

1. a)  $\cos^2 x$       b)  $\sin^2 x$       c)  $\cos^2 x$       d) 1  
e) 1      f)  $\tan^2 x$       g)  $\cot x$       h)  $\tan^2 x$
2. a)  $\frac{3\sqrt{7}}{8}$       b)  $3\sqrt{7}$       c) 8      d)  $\frac{8\sqrt{7}}{21}$
3. a)  $\frac{3\sqrt{159}}{40}$       b)  $\frac{40\sqrt{159}}{477}$       c)  $-\frac{40}{13}$       d)  $-\frac{3\sqrt{159}}{13}$
4. a)  $\frac{2\sqrt{2}}{3}$       b)  $-\frac{1}{3}$       c)  $-2\sqrt{2}$       d)  $-\frac{\sqrt{2}}{4}$
5. a)  $-\frac{3}{5}$       b)  $-\frac{4}{5}$       c)  $-\frac{5}{3}$       d)  $\frac{4}{3}$
6. a)  $\frac{3}{5}$       b)  $-\frac{3}{4}$       c)  $-\frac{5}{4}$       d)  $-\frac{4}{3}$
7. a)  $\left\{-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}\right\}$   
c)  $\left\{-3\pi, -\frac{8\pi}{3}, -\frac{4\pi}{3}, -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{8\pi}{3}, 3\pi\right\}$   
e) No solution.
- b)  $\left\{-\frac{8\pi}{3}, -2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3}\right\}$   
d)  $\left\{-\frac{17\pi}{6}, -\frac{13\pi}{6}, -\frac{3\pi}{2}, -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{2}\right\}$   
f)  $\left\{-\frac{5\pi}{2}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}$

8. a) 1      b)  $\sin^2 x$       c)  $\operatorname{cosec}^2 x$       d)  $\sin^2 x$       e)  $\sec x$       f) 1
9. 
$$\frac{\cos^2 x - \cos^4 x}{\sin^2 x - \sin^4 x} = 1$$

$$\frac{\cos^2 x(1 - \cos^2 x)}{\sin^2 x(1 - \sin^2 x)} = 1$$

$$\frac{\cos^2 x \times \sin^2 x}{\sin^2 x \times \cos^2 x} = 1$$

$$1 = 1$$
10. a)  $\frac{\sqrt{2} + \sqrt{6}}{4}$       b)  $\sqrt{3} - 2$       c)  $\sqrt{6} - \sqrt{2}$       d)  $\frac{\sqrt{2} - \sqrt{6}}{4}$       e)  $2 - \sqrt{3}$       f)  $\sqrt{2} - \sqrt{6}$
11. a)  $\sin^2 x = 1 - \cot^2 x \sin^2 x$   
 $1 - \cos^2 x = 1 - \cot^2 x \sin^2 x$   
 $1 - \cos^2 x \frac{\sin^2 x}{\sin^2 x} = 1 - \cot^2 x \sin^2 x$   
 $1 - \frac{\cos^2 x}{\sin^2 x} \sin^2 x = 1 - \cot^2 x \sin^2 x$   
 $1 - \cot^2 x \sin^2 x = 1 - \cot^2 x \sin^2 x$
- b)  $\sin x \cot x = \cos x$   
 $\sin x \frac{\cos x}{\sin x} = \cos x$   
 $\frac{\sin x}{\sin x} \cos x = \cos x$   
 $\cos x = \cos x$

c)

$$\begin{aligned} \frac{\cot x - \tan x}{\cot x + \tan x} &= 2 \cos^2 x - 1 \\ \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} &= 2 \cos^2 x - 1 \\ \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} &= 2 \cos^2 x - 1 \\ \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \times \frac{\sin x \cos x}{\sin x \cos x} &= 2 \cos^2 x - 1 \\ \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} &= 2 \cos^2 x - 1 \\ \frac{\cos^2 x - \sin^2 x}{1} &= 2 \cos^2 x - 1 \\ \cos^2 x - \sin^2 x &= 2 \cos^2 x - 1 \\ \cos^2 x - (1 - \cos^2 x) &= 2 \cos^2 x - 1 \\ 2 \cos^2 x - 1 &= 2 \cos^2 x - 1 \end{aligned}$$

e)

$$\begin{aligned} (1 - \sin^2 x)(1 + \cot^2 x) &= \cot^2 x \\ \cos^2 x \csc^2 x &= \cot^2 x \\ \cos^2 x \frac{1}{\sin^2 x} &= \cot^2 x \\ \frac{\cos^2 x}{\sin^2 x} &= \cot^2 x \\ \cot^2 x &= \cot^2 x \end{aligned}$$

