

3. Find the zeros of the following functions.

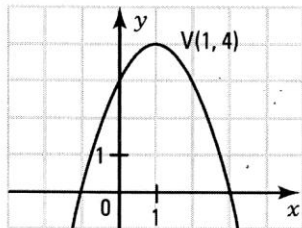
- a) $f(x) = -4(x + 2)^2 + 16$ -4 and 0 b) $f(x) = \frac{1}{2}(x + 3)^2 - 2$ -5 and -1
 c) $f(x) = 2(x + 1)^2 - 10$ $-1 - \sqrt{5}$ and $-1 + \sqrt{5}$ d) $f(x) = (x - 1)^2 - 7$ $1 - \sqrt{7}$ and $1 + \sqrt{7}$
 e) $f(x) = -2(x + 3)^2$ -3 f) $f(x) = 3(x - 2)^2 - 27$ -1 and 5
 g) $f(x) = 3(x - 1)^2 + 6$ none h) $f(x) = -(x + 1)^2$ -1

4. Consider the quadratic function $f(x) = a(x - h)^2 + k$.

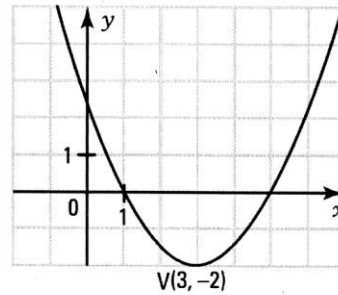
- a) If $a > 0$, indicate the number of zeros when
 1. $k > 0$. none 2. $k = 0$. only one 3. $k < 0$. two zeros
 b) If $a < 0$, indicate the number of zeros when
 1. $k > 0$. two zeros 2. $k = 0$. only one 3. $k < 0$. none

5. Graph the following parabolas.

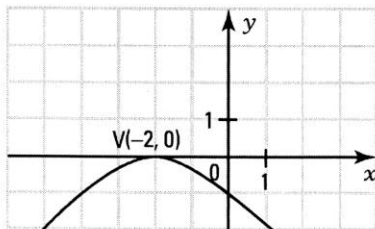
a) $y = -(x - 1)^2 + 4$



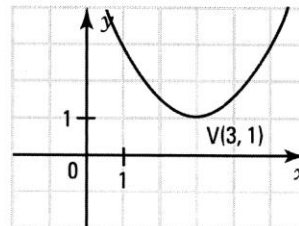
b) $y = \frac{1}{2}(x - 3)^2 - 2$



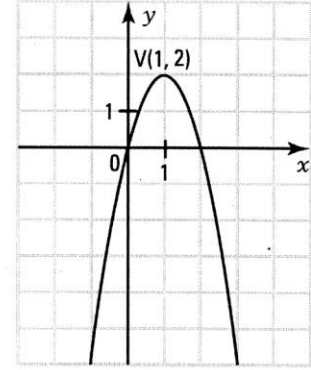
c) $y = -\frac{1}{4}(x + 2)^2$



d) $y = \frac{1}{2}(x - 3)^2 + 1$

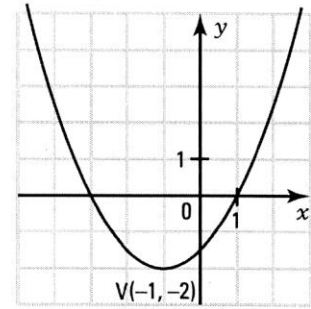


6.) Consider the function $f(x) = -2(x - 1)^2 + 2$ represented on the right.



- a) What is the domain of f ? $\text{dom } f = \mathbb{R}$
- b) What is the range of f ? $\text{ran } f =]-\infty, 2]$
- c) What are the zeros of f ? 0 and 2
- d) What is the y-intercept of f ? 0
- e) What is the sign of f ? $f(x) \leq 0$ if $x \in]-\infty, 0] \cup [2, +\infty[$
 $f(x) \geq 0$ if $x \in [0, 2]$
- f) Complete the study of the variation of f .
 1. f is increasing over $] -\infty, 1]$. 2. f is decreasing over $[1, +\infty[$.
- g) 1. Does function f reach a maximum? If yes, what is it? Yes; $\max f = 2$
 2. Does function f reach a minimum? no

7.) Consider the function $f(x) = \frac{1}{2}(x + 1)^2 - 2$ represented on the right.



