frac{3}{5}, \frac{4}{5}\right)$

**13.** If $P\left(\frac{a+2}{5}, \frac{a+1}{5}\right)$ is a trigonometric point located in the 3rd quadrant, determine $a$.

$\left(\frac{a+2}{5}\right)^2 + \left(\frac{a+1}{5}\right)^2 = 1$; $2a^2 + 6a - 20 = 0$; $a = -5$ or $a = 2$.

*Reject the solution $a = 2$ since the trigonometric point is located in the 3rd quadrant.*

**14.** If $\sin t = \frac{15}{17}$ and $\frac{\pi}{2} \leq t \leq \pi$, find

**a)** $\cos t$ $\frac{-8}{17}$ **b)** $\tan t$ $\frac{-15}{8}$ **c)** $\cot t$ $\frac{-8}{15}$ **d)** $\sec t$ $\frac{-17}{8}$ **e)** $\csc t$ $\frac{17}{15}$

**15.** The function $f$ is given by the rule $f(x) = -4 \cos \frac{\pi}{6}(x - 2) + 2$. Determine

**a)** the amplitude. 4 **b)** the period of $f$. 12 **c)** ran $f$. $[-2, 6]$

**d)** the zeros of $f$. $\{4 + 12n\} \cup \{12 + 12n\}$

**e)** the sign of $f$ over $[2, 14]$. $f(x) < 0 \Leftrightarrow x \in [2, 4[ \cup ]12, 14]$, $f(x) > 0 \Leftrightarrow x \in ]4, 12[$

**16.** Determine the zeros of the following trigonometric functions over $\mathbb{R}$.

**a)** $f(x) = 2 \sin \frac{\pi}{2}(x - 3) - 1$ $\left\{\frac{10}{3} + 4n\right\} \cup \left\{\frac{14}{3} + 4n\right\}$

**b)** $f(x) = -6 \cos \frac{\pi}{3}(x + 1) + 3$ $\{0 + 6n\} \cup \{4 + 6n\}$

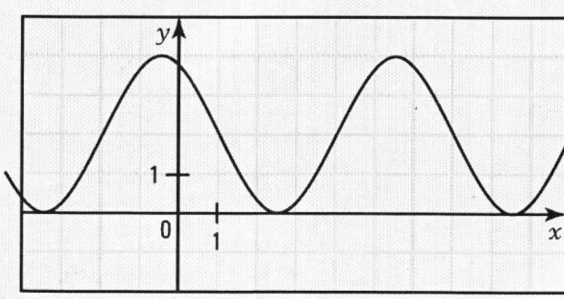
**c)** $f(x) = 2 \tan \frac{\pi}{4}(x - 1) + 2$ $\{4 + 8n\} \cup \{8 + 8n\}$

**d)** $f(x) = 5 \sin \frac{2\pi}{3}(x + 1) + 2$ $\{-1.2 + 3n\} \cup \{0.70 + 3n\}$

**e)** $f(x) = -2 \cos 2(x + 1) + 1$ $\{-0.48 + \pi n\} \cup \{1.62 + \pi n\}$

**17.** Find the rule of the sinusoidal function represented on the right.

$y = -2 \sin \frac{\pi}{3}(x - 1) + 2$



**18.** Solve the following trigonometric equations over $\mathbb{R}$.

a) $2\sin x(\cos x - 1) = 0$ $S = \{0 + 2\pi n\} \cup \{\pi + 2\pi n\}$

b) $(2\sin x + 1)(2\cos x - 1) = 0$ $S = \left\{\frac{7\pi}{6} + 2\pi n\right\} \cup \left\{\frac{11\pi}{6} + 2\pi n\right\} \cup \left\{\frac{\pi}{3} + 2\pi n\right\} \cup \left\{\frac{5\pi}{3} + 2\pi n\right\}$

c) $2\sin^2 x - 5\sin x + 2 = 0$ $S = \left\{\frac{\pi}{6} + 2\pi n\right\} \cup \left\{\frac{5\pi}{6} + 2\pi n\right\}$

d) $\tan^2 x - 1 = 0$ $S = \left\{\frac{\pi}{4} + \pi n\right\} \cup \left\{\frac{3\pi}{4} + \pi n\right\}$

**19.** Solve the following equations over $[0, 2\pi[$.

a) $2\sin^2 x - 1 = 0$ $S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

b) $(2\cos x + 1)(\sin x - 3) = 0$ $S = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

c) $25\sin^2 x - 9 = 0$ $S = \{0.64; 2.50; 3.78; 5.64\}$

d) $(5\cos x + 2)(2\sin x - 1) = 0$ $S = \left\{1.98; 4.30; \frac{\pi}{6}; \frac{5\pi}{6}\right\}$

**20.** True of false?

a) $\sin(a + b) = \sin a + \sin b$ ***False***

b) $\sin 2a = 2\sin a$ ***False***

c) $\sin x^2 = \sin^2 x$ ***False***

d) $\cos 2a = \cos^2 a - \sin^2 a$ ***True***

**21.** A metal ball is attached to the end of a vertical spring. The spring is stretched to a length of 5 cm in relation to its equilibrium point and then released.