g)

$$\begin{aligned} \cos x \sqrt{\sec^2 x - 1} &= \sin x \\ \cos x \sqrt{\tan^2 x} &= \sin x \\ \cos x \tan x &= \sin x \\ \cos x \frac{\sin x}{\cos x} &= \sin x \\ \sin x &= \sin x \end{aligned}$$

d)

$$\begin{aligned} \tan x (\sin x + \cot x \cos x) &= \sec x \\ \frac{\sin x}{\cos x} \left( \sin x + \frac{\cos x}{\sin x} \cos x \right) &= \sec x \\ \frac{\sin^2 x}{\cos x} + \cos x &= \sec x \\ \frac{\sin^2 x + \cos^2 x}{\cos x} &= \sec x \\ \frac{1}{\cos x} &= \sec x \\ \sec x &= \sec x \end{aligned}$$

f)

$$\begin{aligned} \sin^2 x \cot^2 x \sec x &= \cos x \\ \sin^2 x \times \frac{\cos^2 x}{\sin^2 x} \times \frac{1}{\cos x} &= \cos x \\ \cos x &= \cos x \end{aligned}$$

h)

$$\begin{aligned} \tan^2 x + \cos^2 x - 1 &= \sin^2 x \tan^2 x \\ \tan^2 x + \cos^2 x - (\cos^2 x + \sin^2 x) &= \sin^2 x \tan^2 x \\ \tan^2 x + \cos^2 x - \cos^2 x - \sin^2 x &= \sin^2 x \tan^2 x \\ \tan^2 x - \sin^2 x &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x}{\cos^2 x} - \sin^2 x &= \sin^2 x \tan^2 x \\ \sin^2 x \left( \frac{1}{\cos^2 x} - 1 \right) &= \sin^2 x \tan^2 x \\ \sin^2 x (\sec^2 x - 1) &= \sin^2 x \tan^2 x \\ \sin^2 x \tan^2 x &= \sin^2 x \tan^2 x \end{aligned}$$

12. a)  $-\sin x$       b)  $\cos x$       c)  $\sin x$       d)  $\cos x$       e)  $\sin x$   
 f)  $-\cos x$       g)  $-\tan x$       h)  $\frac{1}{\tan x}$       i)  $-\tan x$

### Practice 5.4 (cont'd)

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#### 13. Demonstration 1

- from Step ① to Step ② because  $\cos^2 a = 1 - \sin^2 a$
- from Step ② to Step ③ because  $-\sin^2 a - \sin^2 a = -2\sin^2 a$
- from Step ③ to Step ④ by adding  $2\sin^2 a$  and subtracting  $\cos 2a$  to both sides of the equation
- from Step ④ to Step ⑤ by dividing both sides of the equation by 2
- from Step ⑤ to Step ⑥ by completing a square root on both sides of the equation
- from Step ⑥ to Step ⑦ by applying the value  $\frac{b}{2}$  to the variable  $a$
- from Step ⑦ to Step ⑧ because  $2\left(\frac{b}{2}\right) = b$

#### Demonstration 2

- from Step ① to Step ② because  $\sin^2 a = 1 - \cos^2 a$
- from Step ② to Step ③ because  $\cos^2 a - 1 + \cos^2 a = 2\cos^2 a - 1$
- from Step ③ to Step ④ by adding 1 to both sides of the equation and inverting both sides
- from Step ④ to Step ⑤ by dividing both sides of the equation by 2

- from Step ⑤ to Step ⑥ by completing a square root on both sides of the equation
- from Step ⑥ to Step ⑦ by applying the value  $\frac{b}{2}$  to the variable  $a$
- from Step ⑦ to Step ⑧ because  $2\left(\frac{b}{2}\right) = b = b$

14. a)

$$(\csc x - \cot x)^2 = \frac{1 - \cos x}{1 + \cos x}$$

$$\begin{aligned} \csc^2 x - 2 \csc x \cot x + \cot^2 x &= \frac{1 - \cos x}{1 + \cos x} \\ \frac{1}{\sin^2 x} - 2 \frac{1}{\sin x} \times \frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x} &= \frac{1 - \cos x}{1 + \cos x} \\ \frac{1 - 2 \cos x + \cos^2 x}{\sin^2 x} &= \frac{1 - \cos x}{1 + \cos x} \\ \frac{\cos^2 x - 2 \cos x + 1}{1 - \cos^2 x} &= \frac{1 - \cos x}{1 + \cos x} \\ \frac{(1 - \cos x)^2}{(1 - \cos x)(1 + \cos x)} &= \frac{1 - \cos x}{1 + \cos x} \\ \frac{1 - \cos x}{1 + \cos x} &= \frac{1 - \cos x}{1 + \cos x} \end{aligned}$$