- Find
- a) 1. $\text{dom } f$. \mathbb{R} 2. $\text{ran } f$. $[-2, +\infty[$
- b) 1. the zeros of f . -3 and 1 2. the y-intercept of f . -1.5
- c) the sign of f . $f(x) \geq 0$ over $]-\infty, -3] \cup [1, +\infty[$; $f(x) \leq 0$ over $[-3, 1]$
- d) the variation of f . f is increasing over $[-1, +\infty[$.
 f is decreasing over $]-\infty, -1]$.
- e) the minimum of f . $\min f = -2$

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8. Find the domain and range of the following functions.

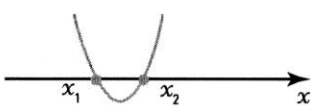
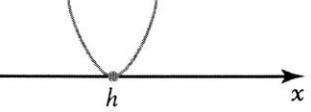
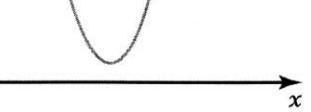
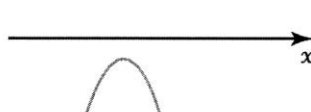
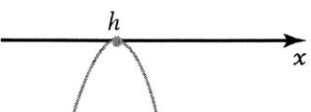
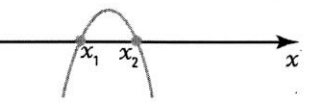
a) $f(x) = -2(x + 1)^2 + 5$
 $\text{dom } f = \mathbb{R}; \text{ran } f =]-\infty, 5]$

b) $f(x) = \frac{3}{2}(x - 1)^2 - 2$
 $\text{dom } f = \mathbb{R}; \text{ran } f = [-2, +\infty[$

9. Determine the zeros of the function $y = -2(x - 3)^2 + 18$. 0 and 6

10. What is the y-intercept of the function $y = -2(x - 3)^2 + 7$? -11

- 11.** The sign of the quadratic function $f(x) = a(x - h)^2 + k$ depends on the signs of a and k . Indicate, in each of the 6 following cases, the intervals where $f(x) > 0$ and $f(x) < 0$.

	$k < 0$	$k = 0$	$k > 0$
$a > 0$	 $f(x) > 0$ if $x \in]-\infty, x_1[\cup]x_2, +\infty[$ $f(x) < 0$ if $x \in]x_1, x_2[$	 $f(x) > 0$ if $x \in \mathbb{R} \setminus \{h\}$	 $f(x) > 0, \forall x \in \mathbb{R}$
$a < 0$	 $f(x) < 0, \forall x \in \mathbb{R}$	 $f(x) < 0$ if $x \in \mathbb{R} \setminus \{h\}$	 $f(x) > 0$ if $x \in]x_1, x_2[$ $f(x) < 0$ if $x \in]-\infty, x_1[\cup]x_2, +\infty[$

- 12.** Determine, in each case, the values of x for which

1. $f(x) > 0$. 2. $f(x) \geq 0$. 3. $f(x) < 0$. 4. $f(x) \leq 0$.

a) $f(x) = 2(x - 1)^2 - 2$

1. $f(x) > 0$ if $x \in]-\infty, 0[\cup]2, +\infty[$

2. $f(x) \geq 0$ if $x \in]-\infty, 0] \cup [2, +\infty[$

3. $f(x) < 0$ if $x \in]0, 2[$

4. $f(x) \leq 0$ if $x \in [0, 2]$

b) $f(x) = -4(x - 3)^2 + 16$

1. $f(x) > 0$ if $x \in]1, 5[$

2. $f(x) \geq 0$ if $x \in [1, 5]$

3. $f(x) < 0$ if $x \in]-\infty, 1[\cup]5, +\infty[$

4. $f(x) \leq 0$ if $x \in]-\infty, 1] \cup [5, +\infty[$

- 13.** Determine the values of x for which $y = 3(x - 1)^2 - 27$ is positive.
 $x \in]-\infty, -2] \cup [4, +\infty[$

- 14.** Study the variation of the following functions.

a) $f(x) = 3(x - 1)^2 - 2$

f is decreasing over $]-\infty, 1]$.

f is increasing over $[1, +\infty[$.

b) $f(x) = -2(x + 1)^2 + 1$

f is increasing over $]-\infty, -1]$.

f is decreasing over $[-1, +\infty[$.