The function which gives the ball's position (in sec) relative to its equilibrium point, as a function of time $t$ (in cm) since the release of the spring is given by the rule: $p(t) = 5\cos \pi t$, $0 \leq t \leq 10$.

At what moments over the first 5 seconds is the spring's position even with its equilibrium point?

$p(t) = 0 \Leftrightarrow 5\cos \pi t = 0 \Leftrightarrow \pi t = \frac{\pi}{2}$ or $\pi t = \frac{3\pi}{2} \Leftrightarrow t = \frac{1}{2}$ or $t = \frac{3}{2}$; period = 2 s.

***The points are: 0.5 s; 1.5 s; 2.5 s; 3.5 s and 4.5 s.***

**22.** The bird population of a certain region varies throughout the year. The population $P(t)$, in thousands, as a function of time $t$ since January 1st is described by the rule: $P(t) = -6.4\cos\left(\frac{\pi}{6}t\right) + 10$.

At what moments in the year are there 6800 birds in this region?

$-6.4\cos\left(\frac{\pi}{6}t\right) + 10 = 6.8 \Leftrightarrow \cos\frac{\pi}{6}t = 0.5 \Leftrightarrow \frac{\pi t}{6} = \frac{\pi}{3}$ or $\frac{\pi t}{6} = \frac{5\pi}{3} \Leftrightarrow t = 2$ or $t = 10$.

***March 1st or October 1st.***

**23.** The current I (in amperes) produced by a generator is given by the rule $I = 20 \sin 30\pi t$ where $t$ represents the time (in seconds) since the moment the generator was turned on. How long, after turning the generator on, do we observe an intensity of 10 amperes

**a)** the first time? ***After $\frac{1}{180}$ seconds.***

**b)** the second time? ***After $\frac{5}{180}$ seconds.***

**24.** Prove the following identities.

**a)** $\frac{1-\sin t}{\cos t} = \frac{\cos t}{1+\sin t}$

$\frac{1-\sin t}{\cos t} = \frac{(1-\sin t)(1+\sin t)}{\cos t\,(1+\sin t)} = \frac{1-\sin^2 t}{\cos t\,(1+\sin t)} = \frac{\cos^2 t}{\cos t\,(1+\sin t)} = \frac{\cos t}{1+\sin t}$

**b)** $\frac{\sin 2t}{1+\cos 2t} = \tan t$

$\frac{\sin 2t}{1+\cos 2t} = \frac{2\sin t\cos t}{1+(2\cos^2 t-1)} = \frac{2\sin t\cos t}{2\cos^2 t} = \frac{\sin t}{\cos t} = \tan t$

**c)** $\cos^4 t - \sin^4 t = \cos 2t$

$\cos^4 t - \sin^4 t = (\cos^2 t + \sin^2 t)(\cos^2 t - \sin^2 t) = 1(\cos 2t) = \cos 2t$

**d)** $\frac{\sec t - 1}{\tan t} = \csc t - \cot t$

$\frac{\sec t - 1}{\tan t} = (\sec t - 1)\frac{\cos t}{\sin t} = \frac{1}{\sin t} - \frac{\cos t}{\sin t} = \csc t - \cot t$

**25.** Calculate

**a)** $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$ $\frac{1}{2}$

**b)** $\cos\left(\sin^{-1}\frac{1}{2}\right)$ $\frac{\sqrt{3}}{2}$

**c)** $\tan\left(\sin^{-1} 0\right)$ $0$

**d)** $\cos\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ $\frac{\sqrt{3}}{2}$

**e)** $\tan\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)$ $-\frac{\sqrt{3}}{3}$

**f)** $\sec\left(\tan^{-1}\sqrt{3}\right)$ $2$

**26.** Solve the following equations.

**a)** $\sin(\arccos x) = \frac{\sqrt{3}}{2}$ $x = \frac{1}{2}$

**b)** $\tan(\arcsin x) = -1$ $x = \frac{-\sqrt{2}}{2}$

**c)** $\sec(\arcsin x) = \frac{2}{\sqrt{3}}$ $x = \frac{1}{2}$

**d)** $\csc(\arctan x) = 2$ $x = \frac{\sqrt{3}}{3}$