c)

$$\begin{aligned} \frac{1 + \tan^2 x}{\csc^2 x} &= \tan^2 x \\ \frac{\sec^2 x}{\csc^2 x} &= \tan^2 x \\ \frac{1}{\frac{\cos^2 x}{\sin^2 x}} &= \tan^2 x \\ \frac{1}{\cos^2 x} \times \sin^2 x &= \tan^2 x \\ \frac{\sin^2 x}{\cos^2 x} &= \tan^2 x \\ \tan^2 x &= \tan^2 x \end{aligned}$$

e)

$$\begin{aligned} \frac{\sin^2 x}{1 - \cos x} &= 1 + \cos x \\ \frac{1 - \cos^2 x}{1 - \cos x} &= 1 + \cos x \\ \frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x} &= 1 + \cos x \\ 1 + \cos x &= 1 + \cos x \end{aligned}$$

g)

$$\begin{aligned} \tan^2 x - \sin^2 x &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x}{\cos^2 x} - \sin^2 x &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x} &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} &= \sin^2 x \tan^2 x \\ \frac{\sin^2 x \sin^2 x}{\cos^2 x} &= \sin^2 x \tan^2 x \\ \sin^2 x \frac{\sin^2 x}{\cos^2 x} &= \sin^2 x \tan^2 x \\ \sin^2 x \tan^2 x &= \sin^2 x \tan^2 x \end{aligned}$$

b)

$$\begin{aligned} \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} &= 1 \\ \frac{1}{\cos x} - \frac{\sin x}{\cos x} &= 1 \\ \frac{1}{\cos x} \times \frac{1}{\cos x} - \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} &= 1 \\ \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} &= 1 \\ \sec^2 x - \tan^2 x &= 1 \\ 1 &= 1 \end{aligned}$$

d)  $\frac{(2 \sin x \cos x - 1)(2 \sin x \cos x + 1)}{(1 + \sin x)(1 - \sin x)} = 4 \sin^2 x - \sec^2 x$

$$\frac{4 \sin^2 x \cos^2 x - 1}{1 - \sin^2 x} = 4 \sin^2 x - \sec^2 x$$

$$\frac{4 \sin^2 x \cos^2 x - 1}{\cos^2 x} = 4 \sin^2 x - \sec^2 x$$

$$\frac{4 \sin^2 x \cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = 4 \sin^2 x - \sec^2 x$$

$$4 \sin^2 x - \sec^2 x = 4 \sin^2 x - \sec^2 x$$

f)  $\sec^2 x (1 - \sin^2 x \cos^2 x - \cos^4 x) = \tan^2 x$   
 $\sec^2 x - \sec^2 x \sin^2 x \cos^2 x - \sec^2 x \cos^4 x = \tan^2 x$   
 $\frac{1}{\cos^2 x} - \frac{1}{\cos^2 x} \sin^2 x \cos^2 x - \frac{1}{\cos^2 x} \cos^4 x = \tan^2 x$   
 $\frac{1}{\cos^2 x} - \sin^2 x - \cos^2 x = \tan^2 x$   
 $\sec^2 x - (\sin^2 x + \cos^2 x) = \tan^2 x$   
 $\sec^2 x - 1 = \tan^2 x$   
 $\tan^2 x = \tan^2 x$

h)

$$\begin{aligned} \sec^2 x + \csc^2 x &= \frac{1}{\cos^2 x \sin^2 x} \\ \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} &= \frac{1}{\cos^2 x \sin^2 x} \\ \frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x} &= \frac{1}{\cos^2 x \sin^2 x} \\ \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} &= \frac{1}{\cos^2 x \sin^2 x} \\ \frac{1}{\cos^2 x \sin^2 x} &= \frac{1}{\cos^2 x \sin^2 x} \end{aligned}$$