- 15.** Determine the interval over which the function $f(x) = 2(x + 4)^2 + 2$ is increasing. $[-4, +\infty[$

- 16.** Determine the values of x for which the function $y = -3(x + 1)^2 + 12$ is increasing.
 $x \in]-\infty, -1]$

17. In each of the following cases, indicate whether the function reaches a maximum or a minimum and determine it.

a) $f(x) = -2(x - 3)^2 - 1$

A maximum: $\max f = -1$

b) $f(x) = \frac{3}{4}(x + 1)^2 - 2$

A minimum: $\min f = -2$

18. Find the extremum and its nature (maximum or minimum) of $f(x) = 3(x - 1)^2 - 4$.

A minimum; -4

19. What is the axis of symmetry of the parabola with equation $y = (x - 1)^2$?

The line $x = 1$

20. Find the values of x for which the function $f(x) = -2(x - 1)^2 - 4$ is equal to -36 .

-3 and 5

21. a) $f(x) = -\frac{2}{3}(x+1)^2 + 4$

b) $f(x) = 2x^2$

c) $f(x) = 4(x-2)^2$

d) $f(x) = \frac{1}{2}x^2 - 1$

22. A parabola with vertex $V(3, 16)$ has a y -intercept equal to 7. What is the y -coordinate of the point A on the parabola whose x -coordinate is 5?

$y = -(x - 3)^2 + 16$; $A(5, 12)$. The y -coordinate of point A is 12.

23. A parabola with vertex $V(3, 8)$ passes through the point $A(6, -10)$. What are the points on this parabola whose y -coordinates are equal to 6?

$y = -2(x - 3)^2 + 8$; $P_1(2, 6)$ and $P_2(4, 6)$

24. What are the zeros of the parabola whose vertex is $V(-1, 12)$ and passes through the point $A(2, -15)$?

$y = -3(x + 1)^2 + 12$. The zeros are -3 and 1.

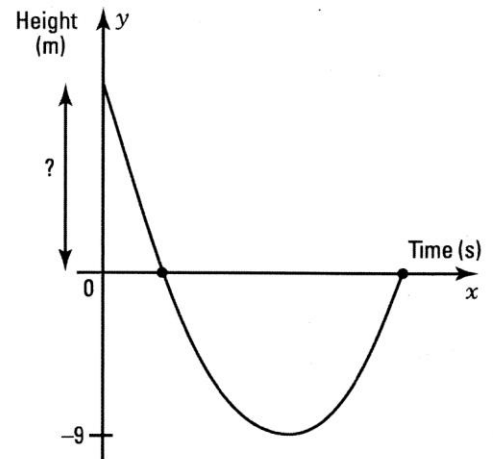
25. A parabola with vertex $V(6, 10)$ passes through the point $P(10, 6)$. What is the initial value of this function?

$f(x) = -\frac{1}{4}(x - 6)^2 + 10$; $f(0) = 1$. The initial value is equal to 1.

- 26.** During a competition, a diver enters the water 2 seconds after jumping from the diving board and reaches a maximum depth of 9 m. The portion of the parabola on the right represents the diver's trajectory. If the diver remains underwater for 6 seconds, determine the height of the diving board.

$$f(x) = (x - 5)^2 - 9; f(0) = 16$$

The diving board is at a height of 16 m.



- 27.** At its purchase, a share is worth \$6. We observe that the function f , which gives the value y of the share as a function of the time x in months since its purchase, is a quadratic function. The share reaches a maximum value of \$8 six months after its purchase. What is the value of this share 9 months after its purchase?

$$f(x) = \frac{-1}{18}(x - 6)^2 + 8; f(9) = 7.5. \text{ The share is worth } \$7.50.$$

- 28.** We have represented on the right the trajectory of two fireworks launched at the same time. The rule $h = -2(t - 4)^2 + 100$ gives the height h , in metres, as a function of the elapsed time, in seconds, since they were launched. Knowing that firework A explodes at a height of 92 m and that firework B explodes 1 second later, determine at what height firework B explodes at.

Firework A explodes 6 seconds after its launch.

Firework B explodes 7 seconds after its launch at a height of 82 m.