15. a)  $\sin 3x = \sin(x + 2x)$   
 $= \sin x \cos 2x + \cos x \sin 2x$   
 $= \sin x \cos(x + x) + \cos x \sin(x + x)$   
 $= \sin x(\cos x \cos x - \sin x \sin x) + \cos x(\sin x \cos x + \sin x \cos x)$   
 $= \sin x(\cos^2 x - \sin^2 x) + 2 \cos x \sin x \cos x$   
 $= \sin x(1 - \sin^2 x - \sin^2 x) + 2 \sin x \cos^2 x$   
 $= \sin x(1 - 2 \sin^2 x) + 2 \sin x(1 - \sin^2 x)$   
 $= \sin x - 2 \sin^3 x + 2 \sin x - 2 \sin^3 x$   
 $= 3 \sin x - 4 \sin^3 x$

b)  $\sin 4x = \sin(2x + 2x)$   
 $= \sin 2x \cos 2x + \sin 2x \cos 2x$   
 $= 2 \sin 2x \cos 2x$   
 $= 2 \sin(x + x) \cos(x + x)$   
 $= 2(\sin x \cos x + \sin x \cos x)(\cos x \cos x - \sin x \sin x)$   
 $= 2(2 \sin x \cos x)(\cos^2 x - \sin^2 x)$   
 $= 4 \sin x \cos x (\cos^2 x - \sin^2 x)$   
 $= 4 \sin x \cos^3 x - 4 \sin^3 x \cos x$

c)  $\sin 6x = \sin(3x + 3x)$   
 $= \sin 3x \cos 3x + \sin 3x \cos 3x$   
 $= 2 \sin 3x \cos 3x$

16. a)  $\frac{\sqrt{2} - \sqrt{2}}{2}$       b)  $\frac{\sqrt{2} + \sqrt{2} + \sqrt{3}}{2}$       c)  $-1 - \sqrt{2}$       d)  $\sqrt{2} - 1$   
e)  $\frac{\sqrt{2} - \sqrt{2} + \sqrt{2}}{2}$       f)  $2 - \sqrt{3}$       g)  $\frac{\sqrt{2} - \sqrt{2} + \sqrt{2}}{2}$       h)  $\frac{-\sqrt{2} - \sqrt{2}}{2}$   
i)  $\frac{-\sqrt{2} - \sqrt{2}}{2}$       j)  $\frac{-\sqrt{2} + \sqrt{3}}{2}$       k)  $\frac{\sqrt{2} - \sqrt{2} - \sqrt{3}}{2 + \sqrt{2} - \sqrt{3}}$       l)  $\frac{-\sqrt{2} + \sqrt{3}}{2}$

17. a)  $\cos(1000\pi - x) = \cos x$   
 $\cos 1000\pi \cos x + \sin 1000\pi \sin x = \cos x$   
 $1 \cos x + 0 \sin x = \cos x$   
 $\cos x = \cos x$

c)  $\sin\left(\frac{\pi}{2} + x\right) = \cos x$   
 $\sin\frac{\pi}{2} \cos x + \sin x \cos\frac{\pi}{2} = \cos x$   
 $1 \cos x + \sin x \times 0 = \cos x$   
 $\cos x = \cos x$

e)  $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$   
 $\cos\frac{3\pi}{2} \cos x - \sin\frac{3\pi}{2} \sin x = \sin x$   
 $0 \cos x - 1 \sin x = \sin x$   
 $\sin x = \sin x$

b)  $\tan(x + 51\pi) = \tan x$   
 $\frac{\tan x + \tan 51\pi}{1 - \tan x \tan 51\pi} = \tan x$   
 $\frac{\tan x + 0}{1 - \tan x \times 0} = \tan x$   
 $\tan x = \tan x$

d)  $\tan(211\pi - x) = -\tan x$   
 $\frac{\tan 211\pi - \tan x}{1 + \tan 211\pi \tan x} = -\tan x$   
 $\frac{0 - \tan x}{1 + 0} = -\tan x$   
 $-\tan x = -\tan x$

f)  $\sin(x - 101\pi) = -\sin x$   
 $\sin x \cos 101\pi - \sin 101\pi \cos x = -\sin x$   
 $-1 \sin x - 0 \cos x = -\sin x$   
 $-\sin x = -\sin x$

18. a)  $\frac{\cot x}{\cosec x - \sin x} = \sec x$

$$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \sin x} = \sec x$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}} = \sec x$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{1 - \sin^2 x}{\sin x}} = \sec x$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}} = \sec x$$

$$\frac{\cos x}{\sin x} \times \frac{\sin x}{\cos^2 x} = \sec x$$

$$\frac{1}{\cos x} = \sec x$$

$$\sec x = \sec x$$

c)  $\frac{\sin x \sec x}{\cosec x \sqrt{1 - \sin^2 x}} = \tan^2 x$

$$\frac{\sin x \sec x}{\cosec x \sqrt{\cos^2 x}} = \tan^2 x$$

$$\frac{\sin x \sec x}{\cosec x \cos x} = \tan^2 x$$

$$\frac{\sin x \frac{1}{\cos x}}{\frac{1}{\sin x} \cos x} = \tan^2 x$$

$$\frac{\sin x}{\frac{\cos x}{\cos x}} = \tan^2 x$$

$$\frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

$$\tan^2 x = \tan^2 x$$

e)  $\frac{\cos^2 x \tan x}{\cot x} = \sin^2 x$

$$\frac{\cos^2 x \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \sin^2 x$$

$$\cos^2 x \frac{\sin x}{\cos x} \times \frac{\sin x}{\cos x} = \sin^2 x$$

$$\sin^2 x = \sin^2 x$$

g)  $\frac{\sin x}{\sec x} \cosec x = \cos x$

$$\frac{\sin x}{\sec x} \frac{1}{\sin x} = \cos x$$

$$\frac{1}{\sec x} = \cos x$$

$$\frac{1}{\frac{1}{\cos x}} = \cos x$$

$$\cos x = \cos x$$

b)  $\frac{\cos^2 x}{1 - \cos^2 x} + \sin^2 x + \cos^2 x = \cosec^2 x$

$$\frac{\cos^2 x}{\sin^2 x} + \sin^2 x + \cos^2 x = \cosec^2 x$$

$$\cot^2 x + \sin^2 x + \cos^2 x = \cosec^2 x$$

$$\cot^2 x + 1 = \cosec^2 x$$

$$\cosec^2 x = \cosec^2 x$$

d)  $\cos^2 x \tan^2 x + \cos^2 x = 1$

$$\cos^2 x \frac{\sin^2 x}{\cos^2 x} + \cos^2 x = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

f)  $\sin^2 x \cot^2 x + \sin^2 x = 1$

$$\sin^2 x \frac{\cos^2 x}{\sin^2 x} + \sin^2 x = 1$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 = 1$$

h)  $(1 - \cos^2 x) \cot^2 x = \cos^2 x$

$$\sin^2 x \cot^2 x = \cos^2 x$$

$$\sin^2 x \frac{\cos^2 x}{\sin^2 x} = \cos^2 x$$

$$\cos^2 x = \cos^2 x$$

19. You have:  $\sin a = \frac{\sqrt{3}}{2}$  and  $\sin b = \frac{3}{5}$ .

$$\text{a) } \tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\tan(a - b) = \frac{-\sqrt{3} - \frac{3}{4}}{1 + -\sqrt{3} \times \frac{3}{4}}$$

$$\tan(a - b) = \frac{48 + 25\sqrt{3}}{11}$$

$$\text{c) } \cos(b - a) = \cos b \cos a + \sin b \sin a$$

$$\cos(b - a) = 0.8 \times \frac{1}{2} + 0.6 \times \frac{\sqrt{3}}{2}$$

$$\cos(b - a) = -\frac{2}{5} + \frac{3\sqrt{3}}{10}$$

$$\cos(b - a) = \frac{-4 + 3\sqrt{3}}{10}$$

$$\text{b) } \sin(a + b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a + b) = \frac{\sqrt{3}}{2} \times 0.8 + 0.6 \times -\frac{1}{2}$$

$$\sin(a + b) = \frac{2\sqrt{3}}{5} + -\frac{3}{10}$$

$$\sin(a + b) = \frac{-3 + 4\sqrt{3}}{10}$$

$$\text{d) } \tan 2a = \tan(a + a)$$

$$\tan 2a = \frac{\tan a + \tan a}{1 - \tan a \tan a}$$

$$\tan 2a = \frac{-\sqrt{3} + -\sqrt{3}}{1 - (-\sqrt{3})(-\sqrt{3})}$$

$$\tan 2a = \frac{-2\sqrt{3}}{-2}$$

$$\tan 2a = \sqrt{3}$$

20. The altitude (in km) of the rocket corresponds to the tangent of the angle of elevation. To show that the statement "When the measure of the angle of elevation O doubles, the tangent of this angle doubles" is false, it suffices to find a counter-example.

Let the angle of elevation be  $\frac{\pi}{6}$  rad. The tangent of  $\frac{\pi}{6}$  is  $\frac{\sqrt{3}}{3}$ . The double of  $\frac{\pi}{6}$  rad is  $\frac{\pi}{3}$  rad. The tangent of  $\frac{\pi}{3}$  is  $\sqrt{3}$ .

Since  $\sqrt{3}$  is not double  $\frac{\sqrt{3}}{3}$ , you can confirm that the statement is false.